OptimalLocation of Motorway Interchanges: Social Welfare Gains versus Concessionaires'Profits

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Abstract

Transportation infrastructure is more and more often promoted through public-private partnerships such as build-operate-transfer schemes. Two main parties are involved in such ventures, government and concessionaires, with different perspectives. The former aims to maximize social welfare benefits, while the latter aim to maximize profits. Such distinct goals may be hard to reconcile into an advantageous solution for both parties. This article presents an optimization model for locating motorway interchanges that takes into account both perspectives. The model maximizes social welfare benefits (using a consumers' surplus measure) such that a given level of profit is ensured. The usefulness of the model is demonstratedfor a real case study.

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Introduction

Transportation networks are essential to promote and support economic progress, and to satisfy the increasing demand for travel of a growing population.Before the 1980s,the provision of transportation infrastructure was mainly made by governments, under the argument that most of the benefits of infrastructure provision have a public nature.Thistendency for central planning and control of transportation infrastructure prevented the private sector from participating in such developments (Kumaraswamy and Zhang, 2001). Since then,the governments of many countries have been encouraging the private sector to invest in transportation infrastructure, both in the construction of new infrastructure and in the maintenance and rehabilitation of existing infrastructure (Vickerman, 2007). Several factors have contributed to this change,including the trend towards the deregulation of public monopolies, the belief that the private sector is more efficient than the public sector, the demand for better service, and the shortage of public funds to finance transportation infrastructure, the latter being probably the major contributing factor (Chen and Subprasom, 2007; Yang and Meng, 2000; and Gomez-Ibanez et al. 1991). Financial arrangements that involve private investmentin the funding of public infrastructure are generally designated as Public-Private-Partnerships (PPP). A discussion of the concept of PPP is available in Tang et al. (2010), together with a review of related studies.

A well-known form of PPP arrangement is the Build-Operate-Transfer (BOT) scheme. According to such scheme, the government grants the concession of the transportation infrastructure to a private investor, who gets the right to build and operate the infrastructure at his own expense, receiving in return toll feesduring the concession period. When the concession period ends, the infrastructure is transferred to the government without remuneration. In the last few decades this scheme has been widely applied worldwide, both in developed and developing countries. Lists of BOT projects can be found in Walker and Smith (1995), Lam (1999), and Subprasom (2004). Among the existing BOT projects there are numerous examples of motorway concessions.

BOT projects generally involve two parties: a public entity(government) and a private investor (concessionaire). The former intends to maximize public benefits (social welfare), while the latter wants to maximize the profit generated from their investments.Such distinct perspectives generally lead to conflicts. Governments may be tempted to encourage BOT projects as a way to subsidizethe development of transportation infrastructure using private funds in order to increase social welfare. However, given the risk involved in such investments, the private sector will only finance a project venture if it is attractive,i.e., if it secures adequate profits. Moreover, as pointed out in Pahlman (1996), if something goes wrong in a BOT project it is the government, and not the concessionaire, who ultimately copes with the costs of failure.For these reasons, when transportation infrastructureis planned, allperspectives should be taken into account in order to achieve a win-win solution (see Kumaraswamy and Zhang, 2001) for both the public and the private parties.

The costs involved in motorway projects depend on infrastructurecharacteristics, and particularly on the number of interchanges (these facilities are quite expensive), while revenues depend on toll fees and on the number of users, which in turn depend on toll fees and route lengths, thus on the location of interchanges. Hence, interchange location plays an essential role to determine costs, demand,revenues, and social welfare.

Thegoal of this article is to introduce an optimization model for helping to define the bestlocation of motorway interchanges in a PPPcontext, taking into accountboth the publicand the privateinterest. The newmodel is related with work presented in two previous articles: Repolho et al. (2010, 2011). In the former article, the motorway is assumed to be toll-free, and the optimization is carried out from the users' standpoint with the objective of minimizing total travel costs. In the latter article, the motorway is assumedto beoperated by a concessionaire whose revenues are obtained from toll fees with the objective of maximizing profit. In the new model, the perspectives of users and concessionaires are dealt with simultaneously. It determines the number and location of motorway interchanges, as well as estimates the traffic flow using the motorwaygiven the location of interchanges, so that social welfare is maximized while ensuring a given level of profit for the concessionaire. As its predecessors, the new model can be classified as a non-strict multiple hub-allocation model (Aykin, 1995; Ebery et al. 2000; O'Kelly, 1996).

The outline of thisarticle is as follows. In the next section,we discuss the valuation of social welfare gainsin transportation projects and the use of the consumers' surplus conceptto measure them. Afterward, the optimization model is introduced and subsequently appliedto a case study (the same used in Repolho et al., 2011). The case study and the respective data set are briefly characterized, and the results obtained through the model are compared with the ones obtained in previous studies for the same data. Concluding remarks are presented in the last section.

Welfare gains

The valuation of social welfare in transportation has been a recurring subject in the literature (e.g., Williams, 1976; Jara-Díaz, 1986; Jara-Díaz and Farah, 1988). Its complexity arises from the fact that transportation is a peculiar economic activity which influences the entire economic system producing a multiplicity of effects. Further information on the subject can be obtained inVickerman (1991), Rietveld and Nijkamp (1993), Rietveld (1994), and Lakshmanan et al. (2001). In the latter article, the assessment of benefits (and costs) in transportationis classified as a 'slippery ice' notion/area.

In a strict sense, the benefits of transportation infrastructure are related with usage, and without users there would be no benefits. Thus, it is logical toassume that the benefits provided by the infrastructure are given by the benefits brought to users over the lifetime of the infrastructure. Most BOT projects use the concept of consumers' surplus to assess the users' benefits (social welfare benefits) derived from the improvement of transportation infrastructure (see e.g.Yang and Meng, 2000; Chen and Subprasom, 2007). Consumers' surplus was defined in Marshall (1920) as the "excess of the price which the consumer is willing to pay for something rather than go without, over that which he actually does pay", i.e., the difference between total willingness to pay and actual payment.

The concept can be better understood through a diagram representing the inverse (aggregate)travel demand function for a given O/D pair ij, D_{ij}^{-1} (Figure 1), where travel costs are a function of traffic flows (lower costs lead to higher flows). In the diagram,q_{0ijmn}denotesthe traffic flow between trip generation centers i and j via motorway segment *mn* when the travel costs are equal to the ones before the construction of the motorway, c_{0ij}; q_{ijmn} denotes the traffic flows between centers *i* and *j* via motorway segment *mn* for the travel costs after the construction of the motorway, c_{ijmn}. According to the definition given above, the consumers' surplus before the construction of the motorway is given by the striped area. After the construction of the motorway the consumers' surplus increases and is given by the striped area plus the grey area. Thus, the consumers' surplus gains due to the construction of the motorway are given by the grey area.

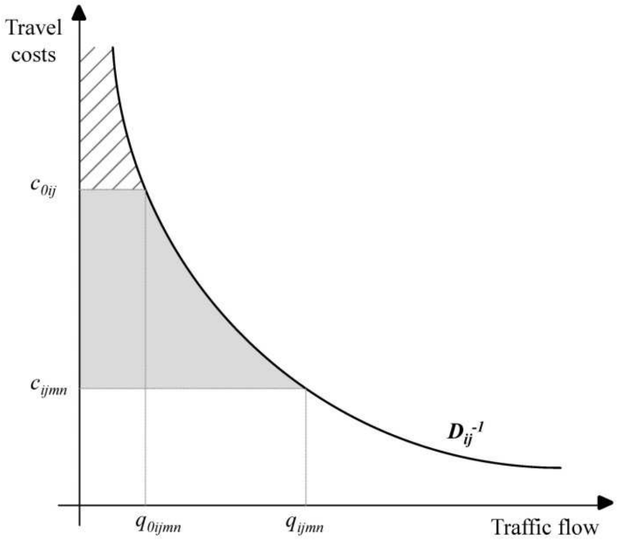

Figure 1 –Consumers' surplus gains for one O/D pair *ij*

It is pertinent at this point to question whether the consumers' surplus reflects social welfare accuratelyor whether should additional external benefits be added? If this was the case, then social welfare benefits would be larger than the willingness to pay of the immediate user. However, as pointed out in Mishan (1976), the addition of external benefits may produce double counting. Jara-Díaz (1986) shows that, at the market level, the net sum of gains and losses are fully reflected by consumers' surplus in a competitive environment, and are approximately reflected in a monopolistic one. Rothengatter (1994) suggests that the external benefits, if they exist at all, would be small. Lakshmananet, al. (2001) analyzed a list of external effects and concluded that no clear and significant case of a positive externality of infrastructure usage was identified.

Given thesebases, we have chosen to use the consumers' surplus concept to measure the increase in social welfare derived from transportation infrastructure investment. No external benefits are considered.The consumers' surplus gains (Δ_{ijmn}) associated with the trips made through the new motorway between a given pair of centers, i and j, can be expressed as follows:

$$\mathit{\Delta}_{ijmn} = \int_{q_{0_{ijmn}}}^{q_{ijmn}} D_{ij}^{-1}(v)dv - c_{ijmn} \times \left(q_{ijmn} - q_{0_{ijmn}}\right) + q_{0_{ijmn}} \times \left(c_{0_{ij}} - c_{ijmn}\right) \qquad (1)$$

The travel demand model we will consider for the computation of consumers' surplus(in theoptimization model presented later in this paper)is the same as the one used in Repolho et al. (2011). In this model, drivers may opt between traveling through the existing road network only and traveling through a route that combines segments of the existing road network with segments of the new motorway. The model further assumes that the introduction of cheaper routes (due to the new motorway) generates additional traffic and that the new motorway is only used if it is less costly. In fact, it is assumed that, because of habit, a fraction of the drivers maycontinue to choose traveling through the existing road network even if it is more costly than the cheapest alternative route using a new motorway segment.According to the model, for a given pair of trip generation centers, i and j, the traffic flow via motorway segment mn, q_{ijmn}, is given by:

$$q_{ijmn} = D_{ij}\left(c_{ijmn}\right) = \frac{c_{0_{ij}}^{2\beta-\beta^2}}{(c_{0_{ij}}^{1-\beta} c_{ijmn}^{\beta} + c_{ijmn})^{\beta} (c_{0_{ij}}^{\beta} + c_{ijmn}^{\beta})^{1-\beta}} q_{0_{ij}} \qquad (2)$$

where q_{0ij} is the traffic flow between centers i and j before the construction of the motorway, and β is a calibration parameter (further details on the travel demand model are available in Repolho et al. 2011).

The previous expression was established considering a power-form impedance function $f(c_{ij})=c_{ij}^{\beta}$, which fits real-world interurban traffic better than the exponential-form impedance function (Fotheringham and O'Kelly 1989; De Vries et al. (2009).

The value of q_{0ij} can be calculated through an unconstrained gravity model (Ortúzar and Willumsen, 2001). Representing with m_i and m_j the "masses" of the origin and destination centers *i* and *j*(measured e.g. with the respective populations)and considering the same power-form impedance function, it is given by:

$$q_{0_{ij}} = \alpha \frac{m_i m_j}{{c_{0_{ij}}}^{\beta}} \tag{3}$$

Optimization model

The optimization model described below is principally based on the Deterministic Motorway Interchange Location Model (DMILM) presented in Repolho et al. (2011).Itincludesall the constraints used in the DMILM model plus a new set of constraints to account for the objectives of concessionaires, but involves a different objective function.All assumptions underlying the DMILM model remain valid. The model optimizes the location of the motorway interchanges such that consumers' surplus gains are maximized while guaranteeing a given level of profit for the concessionaire. By varying the level of profit parametrically, the model can be seen as the constraint form of a multi-objective optimization approach (Cohon, 2004).

As its predecessors, the new model applies to a region where a new motorway will be built over an existing road transportation network. The set of trip generation centers located in the region and the set of candidate interchange locations are known. Drivers are assumed to choose the least cost route according to the travel demand model presented in the previous section.The key decisions to be made through the application of the model are the interchange locations and the traffic assigned to motorway routes. These decisions can be made such that the consumers' surplus is maximized regardless of the corresponding concessionaire's profit or taking into account that a certain percentage of the maximum possible profit must be ensured. The maximum possible profit can be obtained through the DMILM model presented in Repolho et al. (2011). Thus, for a given solution, the percentage of the maximum possible profit is the quotient between the profit corresponding to that solution and the profit corresponding to the solution obtainedthrough the DMILM model.

For formulating the optimization model, consider the following (additional) notation:

Sets

$\boldsymbol{J}$	Set of trip generation centers.
$\boldsymbol{M}$	Set of candidate interchange locations.
$\boldsymbol{R}_{ijmn}=\{v,b\|c_{ijuv}>c_{ijmn}\}$	Set of potential routes ($i\rightarrow u\rightarrow v\rightarrow j$) between centers i and j that use a motorway segment uv and that cost more than the route using motorway segment mn ($i\rightarrow m\rightarrow n\rightarrow j$).

Decision Variables

y_m	Binary variable that takes the value of 1 if a motorway interchange is located at the candidate site m, and zero otherwise.
x_{ijmn}	Fraction of the traffic flow between centers i and j via motorway segment mn.

Parameters

c_{im}	Travel cost between center$i$ and interchange m through the existing road network.
c'_{mn}	Travel cost between interchanges$m$ and n through the new motorway.
$c_{ijmn}=c_{im}+c'_{mn}+c_{nj}$	Travel cost between centers i and j through a route that includes two segments of the existing transportation network, im and nj, and a segment of the new motorway, mn.
d_{mn}	Distance between interchanges$m$ and n through the new motorway.
g^a_m	Upper limit on the traffic flow that may use interchange m as a motorway access.
g^e_n	Upper limit on the traffic flow that may use interchange n as a motorway exit.
t	Toll fee per kilometer.
f	Fixed daily cost for installing and operating an interchange.
w	Fixed daily cost for building and maintaining the motorway.
μ	Minimum percentage of the maximum profit guaranteed to the concessionaire.
π	Maximum profit that can be achieved by the concessionaire.

Since the assignment decision variables (x_{ijmn}) have four indexes, their number can easily become quite large when dealing with real-world problems. In order to mitigate this, the model requires a pre-processing stage where only relevant assignment variables (the ones corresponding to routes that potentially improve the transportation system) are defined. Specifically, a variable x_{ijmn} is only defined in the following circumstances:

- $i<j$ (we assume that both the O/D and the travel cost matrices are symmetric, and only consider their upper triangles).
- $q_{ijmn} > 0$ (the traffic flow on route $i \rightarrow m \rightarrow n \rightarrow j$ is positive).
- $c_{0ij} - c_{ijmn} > 0$ (the travel costs through the motorway route $i \rightarrow m \rightarrow n \rightarrow j$ are smaller than the travel costs between centers i and j using the existing transportation network only).

Given the notation introduced above (and earlier), the model can be formulated as follows:

$$\text{Max } \phi = 2\sum_{i\in J}\sum_{j\in J}\sum_{m\in M}\sum_{n\in M}\Delta_{ijmn}\, x_{ijmn} \qquad (4)$$

s.t.

$$\sum_{m\in M}\sum_{n\in M} x_{ijmn} \le 1 \qquad \forall i, j \in J \qquad (5)$$

$$\sum_{i\in J}\sum_{j\in J}\sum_{n\in M} x_{ijmn} \le g^{a}_{m} y_m \qquad \forall m \in M : x_{ijmn} \text{ exists} \qquad (6a)$$

$$\sum_{i\in J}\sum_{j\in J}\sum_{m\in M} x_{ijmn} \le g^{e}_{n} y_n \qquad \forall n \in M : x_{ijmn} \text{ exists} \qquad (6b)$$

$$\sum_{u\in R_{ijmn}}\sum_{v\in R_{ijmn}} x_{ijuv} + y_m + y_n \le 2 \qquad \forall i, j \in J, \forall m, n \in M : x_{ijmn} \text{ exists} \qquad (7)$$

$$2\sum_{i\in J}\sum_{j\in J}\sum_{m\in M}\sum_{n\in M} t\, d_{mn} q_{0_{ij}} x_{ijmn} - \sum_{m\in M} f y_m - w \ge \mu\pi \qquad (8)$$

$$y_1 = 1 \qquad (9a)$$

$$y_M = 1 \tag{9b}$$

$$x_{ijmn} \geq 0 \quad \forall i, j \in J, m, n \in M \tag{10}$$

$$y_m \in \{0,1\} \quad \forall m \in M \tag{11}$$

The objective function (4) maximizes the consumers' surplus gains, ϕ,made possible by the addition of a new motorway to a road network and is given by the sum of the consumers' surplus gains achieved for all trips made through the motorway between each pair of centers, *i* and *j*(it is multiplied by two to consider both traffic directions). Constraints (5), the assignment constraints, guarantee that trips between each *ij* pair are assigned to at most one route including a motorway segment *mn*. If this is the case, then x_{ijmn}=1 (though, since this is a multiple-allocation hub location model, if there is more than one motorway route with exactly the same lowest travel cost,traffic flows may be distributed among them). If trips are made only through the existing road network, then x_{ijmn}=0. Constraints (6) ensure that trips are only assigned to a motorway segment if there are interchanges at its extremities. Constraints (7), together with expressions (11), ensure that trips are assigned to the least-cost motorway route available. They prevent trips from being assigned to routes with longer motorway segments, thus leading to higher profit for the concessionaire, but which would be more disadvantageous for users than other available routes. The concern with concessionaire's profit is expressed through constraint (8). Itensures that the solution selected guarantees at least μ percent of the maximum profit that the concessionaire could make, π. Profit is given as the difference between total toll fee revenues (multiplied by two to account both traffic directions) and infrastructure costs. These costs are subdivided into costs for installing and operating the interchanges and costs for building and maintaining the motorway. If μ is set equal to one the model becomes equivalent to the DMILM presented in Repolho et al. (2011). Essentially, with μ equal to 1, the model will find the solution which optimizes consumers' surplus subject to achieving the maximum possible profit. If μ is set equal to zero, then we are focusing on the maximization of consumers' surplus gains (the model becomes equivalent – though with a different measure for social welfare benefits – to the models presented in Repolho et al., 2010).Constraints (9a) and (9b) ensure that there will be interchanges located at the endpoints of the motorway. Finally, expressions (10) and (11) define the domain of the decision variables.

Case study

The model proposedin the previous section was applied to the same data set as the one used in Repolho et al. (2011). This data refers to a Portuguese motorway,A25,which plays an important role with respect to both national and international road traffic, and involves 55 trip generation centers and 33 candidate interchange locations (all the currently existing interchanges). The motorwaywas operated from 2006 till the end of 2011 by ASCENDI – the private company that wasresponsible for converting the former "fast" two-way road IP5 into the A25 motorway – as a toll-free road ("virtual" tolls were then paid by the government to the company as an incentive to regional development). Since December 2011, the government stopped paying the virtual tolls and granted ASCENDI permission to operate the motorway as a toll road, charging 0.090€/km on average. The purpose of thestudy is to find whether there were alternative solutions(number and location of the interchanges) that would have ensured similar levels of social welfare and, simultaneously, high revenues for the concessionaire. Figure 2 represents the A25 motorway and the interchange locations, as well as the trip generation centers and the existing road network.

The case study is presented in three parts. First, we briefly describe the data set we have used as input to the optimization model.Next, we specify the objective function corresponding to the demand model adopted. Finally, we present and analyze the results obtained from the application of the optimization model. The benefitsof using the proposed model are illustrated through the comparison of the results obtained for this case study with the ones obtained in Repolho et al. (2011) using the DMILM (with the objective of maximizing concessionaire's profit).

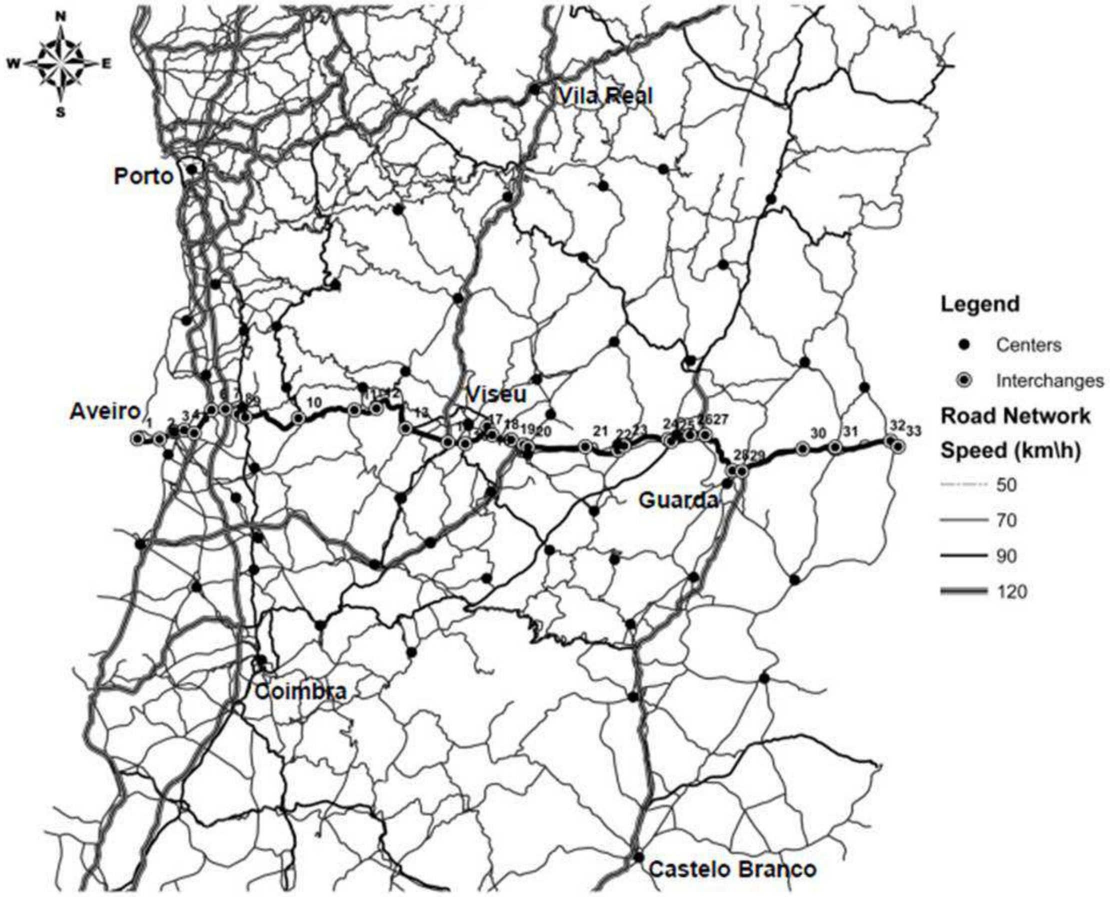

Figure 2 – A25 motorway

Model data

The data needed to run the modelmay be grouped under two categories: data about costs and data about travel demand.There are two types of cost data involved in the model: travel costs and infrastructure costs. The first affects the routes chosen by drivers and the consumers' surplus gains, while the second affects profit.

The computation of costs was made as follows:

1. Travel costs were calculated using the approach presented in Santos (2007), which considers four cost components: vehicle operating costs, accident costs, time costs, and tolling costs. The vehicle operating costs were estimated at 16.811 €/100 km per vehicleusing the HDM-4 approach (World Bank, 2010)

and include fuel consumption, tire usage, vehicle maintenance, and vehicle depreciation. Fuel cost was assumed to be equal to 0.995 and 1.219 €/liter respectively for diesel and gasoline (the same as for the scenario SCN3 defined in Repolho et al.2011). The accident costs were assumed to be equal to 0.01 €/km per vehicle (the same value was used in Santos, 2007). The user time costs were estimated at 7.3306 €/hour based on the HDM-4 approach and the formulation adopted by the Portuguese Road Administration (GEPA, 1995), which takes into account the Portuguese national car fleet (information available in IMTT 2006a and 2006b). Finally, the tolling costs were considered according to the real toll fees currently being applied.

2. Infrastructure costs consist of interchange costs and roadway costs. The cost of an interchange was estimated at 2.00 million €, while the cost of each kilometer of roadway was estimated at 2.85 million €. Considering a lifespan of 30 years and a real discount rate of 4 percent, the daily fixed charges for installing and operating the interchanges and for building and maintaining the roadway are, respectively, f = 305 € and w = 82,495 € (the motorway has 190 kilometers).

The computation of travel demand was made through expressions(2) and (3), considering themasses of the trip generation centers (m_i and m_j) to be given by the respective populations. Calibration parametersα and β were estimated at 1.4 and 1.0, respectively, using O/D traffic information available for the North Region of Portugal and regression analysis.

Objective function

The objective function considered in the model expresses the consumers' surplus gainsmade possible by the construction of the new motorway, and was obtained through the combination of expressions (1) and (2). The travel demand function D_{ij} (c_{ijmn}), represented in expression (2), specifiesthe traffic flows between centers $i$and j via motorway segment mn, q_{ijmn}, as a function of the route's travel costs, c_{ijmn}.

For β=1, the demand function becomes:

$$D_{ij}\left(c_{ijmn}\right)=q_{ijmn}=\frac{c_{0_{ij}}}{2c_{ijmn}}q_{0_{ijmn}} \tag{12}$$

The inverse demand function for an O/D pair *i/j*, D_{ij}^{-1}, is then given by:

$$D_{ij}^{-1}\left(q_{ijmn}\right)=c_{ijmn}=q_{0_{ij}}\frac{c_{0_{ij}}}{2}\frac{1}{q_{ijmn}} \tag{13}$$

With this expression for D_{ij}^{-1}, the objective function (4) becomes:

$$\begin{aligned}\text{Max } \phi = 2\sum_{i\in J}\sum_{j\in J}\sum_{m\in M}\sum_{n\in M}\Bigg[& q_{0_{ij}}\frac{c_{0_{ij}}}{2}\left(\ln q_{ijmn}-\ln q_{0_{ijmn}}\right)\\ & -c_{ijmn}\left(q_{ijmn}-q_{0_{ijmn}}\right)\\ & +q_{0_{ijmn}}\left(c_{0_{ij}}-c_{ijmn}\right)\Bigg]x_{ijmn}\end{aligned} \tag{14}$$

Model results

The results obtained with the modelwere calculated considering three levels of toll fees – 0.040, 0.050 and 0.060€/km. Below or above these values, both social welfare gains and profit are clearly worse. This includes the toll fee of 0.090€/km currently being applied (as shown later in this section).The calculations were made using an Intel Core 2 Quad Processor Q9550 2.84 GHz computer with 4 GB of RAM and employing the FICO Xpress 7.0 optimizer (FICO Optimization, 2009). This is the same computing system as that used in Repolho et al. (2011) for the profit maximization model. The CPU time needed to handle the model was less than 20 seconds for all instances solved.

In Table 1, we summarize the results obtained when only the government objective is considered (μ=0) and when only the concessionaire's objective is considered (μ=1). It can be seen there that when the concessionaire's perspective is discarded from the analysis the solutions found are associated with low levels of profit. The average percentage of maximum profit of the toll fee instances tested is only 40.9%. On the contrary, if the concessionaire's profit is maximized (μ=1) the average percentage of maximum consumers' surplus gains is 78.7 % (when t=0.040 €/km it even ascends to

92.5%). The solutions obtained when μ=1 are the same as the ones given by the DMILM model in Repolho et al. (2011).

Table 1 – Percentage of maximum profit and consumers' surplus gains achieved for μ=0 and μ=1

t	μ = 0		μ = 1	
	Max. ϕ	% of π	% of Max. ϕ	π
0.040	137986	71.1	92.5	22844
0.050	115525	41.3	69.7	41611
0.060	97413	10.2	73.7	20844

Between the two extreme conditions, μ=0 and μ=1, there may be some intermediate solutions capable of ensuring win-win solution for both the public and the private parties, i.e., solutions with high level of consumers' surplus gains and expected profit at the same time.

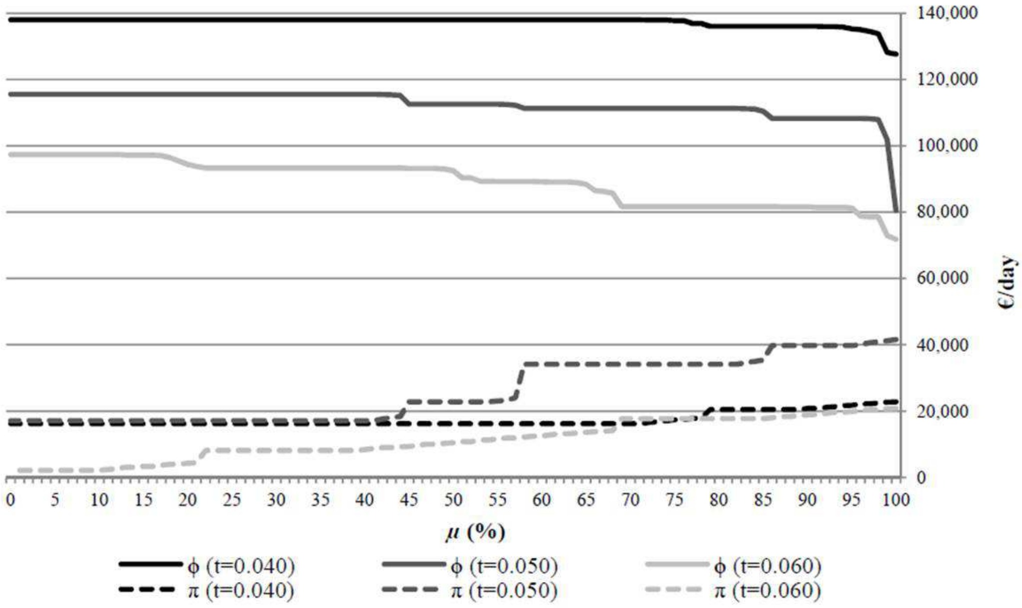

Figure 3 – Relationship between μ and consumers' surplus gains and profit

In order to look for these solutions,we have calculated the optimal solution for each toll fee instance considering all values of μ between 0 and 1 with increments of 0.01. The results obtained for each toll fee instance with respect to consumers' surplus gains and profit are displayed in Figure 3. As expected, consumers' surplus gains increase inversely as the toll fee values decrease. As for the concessionaires' profit, the highest value is obtained for the intermediate toll fee instance 0.050 €/Km. As concluded in Repolho et al. (2011), lower toll fees attract more drivers but paying less, while higher toll fees attract fewer drivers but paying more. The highest profit is obtained from the trade-off between the amount of users and the total toll fees collected. The results obtained in this paper strengthen this conclusion by showing that even when a second objective is considered and the percentage of minimum profit imposed is lower, the highest values are still obtained for the intermediate toll fees. When μ is set equal to a value higher than 0.5 the solutions with highest profit are obtained for the toll fee instance 0.050 (the one that provides the highest profit when μ=1).

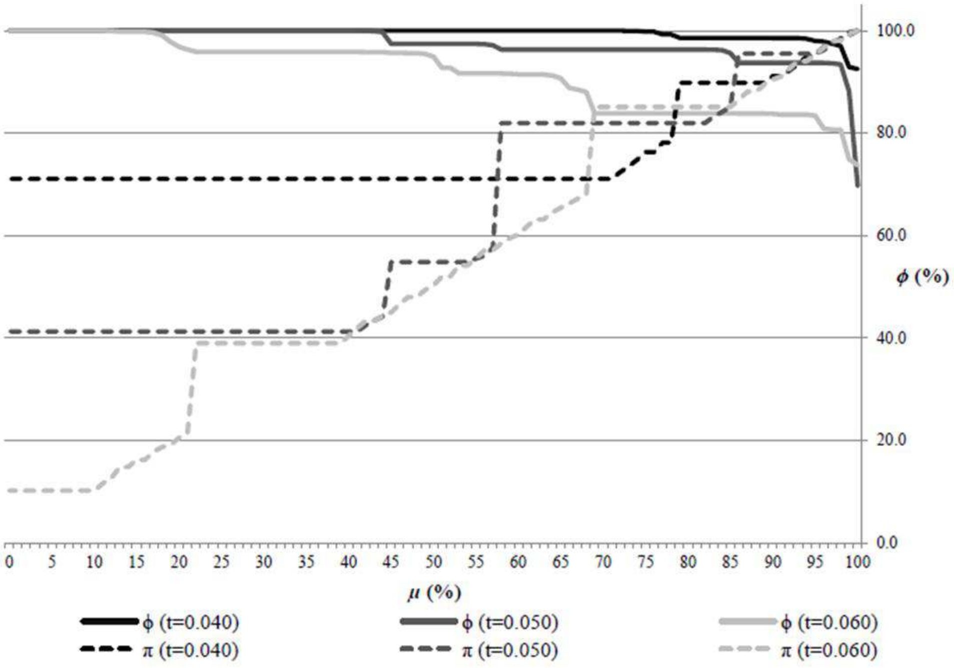

Figure 4 – Percentage of the maximum consumers' surplus gains and profit according to μ

The search for a good compromise solution regarding the objectives of the government and the concessionaire (maximizing consumers' surplus and maximizing profit) can be facilitated if based on a graphic relating the percentage of maximum consumers' surplus gains and maximum profit (rather than absolute values) obtained for each value ofμ.

Figure 4 illustrates the tradeoff between the percentages of consumers' surplus gains and profit achieved for all integer percentages of μ between 0 and 100%.

The results show that it is possible to guarantee very high levels of profitand consumers' surplus gains simultaneously. For the three toll fee instances tested there are indeed solutions that ensure a percentage of consumers' surplus gains and profit higher than 80% of the maximum that could be attained for each objective alone (for the toll fee instances 0.040 and 0.050€/km it is even higher than 90%). For instance, when a minimum of 96% of the maximum profit is imposed (μ=0.96), the percentage of maximum consumers' surplus gains achieved is 97.8%, 93.7% and 80.9% respectively for the toll fee instances 0.040, 0.050 and 0.060 €/km. Additionally, Figure 4 also indicates that when imposing low values of μ the gains in consumers' surplus are quite small when measured against the considerably reduced levels of profit that would be obtained by the concessionaire. Thus, the approach used can indeed help to find solutions that provide highly satisfactory levels of consumers' surplus gains and concessionaire's profit simultaneously.

The consumers' surplus gains and concessionaire's profit obtained for each solution rely on the decisions made with respect to the number and location of interchanges. Thus, it is important to evaluate theimpact of varying the value of μ on these decisions.In this sense, we have analyzed the solutions obtained for the toll fee instance 0.050 €/km (this is the one for which most results are presented in Repolho et al., 2011). The optimum number and location of interchanges, the consumers' surplus gains, the concessionaire's profit, and respective percentages for theμ values for which the solutions change are summarized in Table 2.There, it can be seen that, in general, as we relax the minimum percentage of concessionaire's profit that must be ensured, the number of interchanges located increases. When μ is set equal to zero, 28 interchanges are located. The remaining 5 candidate locations (2, 12, 18, 30, and 32) are never selected as they apparently do not contribute at all to increasing consumers' surplus gains (we say apparently because, as mentioned in Repolho et al., 2010, in reality they serve minor population settlements which were not retained as trip generation centers). The interchanges are located close to large trip generation centers or at the intersection of the new motorway with other major roads, but also in places less obvious and therefore harder to identify. Moreover, the model identifies the best options when there is more than one candidate interchangeclose to an attraction point (trip generation centers or major roads). Most candidate locations that figure in Table 2 (except the five mentioned before) have close alternatives, which helps to understand why their addition or removal from the optimal solution produces little impact on the consumers' surplus gains but may have a large effect on profit.

Table 2 – Model results for *t*=0.050€/km

μ (%)	ϕ (€/day)	π (€/day)	ϕ (%)	π (%)	Number of interchanges located	Potential interchange locations not selected
100	80505	41611	69.7	100.0	20	5, 8, 12, 16, 19, 20, 21, 23, 25, 27, 30,
99	101851	41240	88.2	99.1	21	2, 5, 8, 12, 16, 18, 21, 23, 26, 29, 30, 32
98	107966	40918	93.5	98.3	23	2, 12, 16, 18, 21, 23, 26, 29, 30, 32
97	108154	40630	93.6	97.6	24	2, 12, 16, 18, 21, 23, 29, 30, 32
96	108259	40041	93.7	96.2	25	2, 12, 16, 18, 23, 29, 30, 32
95	108277	39748	93.7	95.5	26	2, 12, 16, 18, 29, 30, 32
85	110445	35423	95.6	85.1	23	2, 8, 12, 16, 18, 21, 23, 26, 30, 32
84	111134	35005	96.2	84.1	25	2, 12, 16, 18, 21, 23, 30, 32
83	111196	34633	96.3	83.2	25	2, 12, 16, 18, 22, 23, 30, 32
82	111257	34123	96.3	82.0	27	2, 12, 16, 18, 30, 32
57	112231	23989	97.1	57.7	24	2, 12, 18, 21, 23, 26, 29, 30, 32
56	112482	23323	97.4	56.1	25	2, 12, 18, 22, 23, 29, 30, 32
55	112524	23109	97.4	55.5	26	2, 12, 18, 23, 29, 30, 32
54	112543	22815	97.4	54.8	27	2, 12, 18, 29, 30, 32
44	115203	18392	99.7	44.2	25	2, 12, 18, 21, 23, 26, 30, 32
43	115402	18065	99.9	43.4	26	2, 12, 18, 21, 23, 30, 32
42	115464	17690	99.9	42.5	26	2, 12, 18, 22, 23, 30, 32
41	115525	17179	####	41.3	28	2, 12, 18, 30, 32
0	115525	17179	####	41.3	28	2, 12, 18, 30, 32

A possible way of selecting the solution to implement is through the use of Goal Programming. This method was introduced in Charnes et al. (1955) and developed in Charnes and Cooper (1961). It applies when a goal or target value can be assigned to each one of several conflicting objectives. The purpose is to determine the compromise solution whose value is the closest to the target values (different weights can be attributed to each objective). Given the objectives of maximizing consumers' surplus gains and maximizing profit, we may define as goals achieving the maximal percentage of consumers' surplus gains and profit (100 percent). In that case, assuming equal weights for the two objectives, the compromise solution is the one that minimizes

$$\delta = \sqrt{(1-\phi)^2 + (1-\mu)^2} \qquad (16)$$

For the toll fee instance 0.050 €/km the minimum δ is obtained for the solution corresponding to μ=0.98. This solution locates 23 interchanges and achieves 93.5% of the maximum consumers' surplus gains and 98.3% of the maximum profit. When compared to the solution that maximizes profit (μ=1.00 – equivalent to the solution obtained with the DMILM model in Repolho et al., 2011) this compromise solution locates three additional interchanges – interchanges 5, 8 and 20 – and replaces interchanges 19, 25 and 27 by nearby locations, i.e., interchanges 18, 26 and 29 respectively.

A final comment should be made on the solution currently adopted by the road administration for the A25 motorway, which involved the location of the 33 interchanges and,since December 2011,charging a toll fee of 0.090 €/km.Despite the consumers' surplus gains of 62740 €/day (still they are 42 percentless than the compromise solution found for μ=0.98 with a toll fee of 0.050 €/km) the solution is not profitable. Indeed, the difference between the toll fee revenues and the fixed costs of the motorway is negative, representing a daily loss of 15193 €.

Conclusion

In this paper we have presented an optimization model for locating motorway interchanges applicable to Build-Operate-Transfer schemes. The objectives of the two parties involved, public entity (government) and private investor (concessionaire), are taken into account. The first aims to maximize public benefit (social welfare), while the second aims to maximize profit. The two objectives considered in the model are then consumers' surplus gains (the measure for social welfare benefits) and profit. The model maximizes consumers' surplus gains such that a given level of profit is guaranteed, combining and extending the work described in REPOLHO et al. (2010, 2011), where a problem of the same kind was approached, respectively, from the users' perspective and the concessionaire's perspective.

With respect to Repolho et al. (2010), we have used a new objective function to measure the public benefits (maximizing total travel cost savings was replaced by maximizing consumers' surplus gains) and employed the travel behavior model introduced in Repolho et al. (2011). Also a major improvement in this work involves the fact that the number of interchanges to be located is no longer used as a proxy for the available budget. In the new model, the costs of the project are actually calculated and the number of interchanges located is determined endogenously from the tradeoff between consumers' surplus gains and concessionaire´s profit.

The application of the model to the A25 case study demonstrated that the use of a model that considers simultaneously the interests of the two main stakeholders involved in the interchange location problem (government and concessionaire) can help identify highly satisfactory solutions for both parties. Thus, it is the authors' belief that this model can be of great utility for both road administrations and motorway concessionaires.

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