

# **URBAN LAND USE EQUILIBRIUM ANALYSES CONSIDERING ADVANCED RAILWAY OPERATIONS**

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## **ABSTRACT**

We formulate a location problem of retail shops by using a land market equilibrium model on railway lines and undertake numerical simulations to understand the effects of advanced railway operations on urban land use. The simulations represent the effects of changes in railway operations on the location patterns of retail shops and land markets consistent with traditional urban economics.

*Keywords: urban land use, advanced railway operation, land market equilibrium*

## **INTRODUCTION**

A solution for Japanese big cities to expand high-density and productive land use may be the highly developed urban railway networks. Japanese urban railway networks are massive and carry a large number of commuters. This often lowers performance owing to the limitations posed by its capacity. Japanese railway systems have improved their performance by employing advanced railway operations such as mutual direct train services and mixed operations with rapid train services. These operations not only mitigate congestion and reduce travel time but also fuel urban development and land use differentiation by changing the nature of transport in urban spaces through hierarchical or diversified railway services.

However, these advanced railway operations have been designed not for urban planning, which controls land use and regional economy, but to benefit private railway

companies. Therefore, it is doubtful whether the operations have been taken full advantage of as measures towards improving urban planning.

This study focuses on the location problem of retail shops on railway lines as a typical example of urban land use in service industries. The relationships between the location patterns of retail shops and railway operations are formulated as a location equilibrium problem. Furthermore, the impact of changes in railway operations on the location pattern of retail shops is subjected to numerical simulations of simplified railway network examples to clarify the effects of advanced railway operations on urban land use.

## **PREVIOUS STUDIES**

Anas (1984) is a representative study integrating the transport network equilibrium model with the urban land use equilibrium model, consistent with urban economics theory. In this study, random utility theory was applied to understand the connection between urban land use and transport choice. Miyagi (1995) reformulated the Lowry-type land use model in accordance with random utility theory, unifying the model as an urban transportation network-land use equilibrium model. Following this, Miyagi and Sawada (2002) analysed the impacts of different urban transportation strategies on urban land use.

Mun (1995) analysed location patterns and social welfare in land markets of the retail industry by applying a location equilibrium model that takes into account the behaviour of standard economic agents such as customers, retailers, developers and land owners. This study considered the positive externality of the accumulation of retail shops and discrimination by an oligopoly of large scale-retail shops. Furthermore, Mun (1997) extended his model, consistent with general equilibrium systems, and investigated the behaviour of a system of cities connected by a linear transportation network, including several industries and economies of scale.

The above studies, while applying location equilibrium models, take into account transportation networks that understand urban spaces as network systems. This study also follows suit. The model here reformulates Mun (1995) by omitting the externality of retail shop accumulation and instead considering consumption frequency and railway services.

## RETAIL LOCATION EQUILIBRIUM MODEL ON RAILWAY LINES

### Consumption behaviour of residents

We make the following assumptions about the consumption behaviour of residents. All residents live along a railway line. They visit retail shops near railway stations several times during a given time period. Each resident chooses a shopping area independently. Their utilities are associated with expense, time cost, and their preference of place. The utility gained by the resident living in area  $i$  and shopping at area  $j$  is given by:

$$V_{ij} = -I - \gamma d_{ij} + \varepsilon_{ij}, \quad (1)$$

where  $I$  represents expense during shopping,  $\gamma$  is value of time,  $d_{ij}$  is travel time and  $\varepsilon_{ij}$  is resident's preference for shopping.

Let  $\varepsilon_{ij}$  be independent and identically distributed. Residents maximize their utility by the choice of shopping area, shown as

$$\max. \sum_{j=1}^n V_{ij}. \quad (2)$$

Then the optimal probability that the resident who lives in area  $i$  shops at area  $j$  is represented by a logit function:

$$P_{ij} = \frac{\exp \mu(-I - \gamma d_{ij})}{\sum_{k=1}^n \exp \mu(-I - \gamma d_{kj})}. \quad (3)$$

The number of consumers who live in area  $i$  and shop at area  $j$  is represented by

$$S_{ij} = \alpha O_i P_{ij}, \quad (4)$$

where  $O_i$  is population in area  $i$  and  $\alpha$  is consumption frequency.

The consumption frequency denoted in (5) is defined as a decreasing function of log-sum type cost, where  $\alpha_0, \alpha_1, \alpha_2$  are positive parameters:

$$\alpha = \alpha_0 - \alpha_1 \left( \alpha_2 - \frac{1}{\mu} \ln \sum_{i=1}^n \sum_{j=1}^n \exp \mu(-I - \gamma d_{ij}) \right). \quad (5)$$

The number of consumers who shop at area  $j$  is the sum of  $S_{ij}$  :

$$D_j = \sum_{i=1}^n S_{ij}. \quad (6)$$

### **Location behaviour of retail sellers**

It is assumed that each retail seller owns one shop, with shop size uniform, regardless of location. The profit of the retail seller operating in area  $j$  is expressed by

$$\pi_j = \frac{eD_j}{n_j + 1} - C - ur_j, \quad (7)$$

where  $n_j$  is the number of retail shops in area  $j$ ,  $e$  is the gross profit for each sale,  $C$  is the management cost of each shop,  $u$  is the floor space of each shop and  $r_j$  is the rental rate for unit floor space.

Latent retail sellers enter area  $j$  only if they expect positive profits. At equilibrium, the following two location patterns come into being:

$$n_j = 0, \quad \text{if } \pi_j < 0, \quad (8a)$$

$$n_j \geq 0, \quad \text{if } \pi_j = 0. \quad (8b)$$

### **Floor space supply behaviour of developers**

Assume one developer exists in each area. The developer supplies retail floor space by providing land  $L_j$  and capital  $K_j$ . The producing technology of retail floor space  $F_j$  is given by:

$$F_j = K_j^\tau L_j^{1-\tau}, \quad (9)$$

where  $\tau$  is a parameter which shows the combination of land and capital in a Cobb–Douglas production function form.

The profit of developer is formulated as

$$\phi_j = r_j F_j - cK_j - \rho_j L_j, \quad (10)$$

where  $c$  is the unit cost of capital and  $\rho_j$  is land rent.

We assume that developers maximize their profit:

$$\max. \phi_j \quad s.t. \quad F_j = K_j^\tau L_j^{1-\tau}. \quad (11)$$

The optimal ratio of capital divided by land is then

$$\frac{K_j}{L_j} = \frac{\tau}{1-\tau} \frac{\rho_j}{c}. \quad (12)$$

The retail floor space demand is constantly less than the supply:

$$un_j \leq F_j. \quad (13)$$

Developers supply retail floor space if they expect positive profits. At equilibrium, we have two supply situations:

$$F_j = 0, \quad \text{if } \phi_j < 0, \quad (14a)$$

$$F_j \geq 0, \quad \text{if } \phi_j = 0. \quad (14b)$$

### **Supply behaviour of landowner**

Consider that each area has one non-resident landowner, who rents land to developers as long as land rents are greater than reservation rents. Let reservation rent be denoted by  $b_j$ . At equilibrium, equations (15a) and (15b) hold.

$$L_j = 0, \quad \text{if } \rho_j < b_j, \quad (15a)$$

$$L_j \geq 0, \quad \text{if } \rho_j = b_j. \quad (15b)$$

### Numerical calculation method for the model

The above equilibrium conditions can be rewritten as <P1> by using complementarity conditions.

< P1 >

$$D_j = \left( \alpha_0 - \alpha_1 \left( \alpha_2 - \frac{1}{\mu} \ln \sum_{i=1}^n \sum_{j=1}^n \exp \mu(-I - \gamma d_{ij}) \right) \right) \sum_{i=1}^n O_i \frac{\exp \mu(-I - \gamma d_{ij})}{\sum_{k=1}^n \exp \mu(-I - \gamma d_{kj})} \quad \forall j \in J \quad (16)$$

$$n_j \left( \frac{eD_j}{n_j + 1} - C - ur_j \right) = 0, \quad -\frac{eD_j}{n_j + 1} + C + ur_j \geq 0, \quad n_j \geq 0 \quad \forall j \in J \quad (17)$$

$$\frac{K_j}{L_j} = \frac{\tau}{1 - \tau} \frac{\rho_j}{c}, \quad \forall j \in J \quad (18)$$

$$F_j = K_j^\tau L_j^{1-\tau}, \quad \forall j \in J \quad (19)$$

$$un_j \leq F_j, \quad \forall j \in J \quad (20)$$

$$F_j (r_j F_j - cK_j - \rho_j L_j) = 0, \quad -r_j F_j + cK_j + \rho_j L_j \geq 0, \quad F_j \geq 0 \quad \forall j \in J \quad (21)$$

$$L_j (\rho_j - b_j) = 0, \quad -\rho_j + b_j \geq 0, \quad L_j \geq 0 \quad \forall j \in J \quad (22)$$

<P1> is a set of simultaneous equations which includes the parameters  $\alpha, O_i, \mu, I, \gamma, d_{ij}, e, C, u, \tau, c, b_j$  and variables  $D_j, n_j, r_j, K_j, L_j, F_j, \rho_j$ . <P1> has a unique solution because the number of equations corresponds to the number of variables. We calculate the equilibrium location patterns by adopting the Fischer–Burmeister function (Fischer, 1992). The function transforms simultaneous equations, including complementarity conditions, into mathematical optimization problems.

## **RETAIL LOCATION EQUILIBRIUM ANALYSES CONSIDERING ADVANCED RAILWAY OPERATIONS**

### **Sensitivity analyses to confirm the validity of the model structure**

To verify equilibrium, we assume a circular railway line, as drawn in figure 1, and the parameters normally discriminated in the areas are equalized in tables 1 and 2. The nodes represent railway stations in each area, and retail shops are located near the stations. The residents visit retail shops within their living areas or in other areas by train, which necessitates access time. For the sake of simplicity, railway fees are not considered, as these are proportional to travel times. Table 1 shows travel times from area  $i$  to  $j$ . The lines are uniform, except while sliding from other lines therefore rendering the total advantage of stations equal. The parameters:  $\alpha_0, \alpha_1, \alpha_2$  are set so that consumption frequency is a decreasing function of goods price and travel time. The populations and reservation rents in each area are also indifferent, as are the other parameters.

Equilibrium is depicted in figure 2 corresponding to the above settings. It is found that the demand of shopping in the areas is equal because total breakdown of consumers who gather each area to shop. The number of retail shops, total floor space for retail shops, and capital and land invested by developers in the areas are uniform, as well. Therefore, the calculation has accurately computed equilibrium.

To control that the model has appropriate properties, the parameters are set in a rational behaviour of the model corresponding to variations in the major parameters which enhance urban and economic environment through sensitivity analysis. The results are shown in table 3. Speeding up of trains seems to shorten travel time, increase average consumption frequency, extend total floor space, and increase capital and land investment. The overall trend in the sensitivity analyses in table 3 shows a favourable outcome, supporting the fact that the model has the potential to describe the relationship between retail location pattern and railway services.

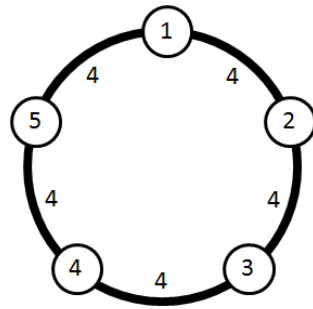


Figure 1 Railway network for sensitivity analyses

Table 1 Travel time

dij	1	2	3	4	5
1	3	4	8	8	4
2	4	3	4	8	8
3	8	4	3	4	8
4	8	8	4	3	4
5	4	8	8	4	3

Table 2 Parameter setting

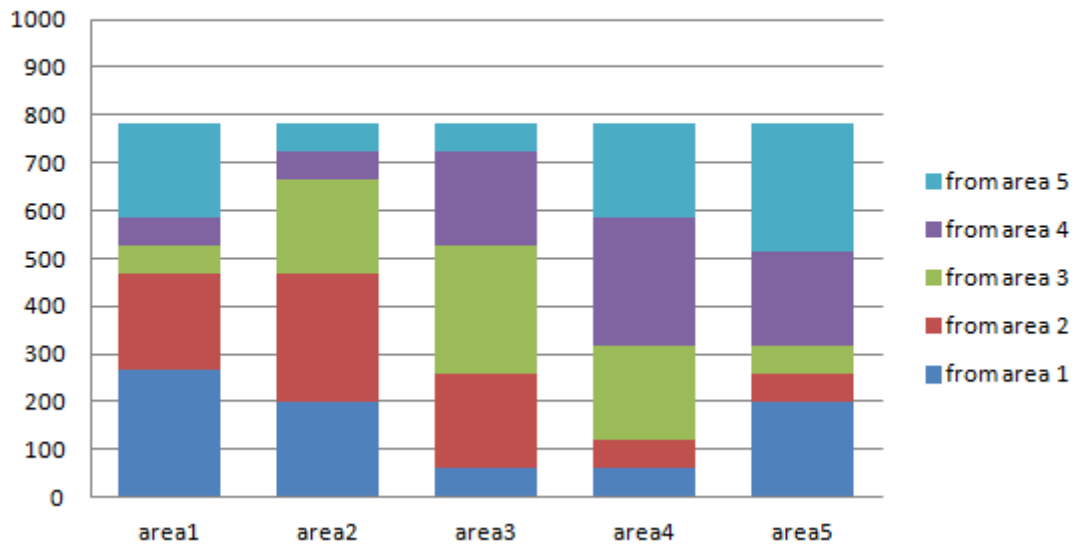
$\gamma$	30	C	10
$\mu$	0.01	u	2
$\alpha_0$	100	$\tau$	0.6
$\alpha_1$	1	c	50
$\alpha_2$	100	$b_1-b_5$	20
$Q_1-Q_5$	10	I	100

Table 3 Model behaviour corresponding to parameter variations

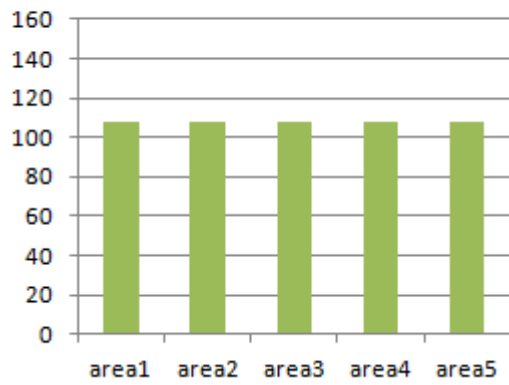
parameter	variation	consumption frequency	number of retail shops	total floor space	capital investment	land investment
train speed	+	+	+	+	+	+
goods price	+	-	-	-	-	-
reservation rent	+	0	-	-	+	-
population	+	0	+	+	+	+
floor space	+	0	-	+	+	+
capital cost	+	0	-	-	-	+
shop managing cost	+	0	-	-	-	-
technology sub.	+	0	-	-	+	-



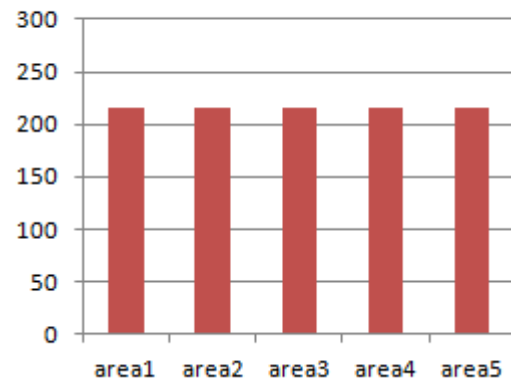
### Breakdown of shopping demand



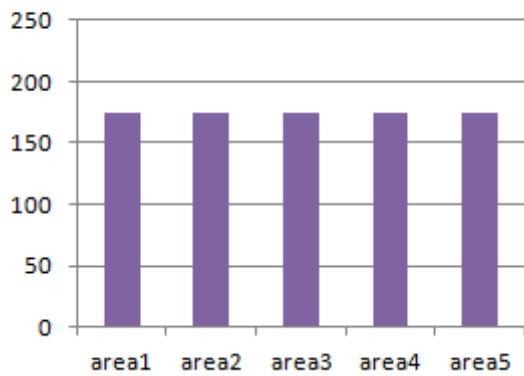
■ number of retail shops



■ total floor space



■ capital investment



■ land investment

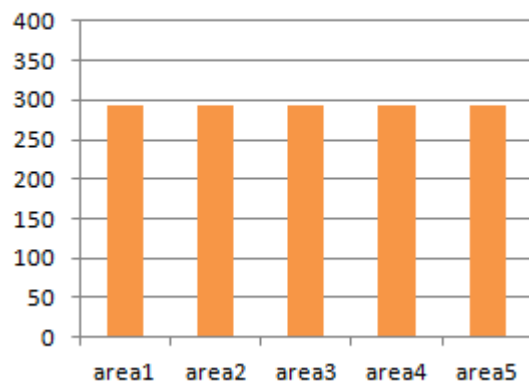


Figure 2 Variables at equilibrium in circular railway line

### **The effect of mutual direct train services on retail shop locations**

In this subsection, we investigate the effects of introducing a mutual direct train on retail shop locations, at equilibrium. We assume that two railway lines connect area 3 (shown on the left side of figure 3), and the transfer requires walking and waiting time. Mutual direct train services are introduced between two railway lines by improving the facilities of stations and rail tracks in area 3 (shown on the right side of figure 3). The parameters, except travel time, are the same as in table 3.

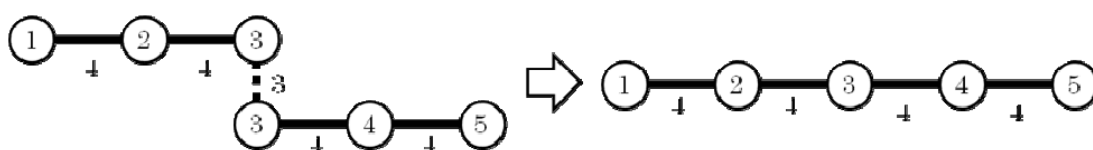
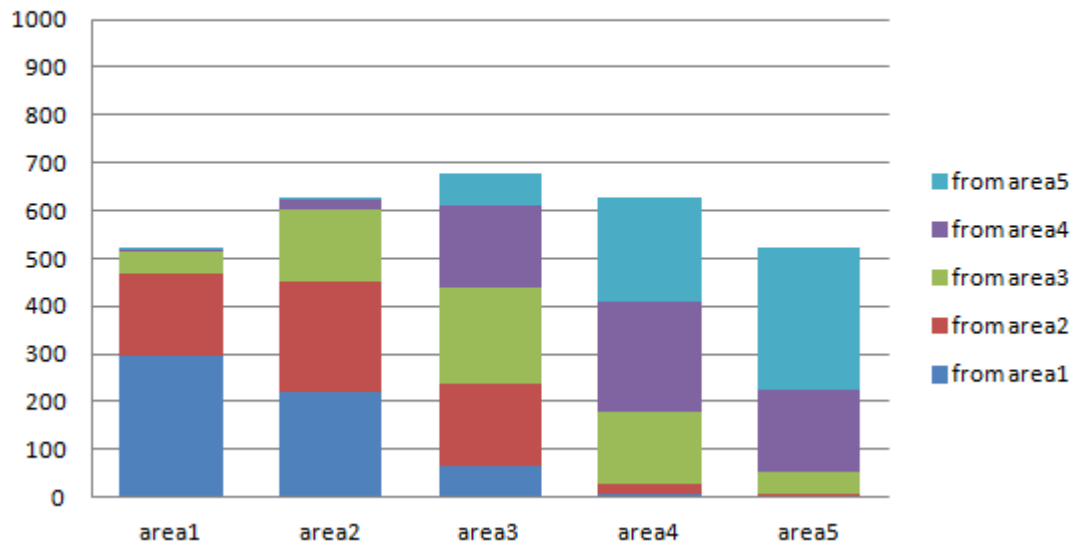


Figure 3 Introduction of mutual direct train service

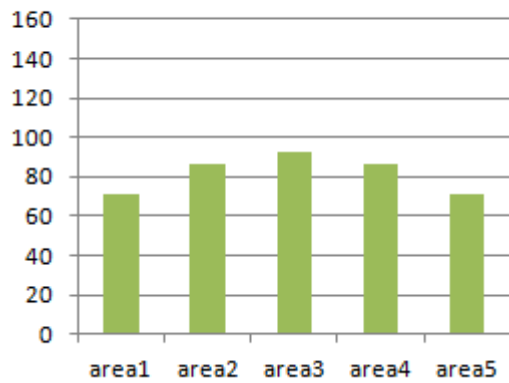
The equilibria corresponding to the two settings—with and without the introduction of mutual direct train services—are simulated using numerical examples. The number of consumers, number of retail shops, total floor space of retail shops, and capital and land investment in areas are depicted in figures 4 and 5. We find that shopping demand in all areas has increased with higher consumption frequency on account of shortening travel times. The increase in demand is to some extent idiosyncratic. That is, demand for retail shops in areas 2 and 4 has increase, whereas the demand concentrated in area 3 has decreased. Hence, it is clearly seen that the terminal area connecting stations of multiple railway lines has had extraordinary appeal in terms of retail shops' location. The introduction of mutual direct train services in this terminal area is relatively less advantageous than other areas instead of relaxation of terminal congestion. On the other hand, areas 1 and 5 are less advantaged on the railway network because demand appears one-sided, owing to which the advantages of introducing a mutual direct train service in these areas are relatively small.

Regarding changes in residents' shopping behaviour, the proportion of customers who travel over terminal area 3 increased with the introduction of the mutual direct train. Furthermore, residents living in area 3 appear to shop more at other areas irrespective of the introduction of the mutual direct train. This is because of their easy access to other area even prior to the introduction. Number of retail shops, total floor space in

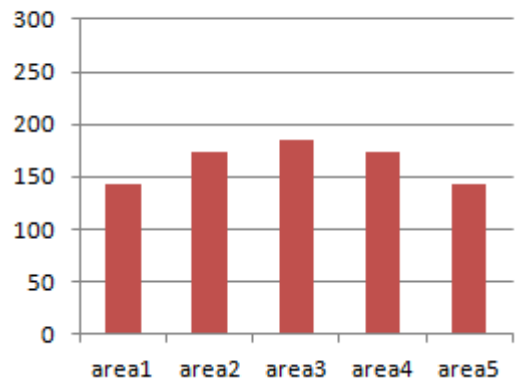
### Breakdown of shopping demand



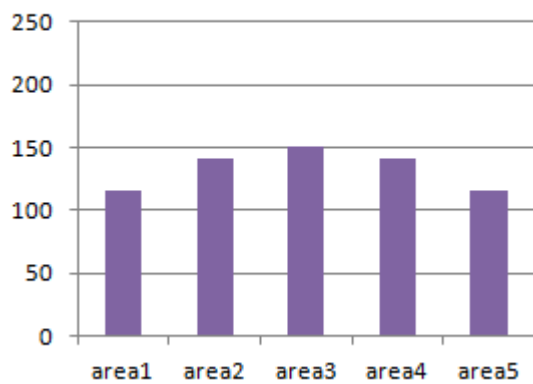
■ number of retail shops



■ total floor space



■ capital investment



■ land investment

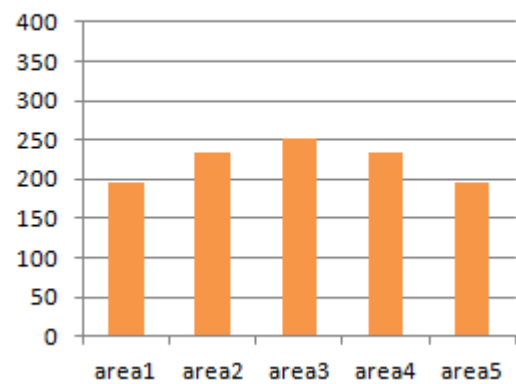
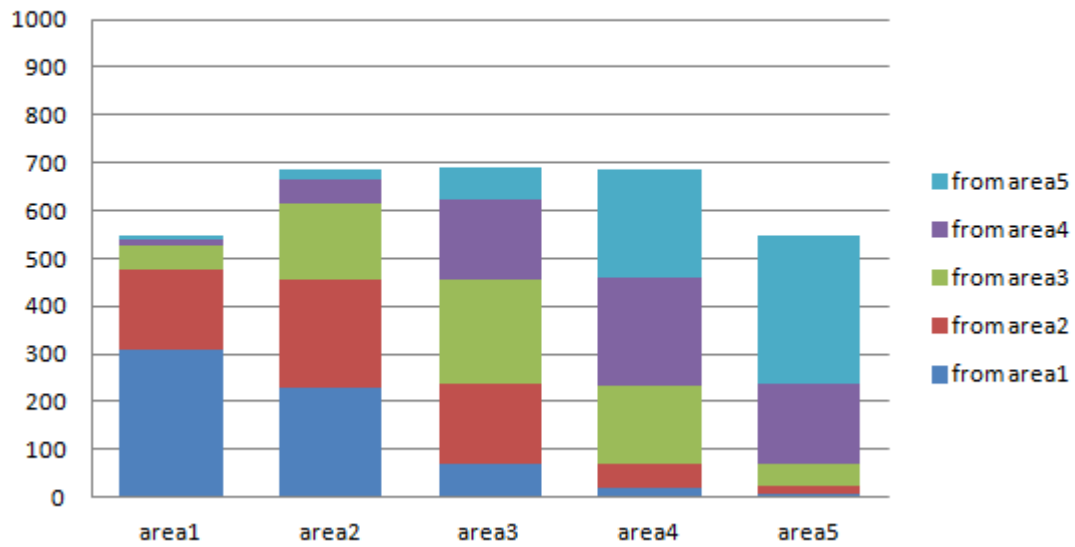
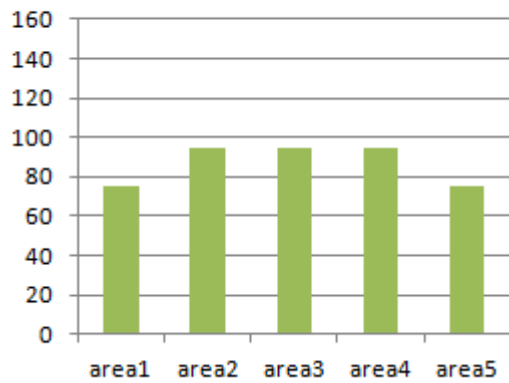


Figure 4 Equilibrium without the introduction of mutual direct services

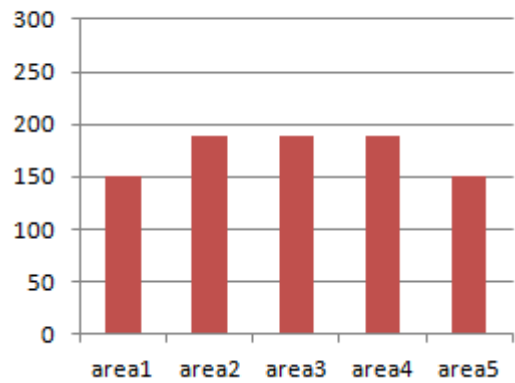
### Breakdown of shopping demand



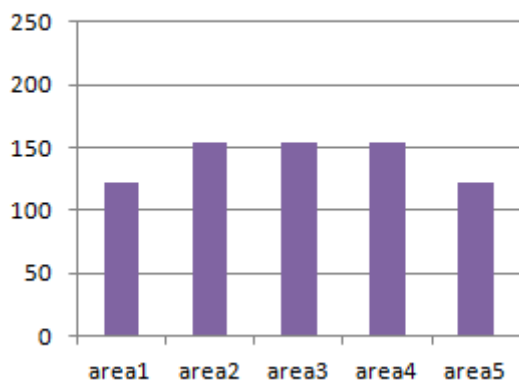
■ number of retail shops



■ total floor space



■ capital investment



■ land investment

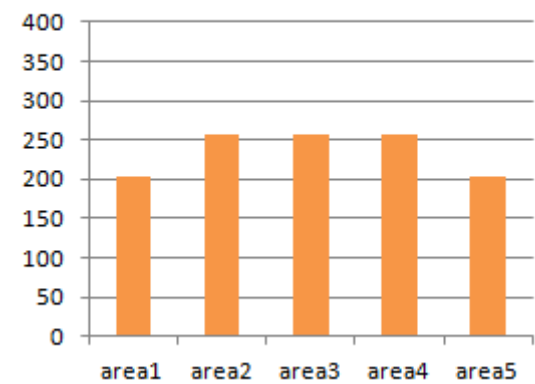


Figure 5 Equilibrium with the introduction of mutual direct services

each area, and capital and land investment are in proportion to shopping demand, demonstrating that economic agents adjust their behaviour to variation in demand. All this implies that the attractiveness of areas other than the terminal area is higher on account of the introduction of mutual direct train services, as the development of the retail industry concentrates in the neighbourhood of the terminal area.

### The effect of mixed operating rapid train services on retail shop locations

In this subsection, we investigate the effect of mixed operating rapid train services on retail shop locations, at equilibrium. We assume the introduction of mixed operation—that is, local train stops at every station and rapid train stops at only stations in areas 1, 3, and 5, with both trains operated simultaneously on the same railway line—as shown as figure 6. It is supposed that the transfer between rapid and local trains requires waiting time to set the travel times between areas. The parameters except travel time retain the same values as in table 3.

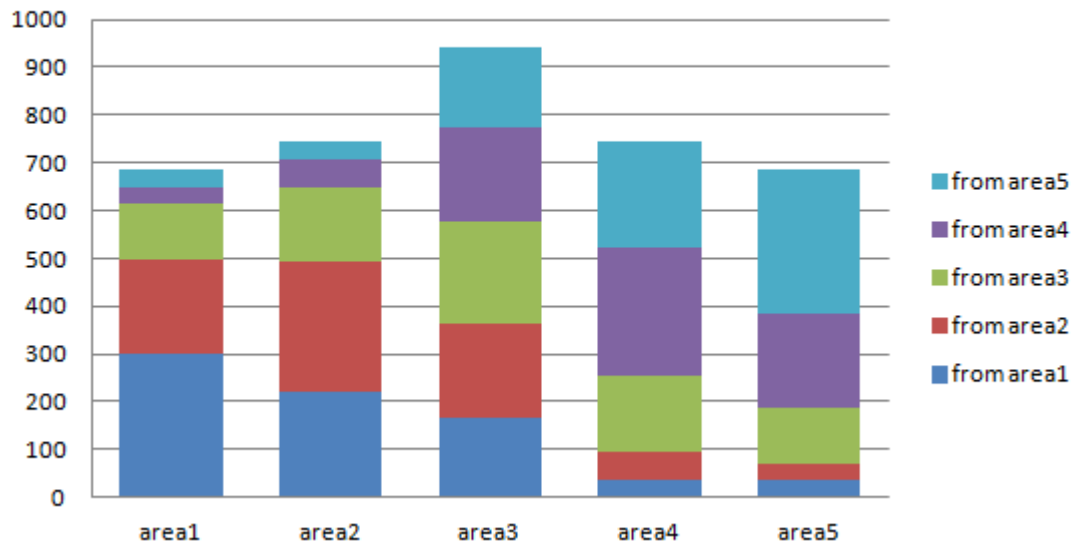
The case without mixed operation is assumed to be the case where the mutual direct train service, mentioned in the previous subsection, is introduced. Then, the values of variables at equilibrium are consistent with figure 5. The values of variables at equilibrium with mixed operation are expressed in figure 7. On comparison with figures 5 and 7, mixed operations shorten customers' commuting time, owing to which the shopping demand expands in all areas. The extent of expansion, however, differs



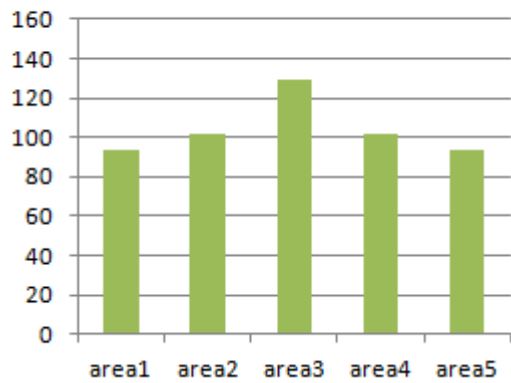
Figure 6 mixed operation rapid train services

among the areas. That is, the increase in demand in areas 2 and 4 (at which only local trains stop) is relatively small, with demand being concentrated in area 3. The attractiveness of area 3 arises from being at the centre of the railway line and a stop for rapid trains. Demand increases further, as compared to the absence of mixed operations in the areas 1 and 5, on account of rapid trains also stopping at these stations. Hence, it is clearly seen that the attractiveness of the area increases when rapid trains stop in these areas.

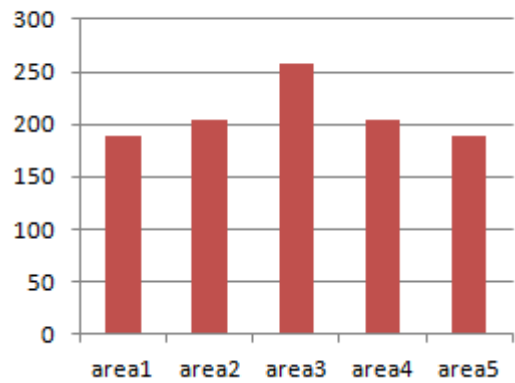
### Breakdown of shopping demand



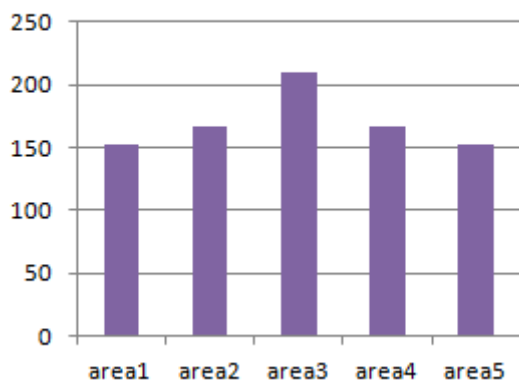
■ number of retail shops



■ total floor space



■ capital investment



■ land investment

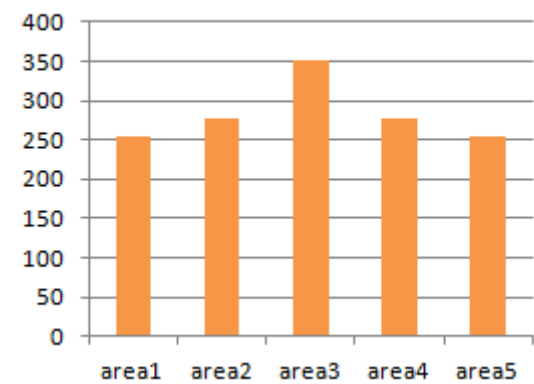


Figure 7 Equilibrium with mixed operating rapid train services

On the other hand, the effect of changes in the demand in other areas at which only local trains stop depend on the design of the mixed railway operations, which is the parameter setting in this particular example.

The volume of customers who commute long distances increases with mixed operations, owing to which mobility is enhanced. It is now obvious that rapid train services make a difference in bringing down demand between the areas.

As for the number of retail shops, total floor space for retail shops, and capital and land investments in each area, all values appear proportionate to shopping demand, implying that economic agents adjust their behaviour to variation in demand. It is also implied that as the attractiveness of the all areas increases, the retail industry is invigorated, especially in area 3.

## **CONCLUDING REMARKS**

We have formulated a location problem of retail shops as a land market equilibrium model on a railway line and undertaken numerical simulations to understand the mechanisms of the effect of advanced railway operations on urban land use. The simulations represented the effects of changes in railway operations on the location patterns of retail shops and land markets, consistent with traditional urban economics.

The advanced railway operations increased the number of new retail shops and revitalized the retail industry from a whole city viewpoint. The effects significantly differ depending on the operations. The introduction of mutual direct train services scattered retail shops from the terminal area. Finally, the mixed operating rapid train services differentiated areas according to types of stopping trains.

Future subjects of study are as follows. The parameters that characterize the service industry need to be detailed further. The competitive situation for renting floor space or land between multiple service industries should be investigated in the case of a certain industry without rent. The advanced railway operation must be refined in accordance with the train operation diagram and managed as a private company.

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