Tour-Based Freight Origin-Destination Synthesis Considering Congestion

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ABSTRACT

The paper proposes an entropy-maximization approach to develop tour-based urban freight travel demand models to obtain origin-destination matrices from traffic counts under congestion (considering both passenger cars and trucks). This process is referred as Multiclass Origin-Destination Synthesis which is developed as a bi-level formulation. The upper level is the entropy maximization function to find the most likely ways to distribute tour flows and car trips if traffic counts are available. The lower level is a variational inequality formulation to guarantee an User Equilibrium condition for trucks and passenger cars in the transportation network.

Keywords: freight modeling, tours, multiclass, origin-destination synthesis

1. INTRODUCTION

The first steps in modeling truck transportation demand were in the 70s. Ogden (1978) used the single gravity model to perform the trip distribution urban truck trips and urban commodity flow using data for Melbourne, Australia. The database for the study was the information collected in a commercial vehicle survey carried out during the Melbourne Metropolitan Transportation Study. The survey included a 10% sample of all trucks registered there. However, the sample was limited because collecting more than 10% sample, will be too expensive. For this reason it is necessary to consider different options for gathering data such as traffic counts because of their availability, low-cost and the non-disruptive method for obtaining freight origin-destination (OD) matrices.

For transport planning and management tasks purposes, the modeler needs appropriate basic data for the estimation of future freight transportation demand and supply. One of the most important elements in such a process is the use of freight origin-destination matrices to represent the travel pattern of truck trips. These freight OD matrices could be estimated from direct samples or from secondary data sources (Holguin-Veras, 2000). OD matrices obtained from the field (from surveys at home, warehouse or at the street) tend to have high cost and require a lot of effort. Moreover, in the case of freight OD matrices the private firms are often unwilling to provide information. Also it has to be taken into account that freight flows vary over time, and hence require repeated surveys (Giuliano et al., 2010). A very attractive option in this regard is the use of traffic counts to estimate OD matrices (also known as origin-destination synthesis - ODS) because of the availability and low cost of this observable information. Traffic counts are routinely collected by many authorities due to their multiple uses in traffic and transport planning tasks (e.g. accident studies, maintenance planning, intersection improvements, traffic studies). Therefore, usually they are available in almost any city. Additionally, the requirements in terms of time, manpower, and management for the traffic counts are inexpensive and can be collected without disrupting travelers (Tamin and Willumsen, 1989, Ortuzar and Willumsen, 2001, Willumsen, 1978). Based on this, ODS produces an estimate of the trip flow OD matrix that matches secondary data (e.g. link traffic counts). Consequently, ODS has the potential to minimize data collection costs and speed up the process of model calibration and updating (Van Zuylen and Willumsen, 1980), obtaining freight demand models of good quality at a low cost.

The major challenge of applying ODS methods to model urban freight travel demand comes from the uncertainty and availability of data. In most urban areas, freight traffic counts are not collected or maintained as well as passenger traffic counts. For that reason, the freight-related ODS methods have to explore all possible data sources. In the case of estimating freight (tons and/or trucks) OD matrices from traffic counts, not much work has been done. Few researchers have studied this interesting topic. However, the current freight origin-destination synthesis – FODS research has some limitations in their ability to depict freight movements, and they do not model commercial vehicle trip-chains. Based on that, it is necessary to produce a new freight modeling approach considering previous techniques developed for passenger and freight demand models to obtain OD matrices from traffic counts.

From other perspective, one of the unique features of urban commercial vehicle movements is trip chaining behavior. Treated similarly as passenger vehicles, trucks were usually assumed to behave independently trying to minimize the transportation costs. In that case, freight vehicles are assumed to make independent trips rather than tours composed of linked trips (Wang, 2008). Nowadays, it is known that trucks make a significant number of trip chains (tours) and the focus of an investigation of freight movements must be addressed on that direction to get more accurate (realistic) results.

In conclusion, it is necessary to employ a new modeling approach in freight transportation considering previous techniques developed for transportation demand models to depict cargo movements. This modeling approach will obtain OD matrices from secondary sources considering trip chains and the relationship in the transportation network with passenger cars. To fulfill this need, a behavioral-based tour choice model is used to generate a sufficient and effective set of tours to estimate the distribution patterns of commercial vehicle tour flows given link traffic counts. Then, the entropy maximization formulations will take in the tour model output in order to perform freight ODS including trip chains under congestion.

2. MULTICLASS ORIGIN-DESTINATION SYNTHESIS USING ENTROPY MAXIMIZATION

Multiclass refers to models that consider two or more classes of travelers with different behavioral or choice characteristics (Boyce and Bar-Gera, 2004, Sheffi, 1985, Dafermos, 1971, Dafermos, 1972, Dafermos, 1973). In this paper the term *multiclass* refers to the combination of autos and trucks in the transportation network.

It is necessary to have a more realistic representation of the movement of commercial vehicles including passenger cars in the modeling of truck behavior. The presence of one or the other affects the assignment of vehicles onto the network. In that case, the congestion effects will depend on the traffic flow (Dafermos and Sparrow, 1969). Following this concept, Dafermos (1972) proposed a multiclass-user model that considers different vehicle combinations sharing a transportation network. Years later, Willumsen et al. (1993) developed a model with an implementation of advanced traffic assignment with consistent treatment of mode, destination and time of day choice with congested networks for passengers. However, trucks were not included in their model.

2.1 Entropy maximization formulations

The concept of entropy is best known by its presence in the second law of thermodynamics: "*The system cannot receive more in energy than the amount of external work supplied"*, where entropy is also known as uncertainty and chaos, and it always increases (Wilson, 1970a). Then the entropy of a probability distribution could be defined in information theory and this could be interpreted as a measure of uncertainty associated with that distribution. The key assumption in entropy maximizing models techniques is that all possible states of the system are equally probable (Wilson, 1970b). Considering the aforementioned assumption, the greatest number of system states is going to produce the most likely estimate of the flows in the system. In other words, the most likely OD trip matrix is the one having the greatest number of microstates associated with it.

The entropy maximizing (EM) models techniques have been used for modeling urban transportation demand (e.g. trip distribution, modal split) by some authors such Wilson (1967, 1970a, 1969, Wilson, 1970b), Fisk (1988), Wong et al. (2005), Zargari and Hamedani (2006), Wang and Holguin-Veras (2009, 2010), Xie et al. (2010), and Xie et al. (2011). The concept of EM can be combined with the methodology of ODS for freight. For example, Rios (2001) developed a model to estimate a multimodal (rail and trucking) OD matrix estimation. The author used a bi-level programming procedure with deterministic user equilibrium and an entropy function. Later, Rios et al. (2002) developed a forecast model for estimating freight OD matrices. For doing this, the authors used the theory of design of experiments as a framework to determine the value of three categories of information: link counts, OD totals, and aggregate OD totals. Munuzuri et al. (2004) presented an EM model procedure to estimate an OD matrix for freight transport in the city of Seville, Spain using different data sources such as population densities, business locations, activity sectors, and number of vans and trucks with license to operate in Seville as an input to their model.

The reasons because the proposed methodology based on EM is used in this study, is as Wong et al. (2005) asserted, EM has better solution properties such as convexity and uniqueness, and it doesn't need statistical information that is difficult to obtain. However, Zargari and Hamedani (2006) and Bera and Rao (2011) pointed out that the main limitation of the EM is that it considers the traffic counts as error-free observations on non-stochastic variables (uncertainties in traffic counts and prior matrix).

The application of the tour-based entropy maximization approach requires a pre-specification of tours potentially visited by trucks and the associated impedances. In this context, a behavioral-based tour choice model will be used to generate a sufficient and effective set of tours as the input to the EM formulations in order to perform freight ODS including trip chains. The EM formulations to be developed here intend to obtain freight OD matrices based on the tour distribution problem of commercial vehicles in an urban area considering congestion. For doing this, the analysis of EM is based on the work of Wilson (1967, 1970a, 1969) and Wang (2008), summarized in Wang and Holguin-Veras (2009).

2.2 Multiclass Traffic Equilibrium

When considering both passenger cars and trucks, it is assumed a condition of congestion in the network given for both classes, then the underlying choice method is an equilibrium assignment (Noriega and Florian, 2007, Willumsen, 2000, Dafermos, 1971, Dafermos, 1972, Dafermos and Sparrow, 1969). The models need functions relating the flow and travel costs on each link. Also, in this study, the study time period will be the peak time *h*. Then, the entropy model will be based on trip attraction and production at peak time *h*.

To build the proposed simultaneous equilibrium EM formulation, the work of Wang (2008) will be used as a basis. Three states for an urban freight system were defined considering that the path flows are going to depend on the trips in the network as shown in Table 1. The proposed multiclass origin-destination synthesis - MODS formulation will consider the states mentioned below with the presence of congestion.

Table 1: States for an urban freight system with simultaneous equilibrium

State	Description
Micro state	Individual commercial vehicle journey starting and ending at a home base (tour flow) by following tour m, and individual car trips t_{ij} in a path with only one origin a one destination
Meso state	t_m is the number of commercial vehicle journeys (tour flows) following tour m, and T_{ij} is the number of car trips between zone i and j
Macro state	O_i is the total number of trips produced by node i (trip production), D_j is the total number of trips attracted to node j (trip attraction), C is the impedance in the network, V_a is the observed traffic counts. All the nomenclature is for both trucks and passenger cars.

Note: $m \in \{1,2,...,M\}$, and M is the total number of tours.

In order to have a sound model considering both passenger cars and trucks is important to have them related under the same objective function to obtain a multiclass equilibrium in the ODS process. In this sense, congestion has an important role in route choice and is interdependent with the trip matrix of both passenger cars and truck (Willumsen, 1982, Dafermos, 1972).

Considering congestion in the ODS formulation implies the use of a capacity restraint element relating the cost of travel on a link to the flow in that link (Willumsen, 1982). The travel cost (travel time) of traveling in link *a* is a non-decreasing function of the flow on that link as shown in Eq. (1).

$$t_a = t_a(x_a) \qquad (1)$$

Typically the cost of travel (travel time) on a link (c_a) in Eq. (1) is obtained using the Bureau of Public Roads - BPR function (Bureau of Public Roads, 1964). However, this function is usually used for passenger cars and not for combination of classes in the transportation network. Holguín-Veras and Cetin (2008) proposed a travel time function that depends on the vector of traffic flows for different vehicle classes: passenger cars, small trucks, and large trucks to find the optimal toll for multiclass traffic. The formula is a second order Taylor expansion of a general link-performance function. The formulation was successfully implemented in the US case with very accurate results. For the aforementioned reason, the formulation proposed by Holguín-Veras and Cetin (2008) will be used in this study, where the travel time in link *a* depends of traffic flow of both trucks and passenger cars, as shown in Eq. (2).

$$t_a(X_t, X_c) = \alpha_0 + \alpha_1 X_c + \alpha_2 X_t + \alpha_3 X_c^2 + \alpha_4 X_t^2 + \alpha_5 X_c X_t \qquad (2)$$

Where:
X_t: Vector of truck traffic flow
X_c: Vector of passenger cars traffic flow

It has to be considered that the travel cost in link *a* for cars and trucks is going to be different. Although the vehicles share the transportation network and the travel time is the same (in equilibrium), the travel cost will be affected by the value of time of each one of the parties. To obtain the travel cost for each one of the vehicle classes per link, the travel time has to be multiplied by the value of time of each class, as follows:

$$c_a^t = \alpha_t \cdot t_a(x_t, x_c) \qquad (3)$$

$$c_a^c = \alpha_c \cdot t_a(x_t, x_c) \qquad (4)$$

Where:
α_t : Value of time for trucks [$]
α_c : Value of time for passenger cars [$]

Considering paths, the cost functions for truck tours and passenger cars trips during the peak time *h* will be given by:

$$C_m = \sum_a c_a^t \cdot \delta_{mah} \qquad (5)$$

$$C_{ij} = \sum_a c_a^c \cdot \delta_{ijah} \qquad (6)$$

Where:
δ_{mah}: A binary variable indicating whether tour m uses link a during the peak time *h*: if yes, $\delta_{mah} = 1$, otherwise $\delta_{mah} = 0$
δ_{ijah} : A binary variable indicating whether trip from i to j uses link a during the peak time *h*: if yes, $\delta_{ijah} = 1$, otherwise $\delta_{ijah} = 0$

Finding a feasible solution satisfying Wardrop's User Equilibrium (Wardrop, 1952) for trucks and passenger cars can be obtained solving the mathematical problem presented below. This problem produces a Wardrop equilibrium solution (Beckmann et al., 1956). As Bliemer and Bovy (2003) mentioned, in a multiclass equilibrium, the cost functions of modes are asymmetric, then the user optimal assignment cannot be written as an optimization problem. In the MODS the cost-functions are not separable because traffic flows interact. There is a mixed traffic of trucks and passenger vehicles. In this case, Beckmann (1956) type Nonlinear program (NLP) formulation does not exist. Then, the UE problem could be addressed using: a) Nonlinear Complementarity Problem – NCP, or b) Variational Inequality Problem - VIP (Dafermos, 1980, Nagurney, 1999, Bliemer and Bovy, 2003). In this study, the VIP approach will be used with EM to solve the MODS with UE due to the link performance function is composed by both passenger cars and trucks flows (asymmetric cost functions). It is worth to mention that for the non-separable cost functions, if the Jacobians of the link cost functions are positive definite, then the solution is unique.

Based on the VI problem and the EM formulation, the MODS formulation considering truck tours (paths) and routes for cars with only one stop, is as follows. The link flow is a combination of paths, then, the VI problem for the tour-based FODS considering congestion is given by:

$$\sum_m C_m(t_m^*, t_{ij}^*)^T \cdot (t_m - t_m^*) + \sum_{ij} C_{ij}(t_m^*, T_{ij}^*)^T \cdot (T_{ij} - T_{ij}^*) \geq 0 \quad (7)$$

Subject to

$$O_i^t = \sum_{m=1}^{M} t_m \delta_{imh} \qquad \forall i \in \{1,2,...,N\} \quad (8)$$

$$D_j^t = \sum_{m=1}^{M} t_m \delta_{jmh} \qquad \forall i \in \{1,2,...,N\} \quad (9)$$

$$O_i^c = \sum_{j=1}^{N} T_{ij} \qquad \forall i \in \{1,2,...N\} \quad (10)$$

$$D_j^c = \sum_{i=1}^{N} T_{ij} \qquad \forall j \in \{1,2,...N\} \quad (11)$$

$$C_m = \sum_a c_a^t \cdot \delta_{mah} \quad (12)$$

$$C_{ij} = \sum_{a} c_a^c \cdot \delta_{ijah} \quad (13)$$

$$X_a^t = \sum_{m=1}^{M} t_m \delta_{mah} \qquad \forall a \quad (14)$$

$$X_a^c = \sum_{ij}^{N} T_{ij} \delta_{ijah} \qquad \forall a \quad (15)$$

$$t_m \geq 0 \quad (16)$$

$$T_{ij} \geq 0 \quad (17)$$

$$X_a^t \geq 0 \quad (18)$$

$$X_a^c \geq 0 \quad (19)$$

Where:

t_m : Number of commercial vehicle journeys (tour flows) following tour m;

T_{ij} : Number of car trips between i and j;

t_m^*: Truck route (tour) flow at UE (optimal solution)

T_{ij}^*: Car route (tour with one origin and one destination) flow at UE (optimal solution)

X_a^t: Truck traffic flow in link a

X_a^c: Passenger cars traffic flow in link a

δ_{mih}: A binary variable indicating whether node *i* is in tour *m* during the peak time *h*: if yes, $\delta_{mih} = 1$, otherwise $\delta_{mih} = 0$

δ_{mjh}: A binary variable indicating whether node *j* is in tour *m* during the peak time *h*: if yes, $\delta_{mjh} = 1$, otherwise $\delta_{mjh} = 0$

2.3 ODS Entropy Maximization Formulation

The proposed ODS formulation is developed as a bi-level formulation. The upper level is an entropy maximization function to find the most likely ways to distribute tour flows and car trips if traffic counts are available. The lower level is the Variational Inequality User Equilibrium condition for trucks and passenger cars presented above in Eqs. (7) to (19).

With the MODS mathematical program, a new equilibrium will be obtained for all the transportation network. The trucks will be loaded onto the network using the developed tour model. They will select the shortest paths between tours (already generated) considering congestion, i.e. the given UE condition for trucks and passenger cars for the peak time *h*. The model includes specific constraints for passenger cars and trucks. The costs are calculated considering the flow of vehicles sharing the same infrastructure at peak time *h*.

Now, the entropy formulation can be written as an equivalent maximization (minimization) program to find the most likely ways to distribute tour flows and car trips if traffic counts are available. Let be $\delta_{ma} = 1$ if tour *m* uses link *a*, and 0 otherwise. Also, let be $\delta_{ija} = 1$ if trip T_{ij} uses link *a*, and 0 otherwise. The traffic counts for trucks and passenger cars can be expressed as:

$$V_a^t = \sum_m t_m \delta_{ma} \qquad \forall a \in \{1,2,...Q\} \tag{20}$$

$$V_a^c = \sum_{ij} T_{ij} \delta_{ija} \qquad \forall a \in \{1,2,...Q\} \tag{21}$$

The entropy maximization formulation for tour-based FODS with congestion is as follows:

$$\text{Max } W = \frac{T_t!}{\prod_m t_m!} * \frac{T_c!}{\prod_{i,j} T_{ij}!} \tag{22}$$

Where:
W: System entropy that represents the number of ways of distributing commercial vehicles tour flows and passenger cars flows
T_t: Total number of commercial vehicle tour flows in the network;
T_c: Total number of passenger cars flows in the network;
t_m : Number of commercial vehicle journeys (tour flows) following tour *m*;
T_{ij} : Number of car trips between *i* and *j*;

Wilson (1967, 1970a, 1969) proposed a methodology where the objective function can be simplified taking logarithms on both sides of the equation. Because (22) is a monotonic function, maximizing the function will lead to the same results of maximizing the logarithm of the function.

$$\text{Max } W = \frac{T_t! \cdot T_c!}{\prod_{m,ij} (t_m! \cdot T_{ij}!)} \tag{23}$$

Taking the natural logarithm, the objective function becomes:

$$\text{Max } z' = \ln(W) = \ln(T_t! \cdot T_c!) - \sum_{m=1}^{M} \sum_{i,j=1}^{T} \ln(t_m! \cdot T_{ij}!) \tag{24}$$

The term ln ($T_t! \cdot T_{ij}!$) in Eq. (24) is constant, then it can be dropped from the objective function, given that the total number of tours flows (T_t) and car trips (T_c) are fixed.

Since the maximization of a function is equivalent to the minimization of the negative of the same function, then *Max z'*= ln (*W*) = *Min z''*= – ln (*W*). Now, the objective function is given by:

$$\text{Min } z'' = \sum_{m=1}^{M}\sum_{i,j=1}^{T}\ln(t_m! \cdot T_{ij}!) \quad (25)$$

Based on Stirling's approximation $(\ln x! = x\ln x - x)$, the expression $\ln t_m! = t_m \ln t_m - t_m$, and $\ln T_{ij}! = T_{ij}\ln T_{ij} - T_{ij}$, then the objective function becomes:

$$\text{Min } z = \sum_{m=1}^{M}\sum_{i,j=1}^{T}(t_m \ln t_m - t_m + T_{ij}\ln T_{ij} - T_{ij}) \quad (26)$$

Considering the previous EM formulation and the variational inequality program for trucks and passenger cars in equilibrium, the following bi-level formulation to obtain tour-based FODS with congestion (MODS) is presented in as follows:

$$\text{Min } z = \sum_{m}\sum_{ij}(t_m \cdot \ln t_m - t_m + T_{ij} \cdot \ln T_{ij} - T_{ij}) \quad (27)$$

Subject to

$$\sum_{m} C_m(t_m^*, t_{ij}^*)^T \cdot (t_m - t_m^*) + \sum_{ij} C_{ij}(t_m^*, T_{ij}^*)^T \cdot (T_{ij} - T_{ij}^*) \geq 0 \quad (28)$$

Where the VI formulation above can be expressed as:

$$0 \leq C_m - C_m^* \perp t_m \geq 0 \quad (29)$$

$$0 \leq C_{ij} - C_{ij}^* \perp T_{ij} \geq 0 \quad (30)$$

Subject to

$$O_i^t = \sum_{m=1}^{M} t_m \delta_{imh} \qquad \forall i \in \{1,2,..., N\} \quad (31)$$

$$D_j^t = \sum_{m=1}^{M} t_m \delta_{jmh} \qquad \forall i \in \{1,2,..., N\} \quad (32)$$

$$O_i^c = \sum_{j=1}^{N} T_{ij} \qquad \forall i \in \{1,2,...N\} \quad (33)$$

$$D_j^c = \sum_{i=1}^{N} T_{ij} \qquad \forall j \in \{1,2,...N\} \quad (34)$$

$$C_m = \sum_{a} c_a^t \cdot \delta_{mah} \quad (35)$$

$$C_{ij} = \sum_{a} c_a^c \cdot \delta_{ijah} \quad (36)$$

$$X_a^t = \sum_{m=1}^{M} t_m \delta_{mah} \qquad \forall a \tag{37}$$

$$X_a^c = \sum_{ij}^{N} T_{ij} \delta_{ijah} \qquad \forall a \tag{38}$$

$$V_a^t = \sum_{m} t_m \delta_{mah} \qquad \forall a \in \{1,2,...Q\} \tag{39}$$

$$V_a^c = \sum_{ij} T_{ij} \delta_{ijah} \qquad \forall a \in \{1,2,...Q\} \tag{40}$$

$$\left| X_a^t - V_a^t \right| \le \theta \cdot V_a^t \qquad \forall a \in \{1,2,...Q\} \tag{41}$$

$$\left| X_a^c - V_a^c \right| \le \theta \cdot V_a^c \qquad \forall a \in \{1,2,...Q\} \tag{42}$$

$$t_m \ge 0 \tag{43}$$

$$T_{ij} \ge 0 \tag{44}$$

$$X_a^t \ge 0 \tag{45}$$

$$X_a^c \ge 0 \tag{46}$$

Where:
M: Total number of possible tours in the system;
N: Total number of nodes in the system;
Q: Total number of links with traffic counts in the system;
t_m : Number of commercial vehicle journeys (tour flows) following tour *m*;
T_{ij} : Number of car trips between *i* and *j*;
t_m^* : Truck route (tour) flow at UE (optimal solution)
T_{ij}^* : Car route (tour with one origin and one destination) flow at UE (optimal solution)
δ_{mih} : A binary variable indicating whether node *i* is in tour m during the peak time *h*: if yes, $\delta_{mih} = 1$, otherwise $\delta_{mih} = 0$
δ_{mjh} : A binary variable indicating whether node *j* is in tour *m* during the peak time *h*: if yes, $\delta_{mjh} = 1$, otherwise $\delta_{mjh} = 0$
δ_{mah}: A binary variable indicating whether tour *m* uses link *a* during the peak time *h*: if yes, $\delta_{mah} = 1$, otherwise $\delta_{mah} = 0$
δ_{ijah} : A binary variable indicating whether trip from *i* to *j* uses link *a* during the peak time *h*: if yes, $\delta_{ijah} = 1$, otherwise $\delta_{ijah} = 0$
X_a^t : Truck traffic flow in link a
X_a^c : Passenger cars traffic flow in link *a*
V_a^t : observed truck traffic count in link *a*.
V_a^c : observed passenger car traffic count in link *a*.
θ : Parameter representing a percentage of traffic counts available (e.g. 10%, θ = 0.1)

As mentioned, the formulation above is a bi-level mathematical program. The objective functions are given in the upper level by an entropy maximization function and in the lower level given by a Variational Inequality problem to obtain a UE condition for passenger cars and trucks. The objective of this problem is to find the most likely ways to distribute tours considering congestion (passenger cars). *The outputs (flows) from the lower level optimization (UE condition) will be used in the upper level (Entropy Maximization) program to achieve the proposed objective of developing Tour-Based FODS considering congestion at equilibrium.* There are six groups of constraints. The first group covers the trip production/attraction constraints. The second group depicts the cost of tour *m* and trip *ij* in the peak time *h*. The third group covers the traffic flows in the network. The fourth group is composed by the observed traffic counts constraints. The fifth group covers the convergence criteria between flows and traffic counts. The last groups are the nonnegative constraints implying tour flows and car trips equal to or greater than zero.

2.4 Existence and uniqueness of the formulation

Based on the tour-based FODS formulation for the congested case, using the existence and uniqueness conditions, it is possible to understand the characteristics of the models for the multiclass freight tour-based model.

As mentioned before, the travel time *t* in link *a* depends of traffic flow of both trucks and passenger cars, as shown in Eq. (47)

$$t_a(X_t, X_c) = \alpha_0 + \alpha_1 X_c + \alpha_2 X_t + \alpha_3 X_c^2 + \alpha_4 X_t^2 + \alpha_5 X_c X_t \tag{47}$$

Where:

$X_c \geq 0$

$X_t \geq 0$

The Jacobian matrix of the link cost function is given by:

$$\nabla t_a(X_t, X_c) = \begin{bmatrix} \dfrac{\partial t_a(X_t, X_c)}{\partial X_t} \\ \dfrac{\partial t_a(X_t, X_c)}{\partial X_c} \end{bmatrix} = \begin{bmatrix} \alpha_2 + 2\alpha_4 + \alpha_5 X_c \\ \alpha_1 + 2\alpha_3 + \alpha_5 X_t \end{bmatrix} \tag{48}$$

It can be observed that $t_a(X_t, X_c)$ is continuously differentiable and the Jacobian is positive definite. Then the function is monotone and the solution is unique.

3. CONCLUSIONS

This study gives a set of mathematical models for conducting tour-based FODS based on ME considering traffic counts.

This review showed that although the freight problem has been identified and documented for more than 30 years, there is only a pitifully small body of research on ODS considering truck characteristics (e.g. trip chain behavior). This was corroborated with the review of the different mathematical formulations produced by the research community, where no formulation included or treated the estimation of trip chains and congestion in the FODS process.

The framework will enable transportation management agencies to estimate freight OD matrices at much reduced cost. The framework will also make it possible to seamlessly integrate freight planning into agencies' transportation system planning.

As future work, the freight origin-destination matrix estimation based on tours could be extended to the dynamic case (considering time of travel) to improve the progress of urban freight demand forecasting towards more realistic representations of freight operations.

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