# A DISAGGREGATE PSEUDO-DYNAMIC ASSIGNMENT FOR THE ACTIVITY-BASED MODEL OF THE GREATER COPENHAGEN AREA

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## ABSTRACT

The COMPAS (Copenhagen Model for Person Activity Scheduling) model is an activity-based model that proposes on the demand side a micro-simulation approach to the representation of activity and travel patterns of Copenhageners as individuals and household members, and on the supply side a disaggregate pseudo-dynamic approach to the assignment of Copenhageners to the multimodal network of the Greater Copenhagen Area. This paper focuses on the development of the framework for the supply side that aims at (i) capturing time-dependent interactions of travel demand and network supply of the network, (ii) representing the network at a disaggregate level, (iii) representing congestion build-up and dissipation; (iv) evaluating the effect of traffic management measures and traffic policies.

*Keywords: activity-based models, traffic assignment, pseudo-dynamic approach, individual-based approach, restricted user equilibrium*

## INTRODUCTION

For nearly 30 years, the planning process in transportation modelling has been dominated by the traditional trip-based approach with the well-known four stages of trip generation, trip distribution, modal split, and trip assignment. While the demand side concerns the first three steps, the supply side concerns the fourth step, and a full feedback execution of the trip-based approach accommodates demand and supply within a joint framework. The COMPAS (Copenhagen Model for Person Activity Scheduling) model being developed for the Greater Copenhagen Area recognizes the limitations to the trip-based approach (see, e.g., Axhausen and Garling, 1992; Jones et al., 1993; Bhat and Koppelman, 2003; Bhat et al., 2004; Vovsha and Bradley, 2005) that have led to an active stream of research focusing on alternative paradigms for predicting travel demand and supply by incorporating more behaviourally realistic methodologies.

On the demand side, the COMPAS model uses an activity-based modelling paradigm that recognizes that travel is a derived demand and that the need to travel stems from the more fundamental need to participate in activities. The activity-based paradigm is conceptually more appealing than the trip-based method to model travel demand for several reasons: (i) the focus is on sequences and patterns of activities and travel rather than on individual trips; (ii) the various activity-travel decisions are recognized as linked rather than as independent; (iii) the emphasis is on individual-level travel patterns rather than on aggregate trips; (iv) intra-household interactions are incorporated and inter-personal and intra-personal consistency measures are considered within the model; (v) time is treated as a continuum or at least a detailed temporal dimension is accounted for within the model; (vi) space-time constraints on activities and travel are included within the model. A number of micro-simulation platforms that employ the activity-based paradigm of transportation demand forecasting have been developed recently in several metropolitan areas, such as Portland (see, e.g., Bowman et al., 1998), San Francisco (see, e.g., Bradley et al., 2001; Jonnalagadda et al., 2001), New York (see, e.g., Vovsha et al., 2002), Columbus (see, e.g., Vovsha et al., 2003; Vovsha et al., 2004), Dallas (see, e.g., Bhat et al., 2004; Pinjari et al., 2006).

On the supply side, the COMPAS model uses a pseudo-dynamic approach that overcomes the limitations of static assignment procedures and exploits the advancements in computing capacity that have allowed the field to move toward more behaviourally realistic traffic assignment models. A dynamic traffic assignment approach is more appealing than the traditional static traffic assignment for several reasons: (i) time-dependent interactions of the travel demand and supply of the network can be captured; (ii) the network representation can be undertaken at a disaggregate level; (iii) congestion build-up and dissipation can be represented; (iv) the effect of traffic management measures (e.g., traffic control, intelligent transport systems) can be evaluated. A number of simulation-based DTA modules have been developed in the recent past such as DYNASMART-P (Mahmassani et al., 1992), VISTA (Waller and Ziliaskopoulos, 1998) and DynaMIT (Ben-Akiva et al., 1998).

The rationale behind the traffic assignment framework within the COMPAS model originates from the consideration that significant advancements have occurred on both the demand side and the supply side, but the progress in the two streams appears to have been pursued and achieved quite independently (see Lin et al., 2008). Inconsistent results would most likely stem from using only one of these frameworks for travel demand and supply modelling without exploiting the true potential of either approach. On the one hand, using an activity-based paradigm with a static process not considering temporal dynamics would negate much of the advantages of predicting travel patterns in continuous time. On the other hand, using a trip-based approach that provides travel demand over a limited number of time periods to develop the inputs for dynamic traffic assignment would cancel out the reasons for which dynamic traffic assignment models have been developed.

In order to obtain consistent results and to realize the benefits of both advancements in demand and supply modelling, a conceptual framework for the traffic assignment has been designed to combine their advantages.

The traffic assignment framework within COMPAS is not based on origin-destination matrices, but instead on the individual activity and travel patterns that are loaded onto the network with the person as the unit. This methodological framework does not require the definition of zones and connectors, since trips start on one of the network nodes and end in another of the network nodes corresponding to parcels (i.e., addresses of households, shops, firms, schools, restaurants, cinemas, etc.).

From a behavioural perspective, this approach allows considering individual preference structures depending on individual attributes such as value-of-time, income, age, trip length, trip purpose, time-of-day, etc. Non-linear terms in the utility functions may be considered because there is not any aggregation process that will affect the assignment results. In addition, choice set generation does not need to consider taste heterogeneity, and route choice does not need to account for distributions of parameters across the population in a certain zone, since neither the zones nor the population are to be considered in the traffic assignment. This implies lesser need for random coefficients to be drawn, and higher precision in the preferences of individuals to be represented, with a large improvement in realism. However, this implies higher complexity in calculating the level-of-service for non-chosen alternatives (e.g., non-chosen modes and non-chosen destinations).

From a computational perspective, the individual-based methodological framework has a complexity similar to the static assignment. When adding the time dimension to the matrix-based assignment, the calculation complexity increases significantly and either a cell-based or a row-based approach is needed. This complexity is even more evident when considering large-scale models where the large number of cells in the origin-destination matrices implies that extremely low number of trips enter each cell. Individual-based approaches have almost the same complexity as static assignment, since only some more updating of the speed-flow and flow-density functions are required. The advantages are that there is not loss of information on the trips from the demand model, there is an increase in the explanation and prediction abilities, and there is an avoidance of aggregation bias of the level-of-service variables in the feed-back to the demand models.

The current paper discusses the issues related to the development of the dynamic assignment model and its remainder presents the different perspectives these issues touch upon. Section 2 presents issues related to the interaction between demand and assignment. Section 3 introduces specific issues related to the dynamic nature of the traffic assignment. Section 4 summarizes the main points for the theoretical derivation and practical implementation of the assignment model within the COMPAS model.

# INTERACTION BETWEEN DEMAND AND ASSIGNMENT

Three main issues concern the interaction between demand and assignment in the COMPAS model: (i) consistency between road and public transport assignment, (ii) integration (and more specifically iteration) between demand and supply, and (iii) calculation of level-of-service for alternative modes and destinations in time and space.

## Consistency between road and public transport assignment

The first issue concerns the necessity of consistency between road and public transport assignment. With respect to public transport assignment, Nielsen and Frederiksen (2006) describe a model able to (i) perform timetable-based assignment, (ii) include feeder modes, (iii) account for heterogeneity in passengers' preferences within and between purposes and classes, and (iv) consider capacity problems at the vehicle level, system level and terminal level. The model is developed and estimated for the Copenhagen-Ringsted Model (CRM), and the Stochastic User Equilibrium (SUE) problem is solved by the Method of Successive Averages (MSA), with initially very long calculation times. However, Nielsen and Frederiksen (2006) present optimisation techniques for the transit assignment models on a large-scale network on the basis of MSA combined with a generalised utility function. Considering that the distributions of the passenger preferences is not necessary in the activity-based framework where each individual is loaded onto the network, and that the purpose of the trip is embedded in the provided activity pattern, the public transport assignment is faster in its implementation in the COMPAS model.

While an individual-based assignment for public transport is not particularly difficult to achieve with a modification of the existing public transport assignment (already implemented in the Danish national model), the key issue concerns the road transport assignment. In fact, loading the individuals onto the public transport network is possible through the "entrance" of the individual in the network at one of the stations close to the origin, the "exit" of the individual from the network at one of the stations close to the destination (see Larsen et al., 2010), and the consideration of access and egress times and modes (see Halldorsdottir, 2010). Consistently, loading the individuals onto the road network is also possible through the "entrance" of the individual in the network at the node closest to the origin and the "exit" of the individual from the network at the node closest to the destination, since the demand model gives the parcel-level and not the zone-level origin-destination. The complexity concerns the two following issues because the integration with the demand framework and the calculation of the level-of-service measures for the non-chosen alternatives appear extremely more complicated with respect to the case of the matrix-based traffic assignment model.

From a general perspective, the dynamic traffic assignment intends to capture the decision by the individual for both the road and the public transport assignment and hence move the private or public vehicle through the network until the individual reaches either an intermediate stop or the final destination. Similar to the concept proposed in Dynasmart (see Abdelghany et al., 2001), vehicles are represented individually but moved consistently with macroscopic traffic flow relations between average speed and concentration on a roadway link. Intersections are not modelled but, in the case intersection modelling is required, vehicles could be moved to the downstream link consistently with the first-in-first-out principle and with the type of intersection control. At intermediate destinations, the vehicle "exits" the network for a pre-specified period equal to the activity duration at this destination. This duration is the result of the activity process that is modelled exogenously to the assignment model. Upon completion of the activity, the individual resumes the trip from this destination to complete the trip according to its exogenously determined activity and travel pattern. When the vehicle is out of the network at any intermediate destination, it has no effect on the traffic in the network. Once the vehicle reaches its final destination, it exits the network and is no longer tracked.

The traffic assignment model produces output statistics at both the aggregate and the disaggregate levels (see Abdelghany et al., 2001). For example, it produces various status descriptors for each link in the network, such as travel times, stop times, speeds, densities, and queues. It also produces space and time trajectories of each vehicle, including travel time on each link, stop time at each node, and activity duration at the intermediate destinations, if any. The model also produces statistics aggregated over all travellers, including average travel time, stop time, and so on.

## Integration between supply and demand

The integration between supply and demand is not straightforward when considering a dynamic traffic assignment model and an activity-based demand framework. The iteration between supply and demand is in fact extremely challenging and has attracted growing interest in recent years. A brief review of some significant efforts is followed by the proposal for the integration between supply and demand in the COMPAS model.

### *Integrating activity-based and dynamic assignment in the past*

An initial attempt to interface household activities, land use distributions, regional demographics and time-dependent transportation networks were described by McNally (1997). The micro-simulation model produced activity patterns in the form of probability distributions of activity dimensions such as purpose, start and duration. Start and duration of the activity were drawn from the distributions associated with the target activity pattern, and then trips were sequentially simulated on the basis of a Monte Carlo approach of potential activity-specific destinations within a range of travel times from the origin location. The network simulation involved the development of dynamic equilibrium (or approximate equilibrium) paths that served for simulated drivers as the simulation model did have stochastic behavioural rules that simulate their actual travel paths, and this was the reason why the dynamic assignment did not need to provide an exact solution. McNally (1997) proposed two approaches (quasi-dynamic and dynamic assignment models) that send the results of the (quasi-)dynamic assignment as feedback to the demand model where additional activity patterns are synthesized according to variations in travel times and activity durations. The feedback of adjusted travel times (and new paths) potentially affects the scheduling of portions of any pattern, and hence impacted patterns are re-synthesized from the point where the first temporal disturbance occurred. McNally (1997) observed that an investigation of this feedback and the implications on process stability and convergence represent a major research task.

Lam and Yin (2001) presented a variational inequality (VI) model for the dynamic user equilibrium activity/route choices and proposed a heuristic solution algorithm based on the nested diagonalisation method proposed by Chen and Hsueh (1998) in a space-time expanded network (STEN) where the queuing phenomenon is not considered explicitly. Considering that travel demand is derived demand because people make trips to satisfy their activity desires or needs, Lam and Yin (2001) defined the utility of a certain activity at a certain time as a function of the satisfaction and the intensity of performing the activity itself. Both the satisfaction and the intensity depend on time, and hence each activity has a time-of-day dependent utility (Axhausen and Garling, 1992), with gains or losses expressed in some unspecified units that could measure the degree of satisfaction and the intensity of the activity on the basis of the activity attributes. Considering the route choice behaviour of travellers, Lam and Yin (2001) adopted the ideal dynamic user equilibrium (ideal DUE) defined by Ran and Boyce (1996) as the condition under which travellers who have the same destination and start their journeys at the same time period will reach the destination simultaneously.

Lam and Huang (2002, 2003) extended the previous work by presenting a combined activity/route choice model and proposed a flow-swapping method for obtaining the dynamic user equilibrium solution on the congested road network with queues. The activities of individuals were characterized by given temporal utility profiles and were analysed only relatively to commuters' behaviour during the morning peak period (i.e., at home activity, non-work activity on the way from home to workplace, work-purpose activity). Lam and Huang (2002, 2003) defined equilibrium as the condition for which each combined activity/travel pattern, in terms of chosen location/route/departure time, has identical generalized disutility (or utility) experienced. This equilibrium was expressed as a discrete-time, finite-dimensional variational inequality formulation and was then converted to an equivalent "zero-extreme value" minimization problem. Lam and Huang (2002, 2003) proposed a solution algorithm that iteratively adjusts non-work activity location, route choice and departure time choices to reach an extreme point of the minimization problem.

While pioneering, with the exception of the attempt by McNally (1996) the aforementioned research efforts did not use the feedback from DTA to update the input information in the demand model framework and hence the demand simulator.

Some efforts have been posed toward addressing integration of supply and demand by multi-agent simulation. Esser and Nagel (2001) presented a multi-agent micro-simulation module that accounts for interaction among activity generation, route assignment and network loading. Raney et al. (2003) illustrated an agent-based simulator that consists of activity generation, mode and route choice, traffic simulation and learning modules. Raney and Nagel (2004) refined the model by including route generation, micro-simulation and feedback module that corrected the process. Rieser et al. (2007) presented a model to couple activity-based demand generation with multi-agent traffic simulations. In general, the multi-agent transportation simulation in which each traveller is represented individually consists of (i) population generation, (ii) activity generation, (iii) modal and route choice, (iv) traffic simulation and (v) learning and feedback (even though the first two modules are not used in the example in the paper). The simulation generates the population from disaggregated demographic data in order to obtain individual households and individual household members with their characteristics, such as street address, car ownership, or household income (Beckman et al., 1996). The activity generation produces a set of activities and their locations for a day (Vaughn et al., 1997; Bowman, 1998). Modes are selected and routes are generated to connect activities at different locations, with a dynamic routing in order to react adequately to time-dependent congestion effects. Traffic micro-simulation executes the behavioural plans of all individuals simultaneously and produces the result of interactions between the plans (e.g., congestion). Feedback makes the modules consistent with each other since, for example, plans depend on congestion but congestion depends on the plans. A widely accepted method to resolve this is systematic relaxation (Kaufman et al., 1991; Nagel, 1995; Bottom, 2000) that consists in making preliminary plans, running the traffic micro-simulation, adapting the plans, running the traffic micro-simulation again, etc., until consistency between modules is reached. The method is somewhat similar to the Frank-Wolfe algorithm in static assignment, or in more general terms to a standard relaxation technique in numerical analysis.

Recent efforts have been further posed toward the solution of the interaction between supply and demand in activity-based models with dynamic traffic assignment.

Liu et al. (2006) utilised an integrated approach to modelling drivers' medium-term travel decisions (choice of route and departure time, based on prior travel experiences) and short-term traffic flow evolution. The approach treats all decisions at the microscopic level and utilises a consistent approach to supply and demand modelling. The approach accommodates time-dependent queues, lane changing problems, real-time information, vehicle emission and dispersion models, and noise models. A very interesting finding was that the approach might be used to test complex measures and obtain forecasts that are beyond conventional equilibrium approaches, such as predictions of policy impacts on the variability in travel times and flows.

Lin et al. (2008) introduced a fixed point formulation of the integrated activity-based and dynamic traffic assignment when a variational inequality formulation of the DUE traffic assignment is incorporated in the model to capture user behaviour. The integration of activity-based and dynamic traffic assignment poses methodological and technical challenges. The activity-based model requires the level-of-service values as input to generate activity travel patterns, but it is possible that these input values do not correspond to the actual travel times. Therefore, the generated activity patterns are translated into O-D matrices by time of day and loaded onto the network to produce the travel times. This clearly highlights the necessity of an iterative procedure between the activity-based and the dynamic traffic assignment model. The convergence criterion was defined as follows (Lin et al., 2008): after every iteration, O-D matrices and travel times of the current and the previous iteration could be compared, and hence potentially there exist (i) trip table convergence and (ii) travel time convergence. The convergence criterion is based on the attribute that is averaged after the iteration (with MSA techniques) and the attribute that needs to converge (across iterations). If the average difference is less than a predefined stopping criterion, the integration is stopped and the results are treated as the converged solution. Lin et al. (2008) pointed out that the MSA has limitations that might be overcome with advanced approaches such as methods based on gap functions.

Adnan et al. (2009) presented a combined home-work tour scheduling model that brings the system in stochastic dynamic user equilibrium (SDUE) and established both analytically and numerically that time-of-day based marginal utility functions cannot integrate morning and evening commute and hence do not serve the purpose of the combined modelling. Adnan et al. (2009) contradicted some earlier findings from the integration of home-work tour scheduling with network congestion (e.g., Zhang et al., 2005; Kim et al., 2006) and established that the duration-based marginal utility, which represents the activity satiation effect, is an important ingredient along with the time-of-day representation, for the integration of the morning and evening commutes. The model presented by Adnan et al. (2009) is general, since different discrete choice models could be incorporated at the demand side and

various dynamic network loading models could be assimilated that account for flow propagation, conservation, first-in-first-out, and causality.
Ramadurai and Ukkusuri (2010) proposed an integrated formulation to obtain equilibrium solutions across multiple dimensions of travel choice. The formulation is based on a Supernetwork representation referred to as Activity-Travel Network (ATN) representation where nodes are activity centres that are joined by travel links, activities are arcs that both originate and terminate in the same node, and activity-travel sequences for an individual are ”routes” that include both travel and activity arcs. Ramadurai and Ukkusuri (2010) modelled the “route” choice in the ATN to represent simultaneously activity location, start time, duration, and route choice decisions. Ramadurai and Ukkusuri (2010) presented a mathematical and operational framework for ATNs based on dynamic user equilibrium behaviour with an embedded cell-based transmission model, obtained the equivalent variational inequality problem, and demonstrated a solution method based on route-swapping algorithm. Even though theoretically sound and numerically demonstrated, the last three described approaches are still to be tested and verified on large-scale networks.

## *Integrating activity-based and pseudo-dynamic assignment in COMPAS*

The reviewed studies show the difficulty in the selection of the iteration method between supply and demand. The approach to the interaction in the COMPAS model is inspired by the work of Cantarella and Cascetta (1995), who discussed the theoretical results of the dynamic framework that processed the interaction between transportation demand and supply. Rich et al. (2010) showed that two different equilibrium mechanisms are involved when considering the interaction between supply and demand: (i) the internal traffic assignment equilibrium, and (ii) the external equilibrium loop between the assignment model and the demand model. Rich et al. (2010) analysed convergence performances of the external loop with the method of repeated approximations (MRA), the MSA (including variations such as weighted MSA, MSA with memory reset, and MSA with Polyak step-size) and polynomial smoothing. Experimental results with a simple synthetic network showed that MSA variations outperforms both MRA and MSA in situation of congestion.
The traffic assignment within the COMPAS model provides level-of-service variables for each chosen trip and each non-chosen trip as feedback to the demand model as: unique trip id, from node, to node, starting time, free flow time, congestion time, time variance, length, cost, toll, etc. It should be noted that initially the assignment model provides all the level-of-service variables with the exception of the variance that will be treated in a second stage.
Similarly, the demand model provides a list of trips and a utility function for each trip to the assignment model as: unique trip id, from node, to node, starting time, free flow value-of-time, congestion value-of-time, value-of-time variance, cost per km, etc. In addition, the demand model provides a list of the desired probes corresponding to the alternatives and utility functions for those.
It should be also noted that, from the software perspective, everything is integrated and hence implemented simultaneously. The convergence of the integration between supply and demand requires a procedure similar to the aforementioned outer-inner loop (Rich et al., 2010) in order to reach faster convergence in the model system.

*Level-of-service for alternative choices (in time and space) in the demand model*

In matrix-based assignment models, the alternatives are the cells other than the one corresponding to the chosen alternative. In the individual-based assignment model, the alternatives cannot be calculated as the average over trips, because these other trips are performed by different persons with different socioeconomic characteristics, budget and time constraints, and hence different utilities.

Other than the convergence issue, the integration between supply and demand presents a main challenge in the provision of the level-of-service variables for all the travel alternatives. The level-of-service variables for the chosen trip are straightforward to calculate, but the same variables for the non-chosen trips (e.g., different mode, different departure time) are not. A suitable solution is the generation of ghost travellers (Teklu et al., 2007) that are generated at a constant rate and loaded onto the network along the actual travellers while experiencing the costs without contributing to congestion. The ghost travellers allow obtaining the cost of travelling over alternative routes with alternative modes, thus providing the demand model with the necessary information regarding the non-chosen alternatives. The ghost travellers allow reproducing the individuals with their characteristics in order for the level-of-service of the alternatives to be accurately represented by the same utility function and the same parameters for the individual of interest.

Ghost travellers are treated in the same way as the other travellers, only that when on the road they are assumed not to increase the traffic volume, and when on the public transport vehicles they are assumed not to take up space (Teklu et al., 2007). Teklu et al. (2007) implemented this solution for public transport assignment and pointed out that, although it would be possible to use actual passenger experiences to build up day-by-day travel costs, such an approach would converge slowly because a route initially perceived as unattractive would not be used for a long period by travellers sequentially, and hence would not provide data on which to update its costs.

Ghost travellers could be adapted to represent not only alternative routes, but also alternative starting times (i.e., early or late departure) and alternative destinations. Ghost travellers could be loaded on the network after running the assignment model until convergence, calculating level-of-service variables, and creating a network state that is an equilibrium state on the basis of the trips that the demand generate. The level-of-service would then be measured by all-or-nothing assignment of each probe in the equilibrium state. Ghost travellers could be used also for forecasting purposes and alternatives need to be recalculated in the demand model with the consequent definition of new ghosts for the update of the level-of-service variables.

## GENERAL ISSUES IN DISAGGREGATED DYNAMIC ASSIGNMENT MODELLING

Several issues concern the disaggregate approach to the assignment model: (i) level of network aggregation, (ii) level of time aggregation, (iii) approach to congestion modelling, (iv) definition of equilibrium, and (v) solution approach.

## Level of network aggregation

The traffic assignment model is applied to a large-scale network, while the majority of the literature presented about the integration of supply and demand concerns small-scale synthetic networks or medium-scale networks. An issue emerges when considering that there are hundreds of similar alternative routes and their consideration within the traffic assignment is questionable.

Conceptually, recent research showed that network reduction could be beneficial to route choice model estimation and traffic assignment. Bierlaire and Frejinger (2008) proposed a framework that exploits network-free data with a network based model by defining the Domain of Data Relevance (DDR) as the physical area where each piece of data is relevant. The DDR depends on the associated concept and extends inversely proportional to the fuzziness of the concept (e.g., “downtown” is a larger area than “Rådhuspladsen”). Bierlaire and Frejinger (2008) elaborated information about trips that contains details about origin, destination and a maximum of three intermediate locations, and defined the DDR of each location as the area zip code. When linking the network-free data with the network through the DDRs, the precision level of the observations corresponds to the precision level of the network, consequently the transportation network is simplified, especially in urban areas. Moreover, the estimation of a sub-network component model (Frejinger and Bierlaire, 2007) illustrated more accurate estimates after less time consuming data manipulation. Schuessler et al. (2010) proposed a choice set generation method based on breadth-first search combined with link elimination, in which the large-scale network of Zurich was simplified through the aggregation of consecutive links in major arterials.

From a behavioural perspective, these simplification processes are plausible since psychologists and geographers illustrate that individuals recall routes according to origin, destination and significant locations they define as landmarks (see, e.g., Gale et al., 1990; Freundschuh, 1992; Garling and Golledge, 2000). Accordingly, the large-scale network for the traffic assignment model of the Copenhagen region may be left completely disaggregated, but may also be left disaggregated for only nearby origin and destination and aggregated between the two locations to express the choices among major roads. While being behaviourally plausible, this simplification greatly reduces the time to search for alternative paths and hence the computational time over the iterations of the traffic assignment model.

## Level of time aggregation

Within the class of economic models of road traffic congestion, two main streams can be distinguished, namely static and dynamic models.

In static models of road traffic congestion, the time dimension is not explicitly considered, but obviously the term “static” does not imply that time as such does not play any role in these models. As time certainly plays a role, often implicitly, the term “static” only implies that these models do not explicitly study changes over time. An issue concerns about the selection of the output variable in the definition of demand and cost functions: one the one hand there are “flow-based” measures, where the output measure has an explicit “per-unit-of-time” dimension (see, e.g., Else, 1981, 1982; Nash, 1982; De Meza and Gould, 1987; Evans An., 1992, 1993); on the other hand there are “stock-based” measures, where the output measures are numbers of trips or densities (see, e.g., Evans Al., 1992; Hills, 1993; Verhoef et al., 1995ab, 1996ab). Another issue concerns the question of whether the analysis applies to "peak demand" or "continuous demand", with the answer having important implications for the static modelling of congestion. Unfortunately, the question of which of these two types of demand is considered is usually not treated explicitly in such analyses (Verhoef, 1999).

In dynamic models of road traffic congestion, the focus is typically on peak congestion, and the models explicitly describe equilibrium patterns of variables such as speeds, densities, and arrival rates over time during the peak. Two types of such dynamic approaches can be distinguished. The first approach is the "bottleneck approach", originally developed by Vickrey (1969) and later on refined and extended in various directions by Arnott et al. (1993, 1997) and others. The second approach is the "flow congestion" approach, originally proposed by Henderson (1974, 1981), that does not completely eliminate travel delays from the social optimum (see Chu, 1995). In both approaches, the distribution of travel delays and scheduling costs over the peak and the duration of the peak in the unregulated equilibrium and the social optimum are determined endogenously, and both models have in common that the optimal toll is time-dependent, reaching its maximum for the drivers arriving at the desired arrival time.

When considering the underlying assumptions that have caused researchers to present different and often opposing views, some important elements emerge that would perhaps be associated with the dynamic approach, and therefore be ignored in the static approach. Apart from the distinction between peak demand and continuous demand, the question of whether a static equilibrium represents a dynamically stable configuration, namely a "stationary state", may be considered explicitly and be taken as a prerequisite for a static equilibrium to be consistent and meaningful. Furthermore, in the static model of peak demand, the scheduling cost structure necessary to render a static model applicable may be made explicit (see Verhoef, 1999).

An alternative approach that could mediate between static and dynamic approach is a pseudo-dynamic approach that divides the time according to the departure time slice, and obtains the flow on each link for each time slice by taking into account the travel time between the path origin node and the initial node of the link. Another element of decision is the scale of the congestion modelling, whether it is mesoscopic, where vehicles are moved in packets and links are divided into parts that include a moving part and a queuing part to model traffic flow, or microscopic, where vehicles are moved individually and movements such as car following and lane changing are considered. The traffic assignment model in the COMPAS model is microscopic in order to exploit the outcome from the demand model framework generating individual travellers to be loaded onto the network with their specific utility functions for their specific travel alternatives.

## Selection of the choice function

Literature in route choice modelling has shown that the choice function is usually expressing the trade-off between free flow travel time, congested travel time, risk of delay, distance covered, travel cost, and other factors related to the level-of-service of alternative routes.

Several studies have shown a growing interest in accounting for taste heterogeneity and heteroscedasticity in route choice models. Ben-Akiva et al. (1993) estimated a model with two alternatives where the time coefficient is log-normally distributed and a Gaussian quadrature calculates the integral. Dial (1997) formulated a traffic assignment algorithm where drivers have different perceptions about travel times and costs because of habitual behaviour, taste differences or information collection. Nielsen (2000) proposed a stochastic traffic assignment model with differences in the utility functions of passengers and discussed that log-normal and gamma distributions are suitable to simulate their heterogeneous preferences. Han et al. (2001) used uniform and normal distributions for delay and travel time in high traffic conditions to model SP games of pairwise route choices, and commented that the intuitively logical log-normal distribution does not produce satisfactory results. Jou (2001) estimated a random component model with normally distributed travel time parameter to investigate the impact of pre-trip information on route choice behaviour. Lam and Small (2001) modelled the choice between a free and a tolled route by accounting for median travel time and variability of travel time according to the time of day. Nielsen et al. (2002) estimated a model for different driver categories and tested both normally and log-normally distributed coefficients for travel time and cost. Nielsen (2004) presented an SP experiment about road pricing and estimated parameters from RP collected through GPS devices while accounting for heterogeneity in drivers' responses to pricing schemes.

In an individual-based approach, the distribution of the parameters is not an issue because the utility function has a specific form for each individual. This function is "passed from" the demand model framework and is defined for each individual and each travel alternative in order for the assignment model to load onto the network the individual (for the chosen alternative) and the ghosts (for the non-chosen alternatives). In particular, the value-of-time (e.g., for free flow and congested time) is not distributed across the population, but it is specific for each single individual to be loaded onto the network. The simplification related to the non-necessity of parameter distributions in the choice function allows exploring possible non-linearities in the function itself, given that the saved computational cost of drawing from probability distributions is usually superior to the additional computational cost of using non-linear utility functions.

## Definition of equilibrium

The consideration of stochastic elements in the choice function relates to the type of equilibrium that the traffic assignment model intends to achieve. The literature defines several types of equilibrium that are related to the static or dynamic nature of the assignment model and to the consideration or not of stochasticity in the assignment model.

In the static assignment case, most route assignment models either specify user (Nash, Wardrop) equilibrium (UE) or stochastic user equilibrium (SUE). The UE defines a stable condition being reached only when no traveller on the network can reduce his/her travel time by unilaterally selecting an alternative route (Wardrop, 1952). The SUE defines a stable condition being reached only when no traveller on the network believes that his/her travel time can be reduced by unilaterally changing his/her route (Daganzo and Sheffi, 1977). The SUE implies that users have different perceptions of route cost and every user takes the route with the minimum perceived cost. For path-based assignment models, the solution typically involves iterating (i) network loading, (ii) choice set generation and (iii) choice. Choice set generation often considers the new routes being the best paths from the previous iteration, and are generated within the iteration because a priori enumeration of all possible routes is computationally infeasible. Choice among the routes shows that in the UE case the flows on the currently best routes is increased at the cost of the other route flows ("best reply" choice), while in the SUE case the flows are shifted towards the desired route choice distribution (e.g., Dial, 1971, Cascetta et al., 1996, Ben-Akiva and Bierlaire, 1999). For stability reasons, this shift is typically realized in some gradual way that reduces the dynamics of the iterations until convergence is obtained. Convergence corresponds to attaining a fixed point for which no shift between routes takes place, and hence no traveller has an incentive to unilaterally select an alternative route.

In the dynamic assignment case (see Peeta and Ziliaskopoulos, 2001), where both the demand and the network conditions are time-dependent and the time-dependent travel times in the network define a physically meaningful progression of a demand unit through the network, the same applies. With respect to the iterative solution, if the new routes are best replies, if demand is shifted towards these new routes, and if these iterations reach a fixed point, then the procedure is similar and there is dynamic user equilibrium (DUE) where no traveller can reduce his/her travel time by unilaterally deviating to an alternative route. The SUE interpretation carries over in a similar way.

Nagel and Flotterod (2009) discuss the case of individual-based assignment where the notions of UE and SUE carry over if the notion of an O-D pair is replaced by that of an individual particle (i.e., an individual traveller). A particle UE is defined as a system state where no particle can unilaterally improve itself. A particle SUE is defined as a system state where travellers draw routes from a stationary choice distribution such that the resulting distribution of traffic conditions regenerates that choice distribution. The route flows are integer random variables, and consequently the cost structure based on which the individual chooses becomes probabilistic as well (e.g., Cascetta, 1989; Cascetta and Cantarella, 1991).

It is clear that UE models are attractive for large-scale transportation networks as they do not require a pre-specification of 'relevant routes', but implicitly allow some routes to be used for a given trip, while leaving many unattractive routes unused. However, the cutoff is strictly enforced: in a time-only model, if the current equilibrium travel time is 15.3 minutes, then adding a route with travel time of 15.4 minutes will have no impact on routing behaviour, whereas in practice (because of uncertainty, variability and unobserved attributes) the new route is likely to be attractive to some travellers. It is also clear that SUE models allow sort of 'smoothing' this condition, in that routes with higher travel time will be less used. This means that, with a custom specification with a continuous random error term with infinite support, SUE models will assign some flow to all feasible routes. If the set of feasible routes is assumed to be all acyclic routes, then this could be said to be implausible for a different reason to the DUE case: adding any route of any length will have some impact on SUE routing, even if entirely nonsensical for the trip being made. This issue is further complicated by the fact that typically only a sub-set of possible routes will be identified in numerical algorithms solving for SUE.

In the COMPAS model, new alternative forms of SUE conditions are present that permit unused alternatives, accommodate behaviour on used alternatives according to Random Utility Theory, and are generic in the sense that may be applied to any SUE model and any solution method. This new set of conditions is defined as the Restricted Stochastic User Equilibrium (RSUE) conditions, according to which there exists a path flow vector for which for each OD movement (i) the proportion of travellers on a used path is equal to the probability that the used path has a perceived utility greater than or equal to the perceived utilities on the alternative used routes, and (ii) all unused paths have an actual travel cost that is no less than what is specified by $\Phi(\cdot)$, where $\Phi(\cdot)$ refers to a criterion-value to be fulfilled by the paths which are unused.

### Definition of the solution approach

For individual-based traffic assignment models, with either pseudo-dynamic or fully dynamic congestion modelling, it is necessary to decide whether the probabilities from the route choice models are used for drawing individual trips to be assigned to the network, or for summing the probabilities over the routes. In fact, once the routes are generated, the route choice assigns a probability to each route within the choice set and two possibilities are available: (i) a Monte Carlo simulation assigns the "entire" traveller to one of the routes, or (ii) the traveller is "split" across the different routes according to the choice probabilities. The traffic assignment model in ACTUM uses the second approach that guarantees faster convergence because of higher accuracy, especially when more than one route has similar probabilities of being chosen.

Given the definition of the RSUE conditions, the focus shifts on the solution algorithms and on the observation that many SUE solution algorithms are computationally expensive by requiring simulation. Additionally, identifying all possible routes for realistic-scale networks quickly gets intractable, and most algorithms require pre-specification of 'relevant routes', which can be a difficult task. Recognizing the limitations of solution algorithms to the behaviourally sound SUE and the efficiency of solution algorithms to the UE, a transformation of the cost function is introduced. This transformation function opens up a larger array of possible solution algorithms to the SUE, as it allows us to apply any path-based UE solution algorithm and then to obtain a flow solution which satisfies the RSUE on a pre-specified choice set. The underlying choice model is however restricted to being logit-type. Due to the consistency with the IIA property of logit-type models, heuristic solution algorithms are proposed where the direction finding is based on a pairwise path-swapping algorithm. The transformation function also leads to the proposal of a new Relative GAP-measure (convergence measure) valid for any SUE or RSUE solution algorithm based on the logit-type choice models. Numerical tests have indicated that the proposed solution algorithms induce interesting and promising convergence patterns.

## SUMMARY AND CONCLUSIONS

The described framework of the traffic assignment within the COMPAS model proposes interesting insights from a behavioural and a computational perspective. From a behavioural perspective, the framework allows representing individual preference structures depending on individual attributes (e.g., value-of-time, income, age) and incorporating non-linear terms in the utility functions. The calculation of level-of-service for non-chosen alternatives (e.g., non-chosen routes, non-chosen modes, non-chosen destinations) is solved with ghost probes running (but not loading) the network. From a computational perspective, the proposed framework has a complexity similar to the static assignment. While adding the time dimension to a matrix-based assignment increases the calculation complexity significantly, proposing an individual-based approach requires only some more updating of the speed-flow and flow-density functions are required. The advantages are the complexity similar to static assignment, the absence of loss of information on the trips from the demand model, the increase in explanation and prediction abilities, and the avoidance of aggregation bias of the level-of-service variables in the feedback to the demand models.

As aforementioned, the disaggregate pseudo-dynamic traffic assignment allows (i) capturing time-dependent interactions of travel demand and network supply of the network, (ii) representing the network at a disaggregate level, (iii) representing congestion build-up and dissipation, and (iv) evaluating the effect of traffic management measures and traffic policies. When considering the main policies discussed in the Greater Copenhagen Area (e.g., measures of traffic control management, adoption of intelligent transport systems, adoption of road pricing policies), a state-of-the-art instrument such as an activity-based model with an individual-based pseudo-dynamic traffic assignment will prove highly valuable to decision makers.

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