CRITICAL ASSESSMENT AND BENCHMARKING OF TRAIN TIMETABLING METHODS

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ABSTRACT

This paper performs an analysis and comparative assessment of the most important methods for train timetabling. The primary objective is to identify the advantages and disadvantages of existing methods and to investigate their appropriateness for the different fields of real-time railway optimisation operations.
To assess the various timetabling methods a survey was carried out among Infrastructure Managers (IMs) in Europe. The feedback from the survey along with the results of a state-of-art analysis has led to the identification of the most important railway timetabling methods. These have been catalogued according to their technical, usage and managerial properties.
The methods have then been evaluated using a set of criteria, including: plugability, transparency, computing time, data requirements, etc. It shows from the results that timetabling methods aim to optimise nominal usage of network and level of service, while satisfying a given set of operational constraints (a complex set of do-s and don't-s that reflects rules, regulations, limits induced by infrastructure, etc). They also take special provisions to ensure robustness of the produced timetable.
Various methods have been used to optimise timetables, including: Constraint Programming, Mixed Integer Programming, Greedy Algorithms, Integer Linear Programming, Stochastic Programming, etc. The characteristics of these methods are presented in a way that allows comparative conclusions, i.e. macroscopic vs. microscopic, path vs. route level, time slots vs. speed profiles, periodic vs. aperiodic, deterministic vs. stochastic, etc.
The paper provides evidence on the appropriateness of methods and algorithms for the development of timetables, containing margins and allowances that are the minimum needed for realistic management. It contributes to research on integration of timetable planning and railway control & optimisation. As a result, the whitespace in the timetable, the trainpaths and

engineering access are optimised, thereby maximising the network capacity that can be used for train services.

Keywords: train timetabling, train scheduling

1. INTRODUCTION

Given a set of railway stations, a train service intention, (i.e. a set of train lines, frequencies and stopping patterns, sometimes also departure and arrival time windows of trains at stations) and a set of resources, the Train Timetabling Problem (TTP) consists of assigning a departure time and an arrival time to each train at each station, with respect to operational and safety constraints.

In the literature, train timetabling problems are usually classified as follows:

- According to the level of detail that is considered when representing the topology of the railway network and the train movements, two variants can be distinguished: the first one, known as macroscopic timetabling, represents the railway network as a graph, where stations correspond to nodes and tracks to arcs of the graph. In addition, the average movements per train group are considered. The second one, on the other hand, considers an exact reproduction of the infrastructure network and train operation, furnishing accurate railway traffic information and is known as microscopic timetabling.

- According to the main goal of the timetabling, two classes of problems can be considered: the nominal problem and the robust one. The nominal variant aims to find a schedule that guarantees the maximal efficiency of the railway system, defined as satisfaction of constraints given by capacities of the network and security measures, with at the same time potential optimisation of a certain measure of performance (e.g. minimisation of the total travel time for the trains, minimisation of the total waiting time for transit passengers, etc.). The robust variant on the contrary, aims to find a schedule that avoids, in case a small disruption occurs, the propagation of delays, absorbing them as quickly as possible.

- According to the periodicity of train departures and arrivals, the cyclic version of the problem (or periodic) and its non-cyclic one (or aperiodic) are distinguished. In the former case train departures and arrivals are repeated every given time period, while in the latter one every single train is scheduled individually. It has to be mentioned that non-cyclic timetables belong indeed to the periodic family, as for these timetables it can be considered that their periodicity is equal to one day. This means that, generally speaking, the models of the cyclic version can be adapted to the non-cyclic one and vice versa. The distinction between the two versions is mainly due to the different models that traditionally have been used when dealing with a short cycle time with respect to the case of the one-day period [Cacchiani and Toth, 2012].

- According to the level of uncertainty of the input data, two categories can be considered: the deterministic TTP, where all problem data are supposed to be known with certainty in advance, and the stochastic one, which takes under consideration the fact that parts of the input data may be subject to random fluctuations that become known only upon their completion.

- Finally, according to the objective function to be optimised (in case one exists), TTP can be classified as customer-oriented (e.g. when the objective is the minimisation of the total waiting time for transit passengers), train operating company-oriented (e.g. minimisation of train operation times, waiting times or delays) or infrastructure manager-oriented (e.g. minimisation of costs on infrastructure investments). Often, a combination of the above objectives is envisaged.

This paper is structured as follows: in Section 2, a literature review is being conducted and the main timetabling methods are presented, classified into the above analysed categories. Section 3 attempts an evaluation and comparative assessment of the overviewed methods and finally Section 4 draws the main conclusions of the paper.

2. REVIEW OF TIMETABLING METHODS

This Section contains a literature review of the main train timetabling methods, classified as shown in Table 1. It should be mentioned that, as most of the formulations and algorithms considering the nominal version of the TTP are of deterministic nature and most of those considering its robust version are of stochastic one, this type of classification is not determinant. Furthermore, throughout the literature review it has been noticed that the assumed objective function does not affect the solution approach; therefore, neither this classification criterion is taken into account.

Table 1: Timetabling method classification typology

<table>
<tr><th></th><th>Macroscopic</th><th>Microscopic</th><th>Macro-microscopic interaction</th></tr>
<tr><td rowspan="2">Nominal</td><td>Cyclic</td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>Non-cyclic</td></tr>
<tr><td rowspan="5">Robust</td><td>Stochastic Programming</td><td rowspan="5"></td><td rowspan="5"></td></tr>
<tr><td>Light Robustness</td></tr>
<tr><td>Recoverable Robustness</td></tr>
<tr><td>Bi-criteria – Lagrangian Approaches</td></tr>
<tr><td>Meta-heuristics</td></tr>
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2.1 The nominal TTP

2.1.1 Macroscopic

2.1.1.1 Cyclic

Most of the methods dealing with the cyclic timetabling are based on the so-called Periodic Event Scheduling Problem, as introduced by [Serafini and Ukovich, 1989]. In PESP, events (i.e. departure and arrival times of trains at stations) are repeated every given time period and one has to find a feasible schedule subject to periodic-interval safety and operational constraints.

[Schrijver and Steenbeek, 1994] presented an algorithm for solving the PESP, based on constraint propagation and applied it to real-world instances of the Netherlands Railways. They also developed local search techniques to improve a feasible solution.

[Odijk, 1996] proposed a cutting plane algorithm based on the so-called cycle cuts. The ILP model proposed by [Schrijver and Steenbeek, 1994] and by [Odijk, 1996] is simple but rather weak, as it presents continuous (bounded) variables representing arrival and departure times (modulo the period) and binary variables expressing the order of the train arrivals and departures at the stations [Cacchiani and Toth, 2012]. Furthermore, in general, constraint programming suits better to feasibility rather than to optimisation problems.

[Liebchen and Möhring, 2007] proposed to integrate the decisions of network planning, line planning and vehicle scheduling into the task of symmetric periodic timetabling (i.e. a timetable where opposite directions meet at time 0 and T/2, T being the period), modelling the problem as a PESP instance, where symmetry is introduced in its MIP formulation by adding (extra) tension variables and non-PESP constraints. The problem is solved with a commercial LP solver, with strong branching as variable selection strategy and aggressive cut generation. It appeared that the notion of timetable symmetry is not compatible to the PESP; although introducing symmetry speeds up the optimisation process, the feasible solution for the symmetric instance is worse than the feasible solution for the more general (periodic) problem; hence, this speed-up might not serve as a heuristic for quick generation of good (symmetric) solutions for the general problem.

[Engelhardt-Funke and Kolonko, 2004] developed an adaptive multi-objective genetic algorithm enhanced by additional features, trying to integrate delay management in TTP. The considered cost functions include the total scheduled passengers' waiting time, their mean actual waiting time under random delays and the cost for the infrastructure upgrade. The presented approach produces Pareto-optimal timetables with respect to complex cost criteria, yields cost-benefit analyses between the cost functions and allows estimation of the 'turning point' for the synchronisation of the timetables from which on a mere reduction of waiting time decreases the stability of the timetables under random delays. The main drawbacks are that, if waiting time is not well defined, it can reward delays and also that the evaluation is not based on an optimal delay management and solely relies on a limited number of scenarios.

[Nachtigall and Voget, 1996] presented an algorithm that solves the PESP, with the objective to minimise the total weighted waiting time for passengers changing trains, by variation of train departure times. The algorithm, called 'localgen', is a combination of a greedy heuristic and a local improvement procedure with a genetic algorithm. The greedy algorithm produces good solutions very quickly while the pure GA combines good parts of randomly produced solutions which are only improved gradually; localgen combines advantages of both greedy and GA. However, most solutions of the greedy algorithm are sub-optimal and for a genetic algorithm it may be more difficult to optimise on the restricted set of local minima than to optimise on the complete search space. This procedure also depends heavily on the time precision that was chosen for the computation.

[Nachtigall and Voget, 1997] modelled the TTP as a PESP with the objective to minimise the weighted waiting time for passengers changing trains, as well as the investigation costs for reforming the state of certain track segments, which is susceptible to enable a reduction in the trains' running times thus contribute to a further reduction of passengers' waiting time. A hybrid genetic algorithm incorporating a greedy starting method and a local improvement algorithm is introduced and, to take into account the cost-benefit between the two criteria, a fuzzy logic controller is developed in order to control the modifications of the fitness function in each generation. The resulting parametric MIP for minimising building costs for every possible value of the synchronisation degree is difficult to solve so only lower bounds are calculated, using cutting planes. Tests on real-world data showed that the greedy algorithm generates good solutions with only a little amount of computation time.

[Domschke, 1989] presented an approach that minimises total passengers' weighted waiting time. For every railway line running with a certain period and every possible departure time, a binary variable determines if that route departs at that specific time point. The resulting optimisation task is a 0,1-programming problem with quadratic objective which is similar to a quadratic assignment problem. Solution methods include an exact B&B (which however is efficient only for problems of small size), a starting regret-heuristic with different modifications and a solution approach based on simulated annealing.

2.1.1.2 Non-cyclic

[Szpigel, 1973] presented a job-shop scheduling formulation for the aperiodic version of the TTP on a single track line, in which trains (regarded as jobs) are to be scheduled on track sections (regarded as resources), with the goal of minimising the weighted average train travel time. The author presented an exact branch-and-bound algorithm that was applied on small instances.

[Jovanovic and Harker, 1991] considered the construction of feasible timetables and pass plans, focusing on robustness against travel time randomness. The problem was formulated as a MILP with binary variables defining the order of the trains at the meet-points (i.e., the points where overtaking is allowed) and continuous variables for the arrival and departure times. An exact branch-and-bound algorithm was designed, where each level of the decision tree corresponds to the resolution of a conflict between a pair of trains and the decision nodes correspond to meet-points that can be used to resolve a conflict.

[Cai and Goh, 1994] developed a constructive heuristic algorithm based on local optimality criteria for scheduling trains on a single track line, where they are allowed to cross only at one of the passing loops, with the objective to minimise the total cost due to train stopping and waiting in the passing loops. The problem is formulated as a 0, 1 integer program, where a binary variable determines if a train stops or not at a passing loop. The heuristic algorithm computes in very quick time a feasible solution, which however is unlikely to be globally optimal.

A further extension of the approach of [Cai and Goh, 1994] is presented in [Cai et al., 1998], where the requirement that the initial location for a train be at a siding is relaxed. The authors develop a two-step approach with the first phase updating the current time and position (the so-called Position-Time-Pairs) so that all trains are positioned on some stations. The second phase then implements a refined version of the greedy heuristic presented in [Cai and Goh, 1994]. The algorithm is quick and easily adaptive to new constraints.

[Carey and Lockwood, 1995] proposed a heuristic algorithm for determining train paths along single unidirectional railway lines, with the objective to minimise the cost due to deviation from the desired departure, arrival, trip and/or dwell time. The problem is modelled as a 0-1 MIP, where binary variables govern the train order and continuous variables represent arrival and departure times of trains at stations, solved by B&B with branching decisions made on the link variables that specify the sequence order of the trains. The approach is extensible to more general networks (e.g. multiple lines or platforms, route choice), able to incorporate supply-demand interaction and well suited to adding to or modifying existing timetables incrementally. On the other hand, the obtained solution may not be optimal, as it can depend on the order in which trains are selected for pathing.

[Higgins, Kozan and Ferreira, 1997] studied the single line train scheduling problem with the objective to minimise the total weighted travel time for all trains. Binary variables determine train order and continuous ones arrival and departure times. The authors proposed and compared different heuristic methods, namely local search, genetic algorithm, tabu search and two hybrid heuristics (a combination of local search and genetic algorithm and a combination of tabu search and genetic algorithm).

[Oliveira and Smith, 2000] formulated the TTP as a job-shop scheduling problem and solved it by applying Constraint Programming techniques, with the objective of minimising the total delay, while resolve conflicts in chronological order. CP is a promising alternative to MIP for solving real-world instances, as it allows one to describe easily more complex constraints without needing change in the procedure to solve the problem. On the other hand the method does not allow delaying part of the trip to resolve a conflict and also, when solving conflicts, the process does not take into account the global influence of each individual decision. A faster algorithm can be devised and used specifically to find a good initial feasible solution and the presented algorithm could then search for a better solution.

[Caprara, Fischetti and Toth, 2002] described a Langrangian-based constructive heuristic for the train scheduling problem, with the objective to maximise the sum of profits of scheduled trains on a corridor, satisfying operational constraints. The problem is formulated as a time-space directed acyclic multigraph where nodes represent departures/arrivals at a certain station at a given time instant and paths represent feasible timetables. This formulation is used to derive an integer linear programming model, where a binary variable governs whether a certain arc is selected in an optimal solution that is relaxed in a Lagrangian way (combined with subgradient optimisation). The relaxation is then embedded within a heuristic algorithm. Tests on real-world instances showed that finding the best solution is time consuming, although solutions of comparable quality can be obtained in a shorter running time; also, this is a more manageable model than the classical Lagrangian approach.

[Caprara, Monaci, Toth and Guida, 2006] considered a stronger mathematical formulation of the problem as presented in [Caprara et al., 2002], under the assumption that the travel time of each train along each track segment joining two stations is fixed and coincides with that of the ideal timetable. Tests on real-world instances showed that the best solution value found by the heuristic procedure described in [Caprara et al., 2002] is marginally affected by this additional constraint, whereas the corresponding running time is widely reduced.

[Mistry and Kwan, 2003] presented a cooperative coevolutionary algorithm for the automatic generation of the timetables, the idea being the independent, parallel representation and evolution of the problem subcomponents that interact in useful ways to optimise complex higher level structures. Tests on random data showed that the technique is promising, as it facilitates an efficient concentrated exploration of the search space and produces good quality timetables.

[Semet and Schoenauer, 2005] presented an effective memetic evolutionary algorithm for real-world train timetabling, where the main focus is on the reconstruction of the schedule following a small perturbation, with the aim of minimising the total delay. The evolutionary part of the algorithm is used to quickly obtain a good but suboptimal solution, representing the order in which trains should be allowed to use available resources. The best individual in the population after K generations is then fed to a commercial MIP solver as a starting point for its search for the global optimum. The scheduler then reads the permutation of trains and tries to greedily place them in the schedule, node by node, respecting constraints. This method combines the small computation time of the evolutionary algorithms with the capacity of the solver to find optimal solutions. However, optimality is enforced only at the node level as opposed to greedy stricto sensu (which would mean optimal at the train level).

[Borndoerfer, Groetschel, Lukac, Mitusch, Schlechte, Schultz and Tanner, 2005] presented an approach to implement a multi-round combinatorial auction for the simultaneous allocation of interdependent railway slots in an open and competitive market. The optimal track allocation problem that arises is solved at each 2-stage round: firstly, each Train Operator submits a set of bids and then the infrastructure manager computes the set of bids that are accepted in this round, by determining a conflict-free slot schedule that maximises the network profits. The track allocation problem is formulated as a multi-commodity flow problem, where commodities correspond to the bids. The formulation is equivalent to that of [Caprara et al., 2002], with binary variables governing the allocation of arcs to bids. The authors also present an extension of this initial formulation which includes several constraints representing practical restrictions. A commercial IP solver was used to solve this model. The combinatorial auction enables avoiding inefficient allocations, which constitutes an advantage over the highest price procedure.

[Borndoerfer and Schlechte, 2007] proposed two different ILP models for the problem of finding conflict-free train routes in a railway network. The first one is a packing model with flow conservation and clique constraints, whose LP-relaxation can be solved in polynomial time. The second one, based on the concept of feasible arc configurations, guarantees a conflict-free routing by allowing only feasible route combinations instead of excluding the infeasible ones and is amenable to standard column generation techniques, thus well suited to solve large-scale instances.

[Vansteenwegen and Van Oudheusden, 2006] studied the problem of improving passenger service by designing and minimising a waiting cost function that takes into account waiting times and delays. In the first phase of the approach, ideal buffer times are calculated to safeguard connections when the arriving train is late and, in the second phase, standard linear programming is used to construct an improved timetable with well-scheduled connections and, whenever possible, with ideal buffer times. Simulation compares different timetables and optimises the LP timetable. Computational results showed that the technique is promising - even for very extensive railway networks.

Considering the same ILP formulation as [Caprara, Fischetti and Toth, 2002], [Cacchiani, Caprara and Toth, 2008] proposed constructive and local search heuristics and an exact branch-and-cut-and-price-algorithm based on the solution of the LP relaxation of the ILP formulation. Experimental results on real-world instances show that the proposed approach is capable of producing heuristic solutions of better quality than those obtained by the previous approaches and of solving some small-size instances to proven optimality. On the other hand, the corresponding solution times are much larger, which seems to indicate that the Lagrangian approach is preferable for practical purposes.

[Cacchiani, Caprara and Toth, 2010] studied the problem of scheduling as many new freight trains as possible on a railway network, in which passenger trains have already a prescribed timetable that cannot be changed. An ILP formulation is presented and the problem is solved by using a heuristic algorithm based on a Lagrangian relaxation of the track capacity constraints within a subgradient optimisation framework (see [Caprara et al., 2002]). Computational results prove that the method is capable of dealing with fairly large instances within computing times that are acceptable in a planning environment, allowing one to test several different scenarios. The method can be used to build new timetables from scratch, to add new freight or passenger ones) to a prescribed timetable or to reschedule trains in case of delays. In addition, the model and the method can be extended to schedule trains inside a railway node; in this case, the node can be seen as a network with alternative paths.

[Kraay et al., 1991] studied a different version of the problem, which they call train pacing problem, consisting in determining the velocity profile for each train and a feasible meet/pass plan for the line which simultaneously minimises the weighted sum of the objective functions for all trains (fuel consumption, delay, etc.). The authors present a mixed integer nonlinear program for the problem. A branch-and-cut algorithm is proposed to construct a feasible meet/pass plan. The model can, however, take as input any feasible meet/pass plan and in this case the objective is to optimise the train velocities, through the solution of the nonlinear program with the integer variables held fixed. This approach presents important advantages, as it can incorporate very complex constraints and can also be used to evaluate and rank different scenarios. Finally, the authors proposed a rounding heuristic to filter out meet/pass plans and retain only those closest to the optimal solution obtained when ignoring train interactions. Results on instances of a major railroad produced fuel savings in the order of 5%, while standard deviation in train arrival times decreased by more than 19%.

[Salido et al., 2007 (ARR-TR-0077)] modelled the TTP as a Distributed Constraint Satisfaction Problem, with the objective to minimise the journey time of all trains on a single-track line. Whereas a conventional constraint satisfaction model aims to solve the problem with respect to some constraints (including user requirements, traffic rules and topological constraints), the distributed model is developed to distribute the resultant CSP's variables and constraints among independent automated agents, named block agents. Each agent has some variables and attempts to determine their values, using any algorithm he wants. However, there are interagent constraints that the value assignment must satisfy. The main advantage of the distributed models is that they divide the problem into a set of simpler interconnected sub-problems which can be more easily solved and they enhance privacy and security. The distribution can be domain-dependent or not. The CSP are then solved by a CSP solver, called Forward Checking. Tests on random data as well as benchmark problems showed that the domain-dependent distributed model was more efficient than the general one and that the general distributed models had a better behaviour than the centralised model. One critical issue is that, as the size of the partition turned out to be an important factor for the distributed model (a large number of partitions leads to smaller computation times but increases the number of binary constraints), it is necessary to built up a formal relation between the railway topology and the appropriate number of partitions.

[Barber et al., 2009] presented a meta-heuristic approach based on variable ordering for scheduling new trains on a single line occupied by trains in circulation and whose timetables cannot be modified, with the objective to minimise average delay of new trains with respect to their optimum. The problem is modelled as a search tree and the objective is to find a path from the initial node to a final node, so that the order of priorities established by this path produces the minimum average delay. This technique finds optimised solutions with very low computational times.

[Ingolotti et al., 2006 (ARR-TR-0036)] presented a Constraint Satisfaction and Optimisation Problem solved by a Scheduling Order-Based Method for scheduling new trains on a line occupied by trains in circulation and whose timetables cannot be modified. The quality of each solution can then be measured according to other criteria (e.g. average delay, deviation between average delay in the two directions for the new trains, etc.). The approach can be applied to any railway line and does not require a specific configuration in the railway infrastructure; also, the set of constraints can be modified without affecting the solving process. However, the presented technique is conservative because it does not risk a solution until it is sure that it will not be better than the best one obtained up to this point. Also, the order of train selection determines the priority among the trains and the way that each conflict between two new trains will be solved.

[Lova et al., 2007 (ARR-TR-0086)] solve the same problem as [Ingolotti et al., 2006] of scheduling new trains on a heterogeneous high-loaded network, formulated as a Constraint Satisfaction Problem using a different algorithm. The solving method carries out the search assigning values to variables in a given order verifying the satisfaction of constraints where these are involved. When a constraint is not satisfied, a guided backtracking is done. Finally, the resulting timetable is delivered to the user who can interact with it, guaranteeing the traffic constraint satisfaction.

[Tormos et al., 2008 (ARR-TR-0081)] presented a genetic algorithm based on a Job-Shop approach. Each train can be decomposed in a set of ordered Train-track section (T-ts) that has to verify a set of time and resource constraints and optimise a measure of performance with the lowest computational effort. The huge search space to explore when solving real-world instances makes GAs a suitable approach to efficiently solve TTP. The GA approach proposed in this paper might be improved with the use of local search, able to intensify performance around promising regions of local optima.

2.1.2 Microscopic

[Brännlund et al., 1998] presented a novel approach for train timetabling, having as objective the maximisation of the profits of the scheduled trains, which depend on their departure time and waiting time along the track. The problem is modelled as an integer programming problem, where time is discretised in one-minute intervals and upper bounds are obtained by a Lagrangian relaxation of the constraints and by solving, with a dynamic programming recursion, a shortest path problem for each train. Feasible solutions for the original problem are obtained by properly modifying the solutions of the Lagrangian problem. Computational results showed that the approach is able to find feasible solutions within a few percentage of optimality, with rather modest computational times.

[Caimi et al., 2009] proposed a decomposition method for generating conflict-free timetables. The whole network is decomposed into condensation zones (small areas in the vicinity of main stations with complex topology and high train frequencies) and compensation zones (connect condensation zones and have simple topologies with lower train frequencies). The authors focus on the scheduling problem in condensation zones. The problem is modelled as an independent set problem, which is solved by an algorithm specially developed to solve Constrained Semi-Assignment Problems, using a fixed-point iteration heuristic. The procedure does not guarantee to maintain the feasibility of the scheduling problem, but computational results show that feasible timetables can be generated in less than a minute. Two critical points of this approach are, from one hand to find a balance between adequate distribution of slack time in space and time and timetable stability and, from the other hand, the coordination between the condensation and compensation zones, i.e. to find suitable boundary conditions that allow finding feasible schedules for all condensation and compensation zones in a network simultaneously.

[Caimi, Chudak, Fuchsberger, Laumanns and Zenklusen, 2011] modelled the problem of generating conflict-free train schedules on a microscopic model of the railway infrastructure as an integer linear programming problem that explicitly considers at which track sections conflicts occur together with the temporal relation between conflicts occurring at the same place. The resource-tree conflict graph model is developed, which describes all timing and routing alternatives for each train movement in a tree structure and uses conflict cliques to prevent simultaneous blocking of resources (block sections or sections of an interlocked route) by different train paths. It was shown that the maximal cliques can be determined efficiently for each resource and the number of maximal cliques is bounded by the number of considered train paths. The resulting maximum clique integer linear programming formulation is very compact and, because of its strong linear relaxation is very quick to solve, even for large problem instances. Thus, the resource-tree conflict graph model gives a substantial improvement compared to the standard conflict graph formulation, which works with a quadratic number of (pairwise) conflicts and is known to have a weak linear relaxation in general. Computational results on real-world instances showed that the resource-tree conflict graph model outperforms the classic conflict graph model in terms of computing time thanks to the reduced number of constraints and the stronger linear relaxations. The difference between the models becomes more important as the size of instances grows.

[Medeossi et al., 2011] present a method for introducing stochastic blocking times to improve timetable planning in the blocking time model. The approach redefines timetable conflicts by associating a probability with each conflict estimated as a function of process-time variability. A motion equation based on on-board train data is calibrated with different parameters, which are set using a simulated annealing optimisation algorithm. The simulated annealing algorithm is robust, suitable for non-filtered data and provides good quality solutions with a minimum of computing effort.

2.1.3 Macro-microscopic interaction

[Caimi, Fuchsberger, Laumanns and Schuepbach, 2011] address the problem of generating conflict-free periodic train timetables for large railway networks. A two-level approach is followed: firstly, a simplified track topology is used to obtain a macrolevel schedule and then the detailed topology is considered locally on the microlevel. The authors propose an extension of the PESP model that allows generating flexible time slots for the departure and arrival times (lower and upper bounds as new decision variables) instead of exact times. A further generalisation of the Flexible PESP, called Flexbox, makes use of natural dependencies between the events to increase the chance of getting a feasible solution in the microscopic level. The problem can be formulated as a Cycle Periodicity Formulation (CPF) with an integer cycle basis, which is simple and gives good results in many cases. For the present case, it is reported that the CPF formulation with a good cycle basis is more powerful than the original PESP, with only moderately larger computational effort. This flexible periodic event scheduling problem formulation increases the chance to obtain feasible solutions on the microscopic level, particularly in stations with dense peak traffic; also, flexibility helps overcome delay propagation in the network.

[Schlechte et al., 2011] presented an algorithmic bottom-up approach to transform a microscopic railway network to an aggregated macroscopic network model and back. The algorithm starts from a detailed microscopic level as it is used in railway simulation, the network is aggregated to a macroscopic one, sufficient for long-term planning and optimisation and then trains from a given set of requests are added to the existing timetable by solving an optimal train path allocation problem based on time discretisation, in such a way that their sum of utilities is maximised. The optimised schedule is re-transformed back to the microscopic level so that it can be simulated without any conflicts between train paths. The problem is modelled as MIP or LP.

[Lee and Chen, 2009] developed an optimisation heuristic for solving simultaneously the train pathing and train timetabling problem, given a single-track railroad system and a set of services. The optimisation goal is to let the trains depart as close to their target departure time as possible, minimising the operation times of services. Firstly, the heuristic generates a feasible (but unlikely optimal) initial solution, by running the services sequentially, according to the order of their target departure time and afterwards uses a four-step process to derive the solution iteratively. The heuristic produced a timetable that is at least as good as the real schedule when tested on real-world instances. On the contrary, the model uses time-separation for consecutive trains; however, space-separation is also needed, to ensure that a block is cleared before the train reaches the braking distance when approaching it.

2.2 The robust TTP

2.2.1 Stochastic Programming

The Stochastic Programming approach (SP) is a framework for optimisation under uncertainty. In particular, 2-stage Stochastic Programming features a scenario-independent first stage decision (deterministic part of the model) and for each scenario a second stage decision, which is taken after the full, precise data is known and which usually comes at a higher cost (stochastic part of the model). Together the first and the second stage decision must form a feasible solution to the scenario. The 2-stage Stochastic Program optimises a mixed objective, summing the deterministic first stage cost and the expected value of the second stage cost. Sometimes the expectation is replaced by a more sophisticated stochastic function including some risk measure. In all cases the scenario set is assumed to be endowed with a probability distribution [Stiller, 2008].

[Kroon, et al., 2007 (ARR-TR-0031)] presented a Stochastic Optimisation model for improving the robustness of a given cyclic railway timetable against stochastic disturbances, by allocating running time supplements and buffer times to the initial timetable, without modifying its structure. The model is composed of two parts: a timetabling part (which has many similarities with the PESP) and a simulation part for evaluating the robustness of the timetable under construction. Computations on real-world instances resulted to more robust timetables with respect to the original ones when facing small disturbances.

[Fischetti, Salvagnin and Zanette, 2009] presented different approaches to find robust solutions to the non-cyclic TTP. The nominal problem is modelled by adapting PESP to the non-cyclic case [Caprara et al., 2002] and then treated and evaluated within a robustness framework. The authors presented two different stochastic models for obtaining robust timetables, the so-called “fat” and “slim”. The “fat” stochastic model is a standard scenario-based approach, in which time instants of each event are viewed as decision variables to be optimised. The model keeps a copy of the original (linear) model with a modified right-hand-side for each scenario, along with the original model. For realistic instances and number of scenarios this model turns out to be very time consuming (if not impossible) to solve. Therefore, a “slim” model is proposed, with a smaller number of variables that leads to faster solutions, whose robustness is comparable to those of the “fat” model. The robust solutions are then evaluated by a validation tool that estimates the total cumulative delay through simulation under random scenarios of small disruptions and eventually adjusts the robust solution to make it feasible under the examined scenario.

2.2.2 Light Robustness

[Fischetti and Monaci, 2009] presented the notion of Light Robustness (LR), which couples robust optimisation with a simplified two-stage Stochastic Programming approach. It consists of fixing a maximum deterioration of the objective function of the nominal solution and a robustness goal to be achieved and then the problem is modelled by using a classical robust optimisation framework. In his way a robust model with no objective function is obtained, that however is likely to be infeasible. To cope with infeasibility, appropriate slack variables are introduced, allowing for local violations of the robustness requirements and an auxiliary objective function is defined, aimed to minimise the slacks. Slack variables play a role similar to second-stage recourse variables in SP models, as they penalise the corrective actions needed to restore feasibility. In the case of the TTP, this approach was implemented on real-world instances by [Fischetti et al., 2009] and then compared to Stochastic Programming methods (described previously); globally, the method produced good quality solutions when dealing with a reasonable robustness–efficiency trade-off, though with less effort in terms of model formulation and solution time, which makes it appropriate to attack large instances.

2.2.3 Recoverable Robustness

[Liebchen, Lubbecke, Möhring and Stiller, 2007] presented a new concept for optimisation under uncertainty, called Recoverable Robustness (RR), which combines the flexibility of Stochastic Programming with the tractability and performances guarantee of the classical robust approach. As SP is intractable for large instances and robust optimisation gives over-conservative solutions in the TTP context, it is necessary to compute solutions that are robust against a limited set of scenarios and which can be made feasible (recovered) by a limited effort in case a disturbance occurs. One starts from a feasible solution x of an optimisation problem (any version of the deterministic TTP), which a particular scenario s, that introduces imperfect knowledge may turn to infeasible (e.g. uncertainty in the time needed for driving and stopping that can produce a small source delay). The goal is to have handy a recovery algorithm A that takes x and turns it to a feasible solution under s. In other words, in RR there is uncertainty about the feasibility space: imperfect information generates infeasibility and one strives to (re-)achieve feasibility.

[Cicerone et al., 2009] considered the problem of designing recoverable robust timetables subject to bounded delays, with the objective to minimise the total travel time for all passengers, which is then treated within a RR framework, such that a delay on a single activity can affect only a limited number of subsequent events. The authors presented two robust algorithms based on the Critical Path Method for assigning slack times to activities and indicated their optimality fields. The quality of a robust timetable was measured in terms of price of robustness, defined as the ratio between the cost of the recoverable robust solution and that of a non-robust optimal one.

[D'Angelo, Di Stefano and Navarra, 2008 (ARR-TR-0163)] considered the case of planning robust timetables when the input event activity network topology is a tree, with the objective to minimise the total weighted time for all events of the DAG, where the nodes represent events (e.g. arrival or departures of trains) and the arcs represent the activities (e.g. waiting in a train, driving or changing for another train). The algorithm assigns slack times, ensuring that if a delay occurs, no more than Δ activities are affected by the propagation of such a delay. The problem can be solved in pseudo-polynomial time when the maximum number of affected activities is fixed a priori.

[Cicerone et al., 2012] extended the concept of Recoverable Robustness to deal with arbitrarily many recovery steps, having as limitation either the number of events whose scheduled times might be changed during the recovery with respect to the initial timetable or the sum of deviations of the events in the recovered timetable with respect to the initial one. Recoverable robust timetable means that an initial robust solution should not only be recoverable against the first disturbance but the recovered solution should again be recoverable against the next disturbance which may result in another instance, and so on. This means that under all solutions which hedge against a first disturbance, one should choose a solution that is again robust against the next disturbance, and so on. The procedure firstly finds a slack times assignment that ensures robustness for an initial instance and then solves the instance of the non-robust problem by means of algorithms which assign slack time to the minimal duration needed for the completion of activities. These algorithms can be solved by LP or, in special cases by the Critical Path Method.

2.2.4 Bi-criteria Lagrangian approaches

As efficiency and robustness are two objectives in opposition, a trade-off has to be made. Bi-criteria approaches take this fact into consideration, by an appropriate adjustment of the problem's objective function.

[Schöbel and Kratz, 2009] treated an optimisation problem as a bi-criteria problem, adding the robustness of its solution, defined as the largest possible delay such that all transfers are maintained under some given waiting time rule, as an additional objective function. The approach is applied successfully to the aperiodic TTP (with the nominal objective being to minimise the overall travelling time for passengers) and necessary conditions for the resulting Pareto-optimal timetables are derived.

[Borndoerfer and Schlechte, 2008] presented extensions of the integer programming formulation proposed in [Borndoerfer and Schlechte, 2007] for solving the TTP, in order to incorporate the robustness requirement. The two objectives are the maximisation of the number of trains in the schedule and the maximisation of the schedule's robustness. The models are solved through column generation techniques.

Another approach in the direction of finding robust yet efficient solutions to optimisation problems is the use of Lagrangian heuristics. These methods approximately solve a relaxation of the considered problem through an iterative Lagrangian optimisation scheme and apply several times a basic heuristic driven by the Lagrangian dual information (typically, the current Lagrangian costs) as to hopefully update the current best feasible solution. This approach is applied by [Cacchiani, Caprara and Fischetti, 2012] on the case of the non-cyclic TTP, modified by two simple features: (a) the problem formulation is modified by introducing artificial parameters intended to control the solution robustness and (b) during the Lagrangian optimisation, the weight of the control parameters is dynamically changed so as to produce subproblems where robustness becomes more and more important. In this way, during the process a set of Pareto optimal heuristic solutions can be collected, that have a different trade-off between robustness and efficiency, leaving the final user the choice of the ones to analyse in more details, depending on the specific requirements. The Lagrangian relaxation is solved within a simple subgradient optimisation framework to determine near-optimal Lagrangian multipliers. Computational experiments on real-world instances showed that the approach is rather effective and robust solutions require only a small computational overhead with respect to the nominal problem, so large-scale instances can be attacked.

2.2.5 Meta-heuristics

[Tormos, Lova, Ingolotti and Barber, 2008 (ARR-TR-0173)] presented a genetic approach to robust timetabling. The presented algorithm focuses on an efficient allocation of running time supplements and buffer times in such a way that the resulting timetable becomes less sensitive to stochastic disturbances, while keeping the total travel time at satisfactory level. Once the robust timetable has been generated, an empirical procedure based on simulation follows.

3. COMPARATIVE ASSESSMENT OF THE REVIEWED METHODS

This Section assesses the most important train timetabling methods that have been reviewed so far, identifies their advantages and disadvantages and investigates their appropriateness for the different fields of railway optimisation operations. The main assessment criteria include the easiness of implementation, the input data requirements, the obtained solution quality as well as the computational time.

The following Table summarises the obtained results. It is noteworthy that, as these conclusions emanate from the literature review, they tend to be problem-specific; therefore, they should be regarded in a critical way and every attempt of generalisation should rather be avoided.

Table 2: Main train timetabling methods: advantages/disadvantages and applicability field

Method	**Strengths**	**Weaknesses**	**Applicability**
Constraint Programming	▪ Allows description of more complex constraints without modification of the solution process		For feasibility rather than optimisation problems
Exact B&B		Time consuming	For small-size instances
Column generation			For large-scale instances
Simulated annealing	▪ Provides good quality solutions with small computing effort		For non-filtered data
Stochastic Programming	▪ Produces good quality solutions	▪ Leads to large, time-consuming models ▪ Is in conflict with the size of typical instances ▪ The cost of the first stage decision plus the expected recovery cost may be far from the actual cost in certain scenarios ▪ Requires the knowledge of probability and main features of the various scenarios	For few but significantly different scenarios
Light Robustness	▪ Produces good quality solutions ▪ Easy to formulate and solve ▪ Does not need a cumbersome set of second-stage variables and constraints	▪ Requires a level of protection against the data uncertainty	For large-scale instances

Method	**Strengths**	**Weaknesses**	**Applicability**
Recoverable Robustness	▪ Combines the virtues of stochastic programming and classical robust optimisation	▪ Usually more than one disruptions occur	For large-scale instances requiring reliability
Bi-criteria – Lagrangian Approaches	▪ Produces robust solutions with small computational overhead		
Heuristics	▪ Ease of implementation (no need for exact model of the problem) ▪ Guarantee of local optimality ▪ Small computational time	▪ Poor solution quality (no guarantee of global optimality) ▪ Problem-specific	
Meta-heuristics	▪ Able to cope with inaccuracies of data and model, large-scale instances and real-time problem solving ▪ Includes mechanisms to escape from local optima ▪ Ease of implementation (no need for exact model of the problem) ▪ Small computational time ▪ Efficient exploration of big search spaces	▪ Usually no guarantee of optimality	▪ For large-scale instances requiring reliability ▪ For real-time problem solving

It can be seen from the table above that among the methods employed for solving the nominal TTP, one can distinguish three main families: the exact methods, the heuristic approaches and the meta-heuristic ones. Hybrid methods combining characteristics of the above families have also been reported in the literature.

Exact methods include branch-and-bound, branch-and cut, branch-and-price, branch-and-cut-and-price algorithms, MILP and Constraint Programming. Although exact methods try to find an optimal solution to the TTP with respect to the problem's objective function and specific constraints, they are time consuming and thus their potential applicability is restricted on small-size instances. This practically means that they do not represent the most appropriate method for solving the real-world TTP, due to the prohibitive size of the typical instances.

Heuristic approaches (e.g. Lagrangian-based, constructive and local search heuristics) are characterised by implementation easiness (as there is no need for the exact model of the problem), as well as quick production of local optimal solutions. However, there is usually no guarantee of global optimality of the produced solutions, as they tend to get stuck in poor local optima. Furthermore, the solution quality depends heavily on the specific problem that is treated every time.

Meta-heuristic approaches (ex. evolutionary algorithms, genetic algorithms, memetic algorithms, tabu search, simulated annealing) are, similarly to the heuristic ones, easy to implement, as they do not need an exact model of the problem. Their ability to escape local optima and explore better areas of the solution space qualifies them as methods of choice when solving real-world, large-scale instances and also when facing data inaccuracies and real-time problems. On the other hand, there is generally no guarantee of the solution's global optimality. In [Higgins, Kozan and Ferreira, 1997], various heuristic and meta-heuristic approaches are presented and the results compared between them: it was show that the pure genetic algorithm outperformed the tabu search heuristic on most test problems and both meta-heuristics produced better results than the local search heuristic.

According to the literature combinatorial methods are very promising, as they combine advantages of the solution methods presented above: for instance, [Nachtigall and Voget, 1996] proved that a combination of the small computational time of a greedy heuristic with the ability of a genetic algorithm to combine good parts of randomly produced solutions produced quickly timetables of good quality. On the other hand, [Higgins, Kozan and Ferreira, 1997] presented two hybrid heuristics: the first one (HA1) is a combination of local search and genetic algorithm and the second one (HA2) a combination of tabu search and genetic algorithm. In terms of solution quality and number of calculations, the two hybrid algorithms outperformed the other heuristics presented in that paper, namely tabu search, local search heuristic and pure genetic algorithm, with HA2 producing better results than HA1 on most test problems.

As far as the robust TTP is concerned, many methods have been proposed, including Stochastic Programming, Recoverable Robustness, Light Robustness, Bi-criteria/Lagrangian approaches and Meta-heuristic approaches.

Stochastic Programming (in particular, 2-stage SP) seems to produce good quality solutions, although it leads to large, time-consuming models that are almost intractable for large-scale instances. In addition, it requires the knowledge of probability and main features of the various scenarios.

Light Robustness appeared to derive good solutions when dealing with a reasonable robustness–efficiency trade-off, though with less effort in terms of model formulation and solution time with respect to Stochastic Programming. Therefore, it can be assumed that it is an appropriate method when dealing with large instances.

Recoverable Robustness combines the virtues of Stochastic Programming and classical robust optimisation and according to the literature, could be appropriate for solving large-scale instances requiring reliability.

Bi-criteria/Lagrangian approaches seem to be rather effective for coupling timetabling efficiency and robustness. Their solutions generally require only a small computational overhead with respect to the nominal problem, so large-scale instances can be attacked.

Finally, meta-heuristic approaches and especially genetic algorithms are easy to implement and solve and enable to cope with inaccuracies of data and model, large instances as those typically faced in real-world applications and real-time problem solving.

4. CONCLUSIONS

In this paper, a literature review concerning the existing methods for solving the TTP was conducted, with respect to the problem category. The most common of them for the case of the nominal TTP include exact methods (such as MILP models, branch-and-bound / branch-and-cut / branch-and-price algorithms, constraint programming), heuristic methods (such as Lagrangian-based heuristics, constructive heuristics, local search heuristics) and meta-heuristic ones (such as evolutionary algorithms, tabu search algorithms, simulated annealing, genetic and memetic approaches). It can be concluded that heuristic and meta-heuristic approaches are the most appropriate for real-world typical instances, as they are relatively easy to implement and solve, although they do not guarantee that a global optimal solution can ever be obtained. Hybrid methods are, from the other hand a promising alternative, as they tend to combine the virtues of various methods.

Concerning the robust version of the TTP and the ability of a timetable to remain feasible under small disturbances, Stochastic Programming seems almost intractable for the size of the typical instances in real-world railway applications. From the other hand, Light Robustness and Recoverable Robustness seem able to cope successfully with the uncertainty of the input data, while bi-criteria and Lagrangian approaches go in this direction with only a small computational overhead with respect to the nominal problem. Finally, meta-heuristic approaches explore efficiently the search space, thus could be appropriate for real-time railway scheduling.

Generally, the appropriateness of each method depends on the specific problem to be solved; therefore, results should be critically viewed and global generalisations should rather be avoided.

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