PLATOON BASED MACROSCOPIC INTELLIGENT TRAFFIC MODEL

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ABSTRACT

This paper proposes a macroscopic model to describe the operation of intelligent traffic flow where intelligent vehicles are driving closer to each other than manual vehicles and operating in a form of many platoons each of which contains several vehicles. The model is developed from a car-following model which allows us to obtain well tractable macroscopic equations for such platoon based traffic operation. The stability diagrams are constructed from the developed model based on the linear stability method for a certain model parameter set. It is found analytically that intelligent vehicles enhance the stabilization of traffic flow with respect to a small perturbation. Numerical simulation is carried out to support our analytical findings. We have argued that in parallel to microscopic models for intelligent traffic flow, the newly developed macroscopic model will provide a complete insight into the dynamics of intelligent traffic flow.

Keywords: platoon operation, intelligent vehicles, macroscopic traffic model, instabilities

INTRODUCTION

Traffic congestion is one of the main societal and environmental problems in many developed countries. A lot of research has been conducted to seek solutions to the traffic related problems with the ultimate goal of increasing the road capacity. In general, most of the potential solutions are applying either relevant traffic control strategies or information and communication technologies (ICT). This article deals with the potential applications of ICT and investigates their effects on traffic flow dynamics. ICT are now in the early stages of transforming transportation systems by integrating sensors (remote sensing and positioning), control units (traffic signals, message signs) and automatic technologies with microchips to enable them to communicate with each other through wireless technologies. It is expected that in the coming 5 to 10 years ICT will considerably progress worldwide so that intelligent vehicles, in which the driving tasks are shifted from the driver to the vehicle through autonomous vehicle-to-vehicle and vehicle-to-infrastructure communication, will make up a significant share of the traffic flow. Therefore, it is essential to evaluate the effects of intelligent vehicles on traffic flow dynamics well in advance before they are widely implemented and designed so that the negative effects can be minimised.

To date, there have been impressive advances in modelling the dynamics of intelligent traffic systems in the US (Bose and Ioannou, 2003, Davis, 2007), in Europe (Marsden et al., 2001, van Arem and Tampere, 2003, van Arem et al., 2007, Togeren et al., 2007, Kesting et al., 2008) and in Asia (Kikuki et al., 2003), most existing work relies on a microscopic modelling approach which describes traffic flow at a high level of detail such as the movement of individual vehicles . In this paper, we focus on a macroscopic modelling approach depicting traffic flow at a low level of detail via aggregate traffic variables such as flow, mean speed and density. In contrast to microscopic models, macroscopic models are preferred for real-time prediction and control applications due to their fast computational demand and less calibration effort (Zhang and Wang, 2012). To the best of our knowledge, there are very few advances in incorporating the intelligent driving behaviour in macroscopic models. Ngoduy (2012,2013) have developed macroscopic models taking into account the contribution of the penetration of intelligent vehicles (e.g. adaptive cruise control or cooperative adaptive cruise control vehicles). Based on such models, linear and nonlinear instabilities of traffic flow have been analyzed to show the impact of the penetration of intelligent vehicles on traffic dynamics.

In general, either microscopic or macroscopic models show that intelligent vehicles will stabilize traffic flow with respect to both small and large perturbation such as sudden deceleration or lane changes and lead to significantly increased capacity and reduced travel time. However, in intelligent traffic flow systems, the important functionality of ICT is to arrange vehicles in closely spaced groups, which are called platoons. In such traffic systems, the intra-platoon spacing (that is the vehicle spacing within a platoon) is kept smaller and the inter-platoon spacing (that is the spacing between platoons) is kept larger than in manual traffic systems. Consequently, the current macroscopic traffic models are still insufficient as they do not adequately capture the platoon based operation in intelligent traffic systems. In principle, the platoon based operation will model traffic flow to contain many platoons moving together (instead of many individual vehicles moving together as in manual traffic flow). The behavior of vehicles inside the platoon (hereafter the platooning) is determined by that of the platoon leader (hereafter the platooner) whereas the behavior of the (following) platooner is determined by the leading platoon. To contribute to the state of the art in traffic flow theory, this paper will propose a new platoon based model to describe more realistically the dynamics of intelligent traffic flow. To this end, the contribution of this paper is threefold:

1. The generalized force model (GFM) of Helbing and Tilch (1998) is extended to capture the platoon based microscopic driving behaviour.
2. The proposed platoon based GFM is used to formulate a new macroscopic model using a gas-kinetic approach.
3. The analytical properties of the model are derived and numerical simulations are carried out to show the impact of intelligent vehicles on traffic dynamics.

GENERALIZED FORCE MODEL AND GAS-KINETIC EQUATION

Platoon based generalized force model

We have chosen the GFM because it is well suited for the gas-kinetic model as the deceleration (or braking) time is explicitly taken into account and we can derive a macroscopic model which is well tractable and is consistent with the general form of other macroscopic models. According to Helbing and Tilch (1998), the amount and direction of a behavioral change such as the acceleration/deceleration is given by a sum of generalized forces reflecting the different motivations which a driver feels at the same time. Since these forces do not fulfill Newtons laws: *action* = *reaction*, they are called generalized forces. Helbing and Tilch (1998) argued that the success of this approach in describing traffic dynamics is based on the fact that driver reactions to typical traffic situations are more or less automatic and determined by the optimal behavioral strategy. In the model of Helbing and Tilch (1998), the dynamics of a vehicle n with velocity $v_n(t)$ at place $x_n(t)$ and time instant t is given by the equation of motion:

$$\frac{dv_n}{dt} = \underbrace{\frac{v_0 - v_n}{\tau_n}}_{\text{acceleration}} + \underbrace{f_{n,n-1}\left(v_n, x_n, v_{n-1}, x_{n-1}\right)}_{\text{repulsive interaction}} \quad (1)$$

In equation (1), τ_n and v_0 denote the acceleration time and the desired speed, respectively, of vehicle n. The *acceleration* term presents the motivation of vehicle n to reach its desired speed v_0 while the *repulsive interaction* term describes the motivation of that vehicle to keep a safe distance from the leader $n-1$. The *repulsive interaction* term is specified as:

$$f_{n,n-1}\left(v_n, x_n, v_{n-1}, x_{n-1}\right) = \frac{V_e\left(s_n\right) - v_0}{\tau_n} - \frac{v_n - v_{n-1}}{\tilde{\tau}_n} H\left(v_n - v_{n-1}\right) e^{-\left(s_n - s_0\right)/R} \quad (2)$$

where:

- s_0 denotes the speed dependent safe-distance, defined as $s_0 = d+Tv_n$. Here, T is the safe time headway measured by the time the follower is needed to stop completely to avoid a collision with a stopped vehicle in front, while d is the safe distance between the follower's front and the leader's rear when these two vehicles stop completely.
- $s_n = x_{n-1} - x_n$ is the distance headway between vehicle n and its direct leader $n-1$.
- v_{n-1} is the speed of the direct leader.
- $H(.)$ denotes a Heaviside function, a dimensionless quantity.
- R is the distance measured by a range of the braking interaction (m). $\tilde{\tau}_n$
- $V_e(s_n)$ is the headway-dependent equilibrium speed.
- is the braking (deceleration) time. Typically, $\tilde{\tau}_n < \tau_n$ since deceleration capabilities of vehicles are greater than acceleration capabilities.

Note that if we neglect the contribution of the finite deceleration time $\tilde{\tau}_n$, the model of Bando et al. (1995) or OV-type model is obtained. There are some important properties of the *repulsive interaction force* as described in literature (Helbing & Tilch, 1998). First, it will guarantee early enough and sufficient braking in cases of large relative speed (v_n-v_{n-1}). Second, this force increases with growing relative speed (v_n-v_{n-1}), but will only be effective, if the speed of the follower is larger than that of the leader (i.e. $H(v_n-v_{n-1}) = 1$). Third, it will increase with decreasing distance headway s_n, but vanish for large one (i.e. $s_n \to \infty$).

Now let us consider the operations of intelligent vehicles, which are basically moving in a form of many platoons. The dynamics of intelligent traffic flow are described by the interactions between platooners while the platooning will be controlled by their platooner. If we consider each platoon an entity moving together and the interaction between each platoon also be described by the car-following rule, from equation (1) the dynamics of the considered platooner with speed v_p at place $x_p(t)$ and time instant t can be described as:

$$\frac{dv_p}{dt} = \frac{v_0 - v_p}{\tau_p} + f_{p,p-1}\left(v_p, x_p, v_{p-1}, x_{p-1}\right) \quad (3)$$

where p depicts the platoon index and $f_{p,p-1}$ is determined similarly to equation (2):

$$f_{p,p-1}\left(v_p, x_p, v_{p-1}, x_{p-1}\right) = \frac{V_e\left(s_p\right) - v_0}{\tau_p} - \frac{v_p - v_{p-1}}{\tilde{\tau}_p} H\left(v_p - v_{p-1}\right) e^{-\left(s_p - s_p^0\right)/R} \quad (4)$$

Here:

- s_0^p denotes the speed dependent safe-distance between platoons, defined as s_0^p = $d_p+T_pv_p$. Here, T_p is the time headway between platoons and d_p is the safe distance when the platoons stop completely.
- $s_p = x_p - x_{p-1}$ is the distance headway between platooner p and its leading platooner $p-1$
- v_{p-1} is the speed of the leading platooner.

- $V_e(s_p)$ is the headway-dependent equilibrium speed.

Let m denote the number of vehicles within a platoon. In principle, m is dependent on traffic situations. In contrast to the platooners, platooning are moving much closer together we can model the dynamic length of the platoon as:

$$L_p = (m-1)\left(d_0 + T_0 v_p\right) \tag{5}$$

Where L_p is the length of the platoon p, T_0 and d_0 denote the time headway and safe distance at complete stops of the platooning. Practically, $T_0 < T_p$ since the platooning drive very close to each other.

The distance (or space) headway between platoons now defined as $s_p(x_p,t) = s(x+L_p, t)$ where s is the distance headway between the considered platooner and its direct leading vehicle (i.e. the last vehicle of the leading platoon).

Platoon based gas-kinetic equation

In this section, we apply the gas-kinetic equation to vehicular traffic presented in Helbing and Treiber (1998). This equation will serve as the basis for the subsequent derivation of the platoon based macroscopic model in the next section. As we will show in the next section, the derivation method uses some exact integration calculations to end up with the macroscopic variables. Let $\rho_p\left(x, v_p, t\right)$ denotes the phase and space density (PSD) distribution of platooner p driving with speed v_p ($v_p \in \left[v_p, v_p + dv_p\right]$) at location x ($x \in \left[x, x+dx\right]$) and time instant t, where dv_p and dx are small deviation of speed and location of the platooner p, respectively. Note we drop the index p from time to time for the sake of simplicity since the gas-kinetic equation holds for any platooner with respect to its leading platooner. It is also worth mentioning that, in a traffic problem, a phase-space quantity represents all possible states of traffic flow in time and location. For a homogeneous freeway (e.g. without on-and off-ramps), the platoon based gas-kinetic model reads:

$$\frac{\partial \rho_p}{\partial t} + \underbrace{\frac{\partial\left(\rho_p v_p\right)}{\partial x}}_{\text{convection}} + \underbrace{\frac{\partial}{\partial v}\left(\rho_p \frac{dv_p}{dt}\right)}_{\text{interaction}} = 0 \tag{6}$$

In equation (6), the *convection* term describes the changes of the PSD due to the movement of vehicles along the road while the *interaction* term depicts the changes of the PSD due to the acceleration to the desired speed as well as the deceleration.

Let us substitute equations (3)-(4) into the gas-kinetic equation (6) to obtain:

$$\frac{\partial \rho_p}{\partial t} + \underbrace{\frac{\partial\left(\rho_p v_p\right)}{\partial x}}_{\text{convection}} = -\underbrace{\frac{\partial}{\partial v}\left(\rho_p \frac{v_0 - v_p}{\tau_p}\right)}_{\text{acceleration}} - \underbrace{\frac{\partial}{\partial v}\left(\rho_p \frac{V_e\left(s_p\right) - v_0}{\tau_p} - \rho_p \frac{v_p - v_{p-1}}{\tilde{\tau}_p} H\left(v_p - v_{p-1}\right) e^{-\left(s_p - s_p^0\right)/R}\right)}_{\text{deceleration}} \tag{7}$$

In equation (7) the deceleration is determined by the interaction between the following platooner and the leading platoon, speed of which is defined by the leading platooner. This method is different from the other derivations such as the gas-kinetic models of Treiber et al. (1999) because in those models one usually assumes an instantaneous braking to the speed of the slower vehicle in front, if overtaking is not possible. This approximation of the deceleration behavior is justified only if the time scale of the deceleration is significantly smaller than that of the acceleration and therefore is neglected. It leads to the Boltzmann-like braking interaction term as in current gas-kinetic based models. From equation (7), we will generate a macroscopic model in which the platoon based driving behaviour has been incorporated. This is the subject of the next section.

PLATOON BASED MACROSCOPIC EQUATIONS

The derivation method in this section is a so-called method of moments. In principle, the method of moments has been applied widely to obtain macroscopic traffic models from gas-kinetic theory in literature (Treiber et al., 1999, Helbing et al., 2001, Hoogendoorn et al., 2002, Ngoduy, 2008, Ngoduy et al. 2006, 2009). In this paper, we just briefly show the procedure to obtain a macroscopic model using such method. By definition, the macroscopic traffic variables are determined as below:

- Platoon density $r_p(x, t)$ describing the number of vehicle platoons per unit road length $[x,x+dx]$ at time t. Let $g_v(x, v_p, t)$ denote the probability density function of speed v_p, the phase-space density is defined by: $\rho_p\left(x,v_p,t\right)=r_p\left(x,t\right)g_v\left(x,v_p,t\right)$. If we take the integral of this relation over the speed v_p we will have $\int\rho_p\left(x,v_p,t\right)dv_p=r_p\left(x,t\right)\int g_v\left(x,v_p,t\right)dv_p$. By definition, $\int g_v\left(x,v_p,t\right)dv_p=1$, hence:
$$r_p\left(x,t\right)=\int\rho_p\left(x,v_p,t\right)dv_p \tag{8}$$
- Mean speed $V_p(x,t)$. The platoon mean speed $V_p(x,t)$ is defined as: $V_p\left(x,t\right)=\left\langle v_p(x,t)\right\rangle=\int v_p g_v\left(x,v_p,t\right)dv_p$, hence:
$$V_p=\frac{1}{r_p}\int\rho_p\left(x,v_p,t\right)v_p dv_p \tag{9}$$
- Platoon mean speed variance $\Theta_p\left(x,t\right)$. The platoon mean speed variance $\Theta_p\left(x,t\right)$ is defined as: $\Theta_p\left(x,t\right)=\left\langle\left(v_p-V_p\right)^2\right\rangle$, hence:
$$\Theta_p=\frac{1}{r_p}\int\rho_p\left(x,v_p,t\right)\left(v_p-V_p\right)^2dv_p \tag{10}$$

where $\langle.\rangle$ denotes the so-called mean operator, defined as below. For any function $y\left(\alpha\right)$ where α is a variable:
$$\left\langle y\left(\alpha\right)\right\rangle=\int_\alpha y\left(\alpha\right)\psi\left(\alpha\right)d\alpha \tag{11}$$
with $\psi\left(\alpha\right)$ being the distribution function of α.

Let us multiply both sides of equation (7) with $v_p^kG_w\left(w_p,s_p\,|\,x,t\right)$ (k = 0, 1, 2,…), where we define $w_p=v_{p-1}$, then integrate them over all possible values of v_p,w_p and s_p. Here $G_w\left(w_p,s_p\,|\,x,t\right)$ denotes the probability that, given a platooner with speed v_p at location x and time instant t, the leading platooner drives with a speed w_p at a headway distance s_p. Let us apply the factorization approximation $G_w\left(w_p,s_p\,|\,x,t\right)=g_w\left(w_p,x+s_p,t\right)g_s\left(s_p,x,t\right)$ which means that distributions of the speed of the leading platooner w_p and the corresponding distance headway s_p are statistically independent, and independent of the speed v_p of the following platooner. Accordingly, the aggregated left hand side of equation (7) becomes:
$$A_k=\frac{\partial}{\partial t}\left(\rho_p\int dv_pv_p^k\underbrace{\int dw_p\int ds_pg_w\left(w_p,x+s_p,t\right)g_s\left(s_p,x,t\right)}_{=1}\right)=\frac{\partial}{\partial t}\left(r_p\left\langle v_p^k\right\rangle\right) \tag{12}$$

$$B_k = \frac{\partial}{\partial x}\left(\rho_p \int dv_p v_p^{k+1} \underbrace{\int dw_p \int ds_p g_w\left(w_p, x+s_p, t\right) g_s\left(s_p, x, t\right)}_{=1}\right) = \frac{\partial}{\partial x}\left(r_p \left\langle v_p^{k+1} \right\rangle\right) \tag{13}$$

and the aggregated right hand side of equation (7) reads:

$$C_k = r_p k \int dv_p \int dw_p \int ds_p v_p^{k-1} g_v\left(v_p, x, t\right) g_w\left(w_p, x+s_p, t\right) g_s\left(s_p, x, t\right) \left[\frac{V_e\left(s_p\right)-v_p}{\tau_p} - \frac{v_p - w_p}{\tilde{\tau}_p} H\left(v_p - w_p\right) e^{-\left(s_p - s_p^0\right)/R}\right] \tag{14}$$

In the ensuing paper let us assume that at the aggregate level, the model parameters are the same for either platooner or platooning. Therefore we could drop the platoon index *p* for all model parameters.To set *k* = 0 and *k* = 1, respectively, we obtain the dynamic equations for the platoon density (i.e. the conservation law for the number of platoons per road length unit):

$$\frac{\partial r_p}{\partial t} + \frac{\partial\left(r_p V\right)}{\partial x} = 0 \tag{15}$$

and for the flow as detailed below. It is worth noting that $r_p \doteq mr$, where *r* denotes traffic density (i.e. number of vehicles per road length unit). Furthermore, the mean speed of the platooning is as the same as that of the platooner, so we drop the platoon index for the mean speed.

As the conservation law must also hold for individual vehicles, i.e.:

$$\frac{\partial r}{\partial t} + \frac{\partial\left(rV\right)}{\partial x} = 0 \tag{16}$$

a straightforward calculation from equations (15) and (16) leads to:

$$\frac{\partial m}{\partial t} + V\frac{\partial m}{\partial x} = 0 \tag{17}$$

To close the model, we need an assumption for the dynamics of the speed variance, which has been justified to follow an empirical function of density as in literature (Treiber et al., 1999, Treiber and Kesting, 2013). Consequently, the main difficulty is to determine function C_1, which results in the dynamic equation of the flow. That is:

$$\frac{\partial q_p}{\partial t} + \frac{\partial r_p\left(V^2 + \Theta\right)}{\partial x} = C_1 \tag{18}$$

From equation (18) we set $C_1 = C_1^a + C_1^b$ where:

$$C_1^a = r_p \int dv_p \int dw_p \int ds_p g_v\left(v_p, x, t\right) g_w\left(w_p, x+s_p, t\right) g_s\left(s_p, x, t\right) \frac{V_e\left(s_p\right)-v_p}{\tau}$$

$$C_1^b = -r_p \int dv_p \int_{w_p=0}^{v_p} dw_p \int ds_p g_v\left(v_p, x, t\right) g_w\left(w_p, x+s_p, t\right) g_s\left(s_p, x, t\right) \frac{v_p - w_p}{\tilde{\tau}} e^{-\left(s_p - s_p^0\right)/R}$$

It is straightforward to show that:

$$C_1^a = \frac{r_p}{\tau}\left(\int ds_p g_s\left(s_p, x, t\right) V_e\left(s_p\right) - V\right) \tag{19}$$

By definition: $\int g_s\left(s_p, x, t\right) ds_p = 1$ and $\int h\left(s_p\right) g_s\left(s_p, x, t\right) ds_p = h\left(\left\langle s_p \right\rangle\right)$. Let $\bar{s}$ denote the mean distance gap between two consecutive vehicles. A plausible assumption that the traffic density *r*(*x,t*) is a constant between *x* and *x*+$\bar{s}$ leads to: $\bar{s} = 1/r$. By using equation (5) and first order Taylor expansion we obtain the mean distance gap between platoon *p* and its leading platoon *p-1*:

$$\left\langle s_p \right\rangle = \left\langle s\left(x+L_p,t\right)\right\rangle = \left\langle s\left(x,t\right)+\left(m-1\right)\left(d_0+T_0 v_p\right)\frac{\partial s}{\partial x}\right\rangle = \frac{1}{r}-\frac{\left(m-1\right)\left(d_0+T_0 V\right)}{r^2}\frac{\partial r}{\partial x} \quad (20)$$

Applying equation (20) to the gap-dependent equilibrium speed gives:

$$\int ds_p g_s\left(s_p,x,t\right)V_e\left(s_p\right) = V_e\left(\left\langle s_p\right\rangle\right) = V_e\left(r\right)+\left(m-1\right)\left(d_0+T_0 V\right)\frac{dV_e\left(r\right)}{dr}\frac{\partial r}{\partial x} \quad (21)$$

So equation (19) turns to:

$$C_1^a = r_p\frac{V_e\left(r\right)-V}{\tau}+r_p\frac{\left(m-1\right)\left(d_0+T_0 V\right)}{\tau}\frac{dV_e\left(r\right)}{dr}\frac{\partial r}{\partial x} \quad (22)$$

Now let us expand $s_0^p(v_p)$ around $s_0^p(V)$ and substitute it to the function C_1^b:

$$\begin{aligned} C_1^b = -\frac{r_p}{\tilde{\tau}_p}\int dv_p\int_{w_p=0}^{v_p} dw_p\int ds_p g_v\left(v_p,x,t\right)g_w\left(w_p,x+s_p,t\right)g_s\left(s_p,x,t\right) \\ \left(v_p-w_p\right)e^{-\left(s_p-s_p^0\left(V_p\right)\right)/R}\frac{ds_p^0\left(V\right)}{dV}\frac{v_p-V}{R} \end{aligned} \quad (23)$$

By definition $s_0^p\left(V\right)=d_p+T_p V$ we have:

$$\begin{aligned} C_1^b = -\frac{r_p}{\tilde{\tau}_p}\frac{T}{R}e^{\left(\left\langle s_p\right\rangle-s_p^0(V)\right)/R}\underbrace{\int dv_p\int_{w_p=0}^{v_p} dw_p\int g_v\left(v_p,x,t\right)g_w\left(w_p,x+s_p,t\right)v_p\left(v_p-w_p\right)}_{I_1} \\ +\frac{r_p}{\tilde{\tau}_p}\frac{TV}{R}e^{\left(\left\langle s_p\right\rangle-s_p^0(V)\right)/R}\underbrace{\int dv_p\int_{w_p=0}^{v_p} dw_p\int g_v\left(v_p,x,t\right)g_w\left(w_p,x+s_p,t\right)\left(v_p-w_p\right)}_{I_2} \end{aligned} \quad (24)$$

Let us apply the first Taylor expansion to the exponential term:

$$e^{-\left(\left\langle s_p\right\rangle-s_p^0(V)\right)/R} = e^{-\left(1/r-s_p^0(V)\right)/R}\left(1+\frac{\left(m-1\right)\left(d_0+T_0 V\right)}{Rr^2}\frac{\partial r}{\partial x}\right) \quad (25)$$

Details of the exact calculations of the integrals I_1 and I_2 are described in Ngoduy and Wilson (2013). We only show in this paper the main results:

$$\begin{aligned} I_1 &= V_p\sqrt{\Theta+\Theta^+}N\left(z\right)+\left(V^2+\Theta-VV^+\right)\Phi\left(z\right) \\ I_2 &= \sqrt{\Theta+\Theta^+}N\left(z\right)+\left(V-V^+\right)\Phi\left(z\right) \end{aligned} \quad (26)$$

where V^+ and Θ^+ denote, respectively, the speed and speed variance of the platooner determined at advanced location $x+\left\langle s_p\right\rangle$, $z=\frac{V-V^+}{\sqrt{\Theta+\Theta^+}}$, $N\left(z\right)=\frac{e^{-z^2/2}}{\sqrt{2\pi}}$, and $\Phi\left(z\right)=\int_{-\infty}^{z} N\left(y\right)dy$. By substituting equation (26) into equation (24) we obtain:

$$C_1^b = -\frac{r_p}{\tilde{\tau}}\frac{T\Theta}{R}e^{-\left(1/r-s_p^0(V)\right)/R}\left(1+\frac{\left(m-1\right)\left(d_0+T_0 V\right)}{Rr^2}\frac{\partial r}{\partial x}\right)\Phi\left(z\right) \quad (27)$$

By substituting equations (22) and (27) into equation (18) we have:

$$\begin{aligned} \frac{\partial q_p}{\partial t}+\frac{\partial r_p\left(V^2+\Theta\right)}{\partial x} = r_p\frac{V_e\left(r\right)-V}{\tau}+r_p\frac{\left(m-1\right)\left(d_0+T_0 V\right)}{\tau}\frac{dV_e\left(r\right)}{dr}\frac{\partial r}{\partial x} \\ -\frac{r_p}{\tilde{\tau}}\frac{T\Theta}{R}e^{-\left(1/r-s_p^0\left(V_p\right)\right)/R}\left(1+\frac{\left(m-1\right)\left(d_0+T_0 V\right)}{Rr^2}\frac{\partial r}{\partial x}\right)\Phi\left(z\right) \end{aligned} \quad (28)$$

Further linearization of the error function $\Phi(z)$ leads to:

$$\Phi(z) = 0.5\left(1 + \frac{2z}{\sqrt{2\pi}}\right) \tag{29}$$

which is then substituted into equation (28) to obtain:

$$\begin{aligned}\frac{\partial q_p}{\partial t} + \frac{\partial r_p\left(V^2+\Theta\right)}{\partial x} &= r_p \frac{\tilde{V}_e\left(r,V,V^+,\Theta\right)-V}{\tau} + r_p \frac{(m-1)\left(d_0+T_0V\right)}{\tau}\frac{dV_e(r)}{dr}\frac{\partial r}{\partial x} \\ &- \frac{r_p}{2\tilde{\tau}}\frac{T\Theta}{R}\left(1+\frac{V-V^+}{\sqrt{\pi\Theta}}\right)e^{-\left(1/r-s_p^0(V)\right)/R}\frac{(m-1)\left(d_0+T_0V\right)}{Rr^2}\frac{\partial r}{\partial x}\end{aligned} \tag{30}$$

where $\tilde{V}_e\left(r,V,V^+\right) = V_e(r) - \frac{\tau}{2\tilde{\tau}}\frac{T\Theta}{R}\left(1+\frac{V-V^+}{\sqrt{\pi\Theta}}\right)e^{-\left(1/r-s_p^0(V)\right)/R}$.

By substituting equations (16) and (17) into equation (30) we obtain:

$$\begin{aligned}&\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = \frac{\tilde{V}_e\left(r,V,V^+\right)-V}{\tau} - \frac{\Theta}{m}\frac{\partial m}{\partial x} \\ &-\frac{1}{r}\left[\frac{dP}{dr} - \frac{(m-1)r\left(d_0+T_0V\right)}{\tau}\frac{dV_e(r)}{dr} + \frac{T\Theta}{2\tilde{\tau}}\left(1+\frac{V-V^+}{\sqrt{\pi\Theta}}\right)e^{-\left(1/r-s_p^0(V)\right)/R}\frac{(m-1)\left(d_0+T_0V\right)}{R^2 r}\right]\frac{\partial r}{\partial x}\end{aligned} \tag{31}$$

where $P = P(r) = r\Theta$ denotes traffic pressure term (Treiber et al., 1999; Treiber & Kesting, 2013). For the sake of simplicity, let us assume that $m = m(r)$, equation (31) turns to:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = \frac{\tilde{V}_e\left(r,V,V^+\right)-V}{\tau} - \frac{1}{r}\frac{d\tilde{P}}{dr}\frac{\partial r}{\partial x} \tag{32}$$

where

$$\frac{d\tilde{P}}{dr} = \frac{dP}{dr} + \frac{P}{m}\frac{dm}{dr} - \frac{(m-1)r\left(d_0+T_0V\right)}{\tau}\frac{dV_e(r)}{dr} + \frac{T\Theta}{2\tilde{\tau}}\left(1+\frac{V-V^+}{\sqrt{\pi\Theta}}\right)e^{-\left(1/r-s_p^0(V)\right)/R}\frac{(m-1)\left(d_0+T_0V\right)}{R^2 r}$$

This model equation is consistent with the general equation for a second order model presented in Helbing and Johansson (2009) and Treiber and Kesting (2013). It is worth noticing that, in our model, the finite deceleration time and the multi-anticipative behaviour both contribute to the modified traffic pressure terms and the density-and-speed dependent equilibrium speed. Since the safe speed-dependent distance (i.e. $s_0^p(V)$) and the relative speeds (through the Normal error function terms) are all considered in the model, our model will have non-local interaction properties.

MODEL PROPERTIES

This section investigates analytically the property of the introduced model to reflect the effects of multi-anticipations on traffic flow stability. In order to do so, we derive stability conditions based on the linear method for the introduced (macroscopic) model. The linear method refers to linear Taylor approximations, which are used throughout the analysis. The consequence of these approximations is that the conditions that are stable according to this analysis might actually still show non-linear instability. However, in general the linear analysis gives sound insights in the general behavior of the model in the presence of reaction time.

To follow the derivation in Helbing and Johansson (2009) and Treiber and Kesting (2013), we rewrite equation (32) in the following form:

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}=A\left(r,V,V^{+},\frac{\partial r}{\partial x}\right) \quad (33)$$

To perform the linear stability analysis let us consider small perturbations around the homogeneous and stationary solutions [r_e,V_e] (where $V_e = V_e(r_e)$, denoted by $\left[\delta r,\delta V\right]$:

$$r=r_e+\delta r, V=V_e+\delta V \quad (34)$$

Let these perturbations be determined as corresponding cosine functions with frequency λ and wave number ω as shown below:

$$\delta r=\delta r_0 e^{\lambda t+i\omega x}, \delta V=\delta V_0 e^{\lambda t+i\omega x} \quad (35)$$

where *i* denotes the imaginary unit, δr_0 and δV_0 are constants.

By substituting equations (34) and(35) into equations (16) and (33), performing the linear analysis as detailed in Helbing and Johansson (2009) and Treiber and Kesting (2013) we obtain the following linear stability condition:

$$\left(r_e\frac{dV_e}{dr}\right)^2-\frac{d\tilde{P}_e}{dr}+\frac{dV_e}{dr}\frac{\partial A}{\partial V^{+}}\le 0 \quad (36)$$

where $\frac{\partial A}{\partial V^{+}}=\frac{1}{\tau}\frac{\partial \tilde{V}_e}{\partial V^{+}}=\frac{1}{2\tilde{\tau}}\frac{T}{R}e^{-\left(1/r-s_p^0(V_e)\right)/R}\sqrt{\frac{\Theta_e}{\pi}}$

Let us graphically illustrate the derived linear stability conditions using the space headway dependent equilibrium speed formula in Helbing and Johansson (2009): $V_e(s)=\frac{V_0}{2}\left[\tanh\left(\frac{s-d_0}{s_c}-1.2\right)+\tanh(1.2)\right]$ where V_0 = 115km/h, s_c = 40m, d_0 = 4m. Other parameters are T_0 = 0.7sec, $\tau=3\,sec$, R = 75m, $\tilde{\tau}=0.75$ sec, d_p=20m, T_p=1.5sec, m = 4 (i.e. a constant for the analytical study). Note by definition: $V_e(r)\doteq V_e(1/r)=V_e(s)$ so equation (36) can easily be re-written in the following form:

$$\left(\frac{dV_e(s)}{ds}\right)^2+\frac{dP_e(s)}{ds}+\frac{(m-1)\left(d_0+T_0V_e(s)\right)}{\tau s}\frac{dV_e(s)}{ds} \\ +\frac{T\Theta_e}{2\tilde{\tau}}e^{-\left(s-s_p^0(V)\right)/R}\frac{(m-1)\left(d_0+T_0V_e(s)\right)s}{R^2}-\frac{1}{2\tilde{\tau}}\frac{T}{R}e^{-\left(s-s_p^0(V_e)\right)/R}\sqrt{\frac{\Theta_e}{\pi}}\frac{dV_e(s)}{ds}\le 0 \quad (37)$$

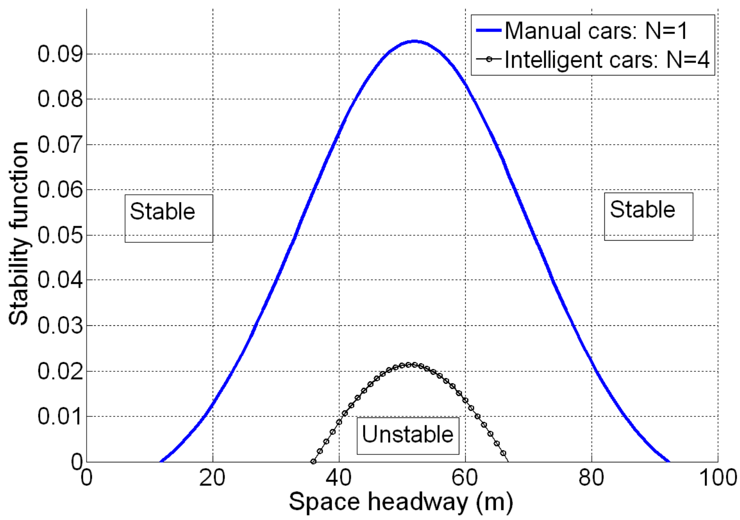

Figure 1. Stability diagram

Figure 1 shows the natural stability curves for the given parameters of the proposed model using the linear stability condition (37), where the stability function is measured by the left hand side of equation (37). Accordingly, traffic is linearly unstable for space headway range inside the stability curves and becomes linearly stable (either free-flow stable or congested stable) for space headway range outside the stability curves. This figure illustrates that platoon based operation of intelligent vehicles leads to stabilization of traffic flow.

We are currently working on the simulation of the model to investigate the effect of the platoon based operation on the capacity at bottlenecks and on the formation and dissipation of stop-and-go traffic jams on a freeway with an on-ramp. Such extensive simulation results will be presented at the conference.

CONCLUSIONS

Although much effort has been undertaken to develop microscopic models for intelligent traffic flow dynamics, there has been few advances in developing a model which can capture realistically the characteristics of such intelligent traffic flow. More specifically, in intelligent traffic flow, vehicles tend to move in a platoon in which the vehicles inside the platoon are moving rather close together and follow the driving behavior of the platoon leader. Therefore, intelligent traffic flow is considered to contain many platoons moving together, rather than many individual vehicles moving together. This paper has presented a new approach to develop a platoon based macroscopic model to describe more realistically such driving behavior. Particularly, our model has been developed from a gas-kinetic model in which the platoon based driving behaviour is explicitly described through an extended (microscopic) generalized force model. The developed model is non-local in its derivation process (i.e. thank to the speed-dependent safe distance and Normal error function terms) and has a structure similar to a generalized macroscopic model (Helbing and Johansson, 2009, Treiber and Kesting, 2013) with modified pressure terms accounting for the finite deceleration time and platoon effect so it could also capture the transitions between meta-stability and the stop-and-go waves in real traffic flow. The proposed methodology could be generalized to the *multi-class* model to investigate the effect of the intelligent vehicles which allow for wider anticipations with roadside infrastructure. Linear stability analysis is performed to show that platoon based operation leads to stabilization of traffic flow. Nevertheless, it would still need further real-life experiments to establish a unique and calibrated/validated model performance. This calibration/ validation work will be left in our future research.

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