SOCIAL NETWORKS, GEOGRAPHY AND TRAVEL DEMAND: A THEORETICAL MODELING APPROACH

Yuki Ohira, Kobe University

Toshimori Otazawa, Kobe University, E-mail: ota@opal.kobe-u.ac.jp

ABSTRACT

Social networks affect activity-travel behavior. People and firms make trips in order to have the face-to-face communications with friends and business partners with whom they have social links. Locations in a geographical space also affect activity-travel behavior. Physical proximity enhances joint activities of agents and increase frequency of traveling each other. In this paper, we incorporate both social and transportation networks into equilibrium model of social interactions and examine how activity-travel behavior depends on topologies of these networks. By analyzing the model, we obtain the Nash equilibrium trip demand which is achieved as a result of the utility-maximizing behavior of the agents. We show that the market equilibrium is not optimal due to the existence of externalities. We also examine the value of the tax/subsidy that could support the first-best allocation as an equilibrium.

Keywords: Social Network, Travel Demand, Social Externality, Congestion

INTRODUCTION

Social networks are originally theoretical constructs to comprehend social interactions between people/ firms by depicting relations between them as graphs. Thus, they play a central role in various kinds of social and economic interactions between people/ firms.People and firms interact each other for communications or trades of goods and services. Face-to-face contacts are the most effective way of exchanging informations, knowledge and skills and they are inevitably accompanied by trips. In this sense, connections in social networks intrinsically induce people to travel.

Activity-travel behavior is also affected by their locations in the geographical space. Physical proximity enhances joint activities of agents and increases their frequencies of traveling each other. Conversely, physical distance is an impediment to interaction, that is, discourages to active interaction between people/ firms. In urban economics literatures, face-to-face contacts between economic agents are regarded as a fundamental of cities (Fujita and Ogawa, 1982[1)] ; Fujita and Thisse, 2002[2)] , etc.). One of reasons is that the agglomeration force is supposed to arise from external effects of social interactions such as knowledge spillover, matching and generation (Duranton and Puga, 2004[3)]).

Ballester, Calvò-Armengol and Zenou (2006)[4)] propose a simple model with strategic complementarities, in which the equilibrium effort level of agents on a social network coincides with Bonacich network centrality, which is a useful measure in game theoretic applications. Helsley and Zenou (2012)[5)] combine a geographic space into this model and examine how interaction choices depend on the interplay of social and physical distance. They show that there is a tendency for those who are more central in the social network to locate closer to the geographical center. However, they assume that the geographical space consists of only two locations, one is the city that is exogenously given as the interaction center and the other is the periphery.Due to these assumptions, they can examine a specific geographical situation.
In this paper, we consider general transportation networks and explicitly incorporate both social and transportation networks into the equilibrium model of social interactions. Our aim is to examine how position in the social network and geographical location determine the equilibrium level of travel demand0 and how social interactions depend on topologies of these networks. Moreover, we consider the positive externality brought by social interactions and the negative externality caused by the traffic congestion, and then analyze social welfare in scenarios where each and both externalities are internalized.
The main novel points of this paper are as follows. First, we extend Helsley and Zenou (2012) ‛s social interaction model into a multi-locational framework in order to declare the interdependence between spatial interaction and social connectivity. Second, we incorporate traffic congestion into the model and conduct a welfare analysis under the existence of both positive and negative externalities.
This paper is organized as follows. In the next section, we develop Helsley & Zenou's model and show that the equilibrium travel demand is expressed in a way quite similar to the Bonacich network centrality measure. It is also shown that trip distribution depends on the topology of the social network.Section 3 considers the traffic congestion and extends the model to analyze the relationship between social interaction and traffic congestion externality. We determine the value of the tax/subsidy that could support the first-best allocation as an equilibrium. Section 4 shows numerical analysis to illustrate the results obtained in the previous section. First, we analyze the effects of topologies of social networks on equilibrium interactions and we examine the relation between growth of social network and transportation network improvement. Second, we compare the equilibrium outcomes and the first-best outcome and discuss the alternative tax/subsidy policies. Concluding remarks are presented in section 5.

TRAVEL DEMAND MODEL BASED ON SOCIAL NETWORK THEORY

Settings

There are n agents in the economy, each of whose benefits are derived from interacting with others. The geographical space is expressed by a general transportation network, which consists of m zones and links between them. Each agent resides in one of these zones and it is assumed to be exogenously given. An agent travels to other agents' places and communicates with them to whom she is directly connected in the social network. An agent i $(i = 1, ..., n)$ determines her travel demand v_{ij} $(i \neq j)$ to maximize her utility subject to the budget constraint. The social network g is a set of identical agents $N = \{1, ..., n\}$, $n \geq 2$, and a set of *links* or *direct connections* between them. The adjacency matrix $\mathbf{G} = [g_{ij}]$ keeps track of the direct connections in the network. By definition, agents i and j are directly connected if and only if

$g_{ij} = 1$; otherwise, $g_{ij} = 0$. We assume that if $g_{ij} = 1$, then $g_{ji} = 1$, so the network is reciprocal. By convention, $g_{ii} = 0$. $\mathbf{G}$ is thus a square $(0, 1)$ symmetric matrix with zeros on its diagonal.

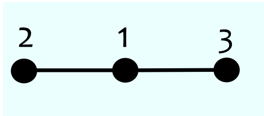


Figure 1: A social network g with 3 individuals

For example, the adjacency matrix $\mathbf{G}$ for the social network in Figure1 is as follows:

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

The geographical space is also a network. The geographical network r consists of m discrete zones and no or single link between any two of them. The residental location of agent i is denoted by x_i and all the agents' location $\mathbf{x} = \{x_1, ..., x_n\}$ is exogenously given. Consider the $n \times m$ matrix $\mathbf{X} = [\tilde{x}_i^h]$ ($\forall i = 1, ..., n, \forall h = 1, ..., m$), where $\tilde{x}_i^h = 1$ if and only if $x_i = h$ i. e. the agent i locates at zone h; otherwise, $\tilde{x}_i^h = 0$. $\mathbf{X}$ represents where each agent resides in the geographical space.
Using $\mathbf{X}$, we define the matrix $\mathbf{L} = [l_{ij}^{hh'}]$, which provides a mapping between any two agents, i and j, who have a direct link between them ($g_{ij} = 1$) and their locations. Its elements $l_{ij}^{hh'}$ is equal to 1 if and only if agent i's location is h and that of her friend j is h', and is equal to zero otherwise:

$$l_{ij}^{hh'} = \begin{cases} 1 & (\text{if and only if } g_{ij} = 1 \text{ and } \tilde{x}_i^h = \tilde{x}_j^{h'} = 1) \\ 0 & (\text{otherwise}) \end{cases}.$$

Every time an agent is to interact with her friend, she makes a trip to the friend's place using some of transportation links, which are labeled $a, b, ..., \Xi$. We assume that there is a shortest path, which are given, for each pair of origin and destination on the transportation network r.$\mathbf{R} = [r_{hh'}^{\xi}]$ ($\forall h, h' = 1, ..., m$, $\forall \xi = a, ..., \Xi$) is the *path-link incidence* matrix where

$$r_{hh'}^{\xi} = \begin{cases} 1 & (\text{if and only if the link } \xi \text{ is included in the path from } h \text{ to } h') \\ 0 & (\text{otherwise}) \end{cases}.$$

Assume $\mathrm{p}_{ik\xi} = \sum_{h=1}^{m} \sum_{h'=1}^{m} l_{ik}^{hh'} r_{hh'}^{\xi}$ denotes whether the transportation link ξ is included in the only path from the location of the agent i to that of the agent k. Then, the matrix $\mathbf{P} = [\mathrm{p}_{\xi ij}]$ is given by the simple matrix algebra, $\mathbf{P} = \mathbf{LR}$. Let ξ and $\mathbf{t} = [t_\xi]$ represent the marginal transport cost of link and t_{xi} ($xi = a, b, ..., \Xi$) and the Ξ-dimensional vector with coefficients t_ξ. Using these matrices, we can express the geographical space also as a graph.

Preferences

Agents derive utility from a numeraire good z and face-to-face contacts with others. The utility function is denoted by

$$U_i(\mathbf{v_i}, \mathbf{v_{-i}}, g, \mathbf{x}, r) = z_i + u_i(\mathbf{v_i}, \mathbf{v_{-i}}, g), \tag{1}$$

where $\mathbf{v_i}$ denotes the set of the number of agent i's visit to agent k: v_{ik}, $\mathbf{v_{-i}}$ is the corresponding vector of travel demands for other $n-1$ agents, and $u_i(\mathbf{v_i}, \mathbf{v_{-i}}, g)$ is the subutility function of interactions that is identical across individuals. We imagine that each activity-travel behavior results in a unit of interaction, so that the aggregate travel demand is a measure of aggregate interactivity. We assume that the subutility function takes the following form

$$u_i(\mathbf{v_i}, \mathbf{v_{-i}}, g) = \alpha \sum_{k=1}^{n} g_{ik} v_{ik} - \frac{1}{2} \sum_{k=1}^{n} g_{ik} v_{ik}^2 + \theta \sum_{k=1}^{n} \big(g_{ik} v_{ik} \sum_{l=1}^{n} g_{kl} v_{kl} \big), \tag{2}$$

where $\alpha > 0$ and $\theta > 0$ and both are constant. In the first two terms, we reflect a reasonable assumption that agents prefer to interact with more various agents (preferences for variety). Note that the marginal utility of v_{ik} is increasing in the travel demand of another with whom i is directly connected, $\partial^2 U_i / (\partial v_{ik} \cdot \partial v_{kl}) = \theta > 0$, for $g_{ik} = g_{kl} = 1$. Thus, v_{ik} and v_{kl} are strategic complements from i's perspective when $g_{ik} = 1$ as we assume that agents prefer to interact with other agents who have more interactions. When agents i and k have no direct link, i.e. $g_{ik} = 0$, this cross derivative is equal to zero. The parameter θ is a decay parameter that scales down the relative weight of longer walks in the social network.
Let y represent income and p_f represent the unit opportunity cost for face-to-face contacts. Recall that t_ξ represents the marginal transport cost of geographical link ξ, then budget balance is

$$y = z_i + p_f \sum_{k=1}^{n} v_{ik} + \sum_{k=1}^{n} \tau_{ik} v_{ik}, \tag{3}$$

where $\tau_{ik} = \sum_{h=1}^{m} \sum_{h'=1}^{m} \sum_{\xi=a}^{\Xi} t_\xi l_{ik}^{hh'} r_{hh'}^{\xi}$, which represents the marginal transport cost which agent i bears to visit agent k. Using this expression to substitute for z_i in (1), and using (2), gives

$$\begin{aligned} U_i(\mathbf{v_i}, \mathbf{v_{-i}}, g, x) = y - p_f \sum_{k=1}^{n} v_{ik} - \sum_{k=1}^{n} \tau_{ik} v_{ik} \\ + \alpha \sum_{k=1}^{n} g_{ik} v_{ik} - \frac{1}{2} \sum_{k=1}^{n} g_{ik} v_{ik}^2 + \theta \sum_{k=1}^{n} g_{ik} v_{ik} \sum_{l=1}^{n} g_{kl} v_{kl}. \end{aligned} \tag{4}$$

We assume $\alpha > p_f$ and also $(\alpha - p_f) > \tau_{ik}$ ($\forall ik = 11, 12, ..., nn$). Each agent i chooses v_{ik} to maximize (4) taking the structure of the social network and transportation network and the visit choices of other agents as given.

Nash equilibrium travel demand

The optimal conditions for a maximum of (4) with respect to v_{ij} are that for each $ij = 11, 12, ..., nn$:

$$\frac{\partial U_i}{\partial v_{ij}} \cdot v_{ij} = 0, \tag{5}$$

$$\frac{\partial U_i}{\partial v_{ij}} = g_{ij} \Big(\alpha - v_{ij} + \theta \sum_{l=1}^{n} g_{jl} v_{jl} \Big) - p_f - \tau_{ij} \leq 0, v_{ij} \geq 0. \tag{6}$$

This equation gives the best-response equation for $g_{ij} \neq 0$, i. e., $g_{ij} = 1$:

$$v_{ij} = \alpha - p_f - \tau_{ij} + \theta \sum_{l=1}^{n} g_{jl} v_{jl} \tag{7}$$

On the other hand, for $g_{ij} = 0$, $\partial U_i / \partial v_{ij} < 0$ and thus $v_{ij} = 0$ is the best response.
For all pairs of agents i, j who are directly connected in the social network, equations (7) need to be satisfied. Simultaneously solving these linear equations in terms of v_{ij}, we obtain the Nash equilibrium level of travel demand.
We now define the variables $\hat{g}_{ij,kl}$, which express adjacency relations of links in the social network g.

$$\hat{g}_{ij,kl} = \begin{cases} g_{ij} \times g_{kl} & (\text{if } j = k) \\ 0 & (\text{otherwise}) \end{cases}.$$

Consider a matrix $\hat{\mathbf{G}}$ with coefficients $\hat{g}_{ij,kl}$ for all set of links ij, kl which satisfy $g_{ij} = g_{kl} = 1$. $\hat{\mathbf{G}} = [\hat{g}_{ij,kl}]$ $(\forall i, j, k, l \in \mathbf{N})$ keeps track of the ripple of links in the network: if there is a direct link between agents i and j, i.e. agents i and j are directly connected, the flow from i to j is influenced by that from j to l for all $g_{jl} = 1$. Reconstructing the social network g of Figure 1 by replacing links with nodes and vice versa, we can depict the network $\hat{g}$ as Figure 2 and then show the link-based adjacency matrix $\hat{\mathbf{G}}$.

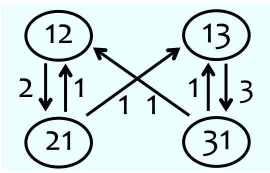


Figure 2: An Example of Link-Based Network $\hat{g}$

$$\hat{\mathbf{G}} = \begin{array}{c} \\ 12 \\ 13 \\ 21 \\ 31 \end{array} \begin{array}{c} \begin{array}{cccc} 12 & 13 & 21 & 31 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array} \tag{8}$$

In matrix form the system in (7) becomes

$$(\alpha - p_f)\mathbf{1} - \boldsymbol{\tau} - \mathbf{v} + \theta \hat{\mathbf{G}} \mathbf{v} = \mathbf{0}, \tag{9}$$

where $\boldsymbol{\tau} = \mathbf{Pt}$ is the transport cost vector with coefficients τ_{ij}, and $\mathbf{0}$ and $\mathbf{1}$ is the vector of zeros and ones.
Solving for $\mathbf{v}$ gives the Nash equilibrium vector $\mathbf{v}^*$:

$$\mathbf{v}^* = [\mathbf{I} - \theta \hat{\mathbf{G}}]^{-1} \cdot \{(\alpha - p_f)\mathbf{1} - \boldsymbol{\tau}\}. \tag{10}$$

Moreover, the Nash equilibrium level of travel demand from agent i to j is given by:

$$\begin{aligned} v^*_{ij}(\mathbf{x}, \hat{g}) &= \sum_{k=1}^{n}\sum_{l=1}^{n}\sum_{\gamma=0}^{+\infty} \theta^{\gamma} g^{[\gamma]}_{ij,kl}\alpha_{kl} \\ &= b_{\alpha_{ij}}(\hat{g}, \theta) \end{aligned} \tag{11}$$

where $\alpha_{kl} = \alpha - p_f - \tau_{kl}$ indicates geographical proximity between agents i and j. In comparison to (30) and (29) (see Appendix I.), equilibrium level of travel demand v^*_{ij} is expressed by Bonacich centrality measure of the directed link $i \rightarrow j$, that incorporates individuals' preference(love of variety and strategic complementarity) and both of opportunity and transportation cost accompanying face-to-face meetings.

Proposition 1 (Equilibrium travel demand) *Assume $\theta\rho(\hat{\mathbf{G}}) < 1$ is satisfied. Then, for any network g, there exist a unique, interior Nash equilibrium in travel demand choices. Under the link-based adjacency graph $\hat{g}$, the level of interactivity from any agent i to any other agent j equals to the Bonacich centrality of the directed link from i to j, weighted by the geographical proximity α.*

Let $V^*_i(g)$ represent the equilibrium aggregate level of travel demand of agent i. From (11), we have

$$\begin{aligned} V^*_i(\mathbf{x}, g) &= \sum_{j=1}^{n} v^*_{ij}(\mathbf{x}, \hat{g}) \\ &= \sum_{j=1}^{n} b_{\alpha_{ij}}(\hat{g}, \theta). \end{aligned} \tag{12}$$

Furthermore, using the first order condition (6), we can write the equilibrium utility level of agent i as following:

$$U^*_i(\mathbf{v}^*_{\mathbf{i}}, \mathbf{v}^*_{-\mathbf{i}}, g, \mathbf{x}, r) = y + \frac{1}{2}\sum_{k=1}^{n} g_{ik} {v^*_{ik}}^2 = y + \frac{1}{2}\sum_{j=1}^{n}\left[b_{\alpha_{ij}}(\hat{g}, \theta)\right]^2. \tag{13}$$

This expression, where the utility of agent i is specified by her own travel demand is to be useful for the analysis of location choices of agents in the future applications.
In addition, tallying v^*_{ij} according to each origin and destination pair gives the OD travel demand V^*_{od}:

$$V^*_{od} = \sum_{i=1}^{n}\sum_{j=1}^{n} l^{od}_{ij} v^*_{ij},$$

where o and d correspond to one of zones $1, ..., m$, $o \neq d$, and doing so to each link gives the link travel demand V^*_{ξ} as below:

$$V^*_{\xi} = \sum_{h=1}^{m}\sum_{h'=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} l^{hh'}_{ij} r^{\xi}_{hh'} v^*_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{p}_{\xi ij} v^*_{ij}.$$

These results are basically of the same kind as Helsley and Zenou's, but our developed framework enables us to deal with general transportation networks.
In order to illustrate previous results, we consider the simple example of the social network g with three agents shown in Figure 1 and the transportation network r with three zones (i. e. $m = 3$) and 2 links in Figure 3, where agent 1 resides in zone 1, agent 2 does in zone 2, the geographical centre, and agent 3 does in zone 3.

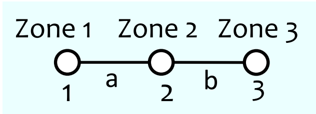


Figure 3: A transportation network with 3 zones

Under this assumption we can compute the simple commuting cost vector $\boldsymbol{\tau}$ as

$$\boldsymbol{\tau} = \mathbf{LRt} = \begin{bmatrix} \tau_{12} \\ \tau_{13} \\ \tau_{21} \\ \tau_{31} \end{bmatrix} = \begin{bmatrix} t_a \\ t_a + t_b \\ t_a \\ t_a + t_b \end{bmatrix}. \tag{14}$$

$\mathbf{L}, \mathbf{R}$ and $\mathbf{t}$ are described in detail in Appendix III. Supported by these matrices, we now can compute the activity-travel demand $\mathbf{v}^*$ as following:

$$\begin{aligned} \begin{bmatrix} v^*_{12} \\ v^*_{13} \\ v^*_{21} \\ v^*_{31} \end{bmatrix} &= \begin{bmatrix} b_{\alpha_{12}}(\hat{g}, \theta) \\ b_{\alpha_{13}}(\hat{g}, \theta) \\ b_{\alpha_{21}}(\hat{g}, \theta) \\ b_{\alpha_{31}}(\hat{g}, \theta) \end{bmatrix} \\ &= \frac{1}{1-2\theta^2} \begin{bmatrix} 1-\theta^2 & \theta^2 & \theta-\theta^3 & \theta^3 \\ \theta^2 & 1-\theta^2 & \theta^3 & \theta-\theta^3 \\ \theta & \theta & 1-\theta^2 & \theta^2 \\ \theta & \theta & \theta^2 & 1-\theta^2 \end{bmatrix} \cdot \begin{bmatrix} \alpha_{12} \\ \alpha_{13} \\ \alpha_{21} \\ \alpha_{31} \end{bmatrix} \\ &= \frac{1}{1-2\theta^2} \begin{bmatrix} (1+\theta)(\alpha - p_f - t_a - \theta^2 t_b) \\ (1+\theta)\{\alpha - p_f - t_a - (1-\theta^2)t_b\} \\ (1+2\theta)(\alpha - p_f - t_a) - (\theta+\theta^2)t_b \\ (1+2\theta)(\alpha - p_f - t_a) - (1+\theta-\theta^2)t_b \end{bmatrix} \end{aligned} \tag{15}$$

We see from (15) that the commuting cost t_b affects the level of travel demand between agents 1 and 2, who does not use link b in visiting each other. In this case, under the condition for the existence of the inverse matrix $[\mathbf{I} - \theta\hat{\mathbf{G}}]^{-1}$, $\theta < 1/\sqrt{2}$ (see Appendix I. for the detail), it is obvious that $v^*_{12} > v^*_{13}$ and $v^*_{21} > v^*_{31}$. It indicates that agents 1 and 2 more actively interact with each other than agents 1 and 3 owing to their physically closer locating.
Consider two social networks g and g', such that for all i, j $g'_{ij} = 1$ if $g_{ij} = 1$, i.e. $g \subset g'$. This implies that the network g' has a denser structure of network links. In other words, some agents who are not directly connected in g are directly connected in g'. We can assume g' as an expanded or developed network of g. The growth of social network g influences not only on agents who has acquired additional friends but also on the whole network and causes an

increase in the equilibrium level of travel demand v_{ij}^* for all i, j and thus from (13) results in a rise in the equilibrium utility level U_i^* for all i.
Next we consider sufficiently small θ and θ' and assume $\theta < \theta'$. From (11), compared to θ, θ' brings about increase in the travel demand between any agents i and j ($\forall i, j = 1, ..., n$). In addition, from (13) the equilibrium level of utility U_i^* increases for all agent i.
We can now suppose the following result:

Proposition 2 (Equilibrium level of interactivity and strategic complementarity) *Assume a sufficiently small θ is given and consider network g and $g', g' \neq g$ as social networks such that $g \subset g'$. Then, given the complementarities in the network, we have*

$$v_{ij}^*(g) < v_{ij}^*(g') \tag{16}$$

and from (13) *and* (16)*,*

$$U_i^*(g) < U_i^*(g') \tag{17}$$

for all agent $i(\forall i = 1, ..., n)$ are induced.
Likewise, consider sufficiently small θ and θ' such that $\theta < \theta'$. Then, from (11) *and* (13) *we have*

$$v_{ij}^*(\theta) < v_{ij}^*(\theta') \tag{18}$$

and

$$U_i^*(\theta) < U_i^*(\theta') \tag{19}$$

TRAVEL DEMAND AND EXTERNALITIES

In this section, we extend the model and consider the externalities and social welfare.We modify the transportation cost in the budget balance (3) to capture the traffic congestion externality on travel demand, while the transportation cost is described in a simple linear form in the previous section. In terms of interest in other agents' behavior, the transportation cost and the factor of strategic complementarity both include technological externalities; The former is a negative externality and the latter is a positive one. This analysis well describe the two essential forces, the attraction force and repulsive force, of spatial interactions. We assume the path between any two agents i and j is fixed and do not deal with traffic assignment problem.

Equilibrium interactions with congestion

This section extends our model of social networks to consider the congestion externality. The budget balance of agent i who is interested in the impact of her own marginal travel on traffic congestion is given by

$$y = z_i + p_f \sum_{k=1}^{n} v_{ik} + \sum_{k=1}^{n} \left(v_{ik} \sum_{\xi=a}^{\Xi} \mathrm{p}_{ik\xi} t_\xi \sum_{k'=1}^{n} \sum_{k''=1}^{n} \mathrm{p}_{\xi k' k''} v_{k' k''} \right). \tag{20}$$

In the way resembling the derivation of (7), we see that

$$g_{ij}\left(\alpha - v_{ij} + \theta\sum_{l=1}^{n} g_{jl}v_{jl}\right) - p_f - \sum_{k=1}^{n} v_{ik}\sum_{\xi=a}^{\Xi}\mathrm{p}_{ik\xi}t_\xi\mathrm{p}_{\xi ij} - \sum_{\xi=a}^{\Xi}\mathrm{p}_{ij\xi}t_\xi\sum_{k'=1}^{n}\sum_{k''=1}^{n}\mathrm{p}_{k'k''\xi}v_{k'k''} = 0 \quad (21)$$

need to be satisfied for $g_{ij} = 1$. The system is represented in the matrix expression:

$$(\alpha - p_f)\mathbf{1} - (\mathbf{\Lambda}' + \mathbf{\Lambda})\mathbf{v} - \mathbf{v} + \theta\hat{\mathbf{G}}\mathbf{v} = \mathbf{0}.$$

$\mathbf{\Lambda} = [\lambda_{ij,kl}]$ is defined by $\mathbf{\Lambda} = \mathbf{P}^T\mathbf{C}(\mathbf{t})\mathbf{P}$ where $\mathbf{C}(\mathbf{t})$ is a Ξ-square diagonal matrix with coefficients t_ξ $(\forall\xi = a, ..., \Xi)$ and the elements of $\mathbf{\Lambda}' = [\lambda'_{ij,kl}]$ is defined as follows:

$$\lambda'_{ij,kl} = \begin{cases}\lambda_{ij,kl} & (\text{if } j = k)\\ 0 & (\text{otherwise})\end{cases}.$$

Solving this for $\mathbf{v}$ gives the unique and interior (since it is a linear system) Nash equilibrium travel demand vector:

$$\mathbf{v}^* = [\mathbf{\Lambda}' + \mathbf{\Lambda} + \mathbf{I} - \theta\hat{\mathbf{G}}]^{-1}\cdot(\alpha - p_f)\mathbf{1}, \quad (22)$$

where the equilibrium level of travel demand is determined in a form similar to the Bonacich centrality measure. The equilibrium level of utility of agent i can be written as:

$$U_i = y - p_f\sum_{k=1}^{n} v_{ik} - \sum_{k=1}^{n}\left(v_{ik}\sum_{\xi=a}^{\Xi}\mathrm{p}_{ik\xi}t_\xi\sum_{k'=1}^{n}\sum_{k''=1}^{n}\mathrm{p}_{\xi k'k''}v_{k'k''}\right) + \alpha\sum_{k=1}^{n} g_{ik}v_{ik} - \frac{1}{2}\sum_{k=1}^{n} g_{ik}v_{ik}^2 + \theta\sum_{k=1}^{n} g_{ik}v_{ik}\sum_{l=1}^{n} g_{kl}v_{kl}, \quad (23)$$

which can be rewritten into, in the same way as (13),

$$U_i^* = y + \frac{1}{2}\sum_{k=1}^{n} g_{ik}v_{ik}^{*2} + \sum_{k=1}^{n} v_{ik}^*\sum_{\xi=a}^{\Xi}\mathrm{p}_{ik\xi}t_\xi\sum_{k'=1}^{n}\mathrm{p}_{\xi ik'}v_{ik'}^*. \quad (24)$$

Like (13), the utility of agent i is specified by her own travel demand. This is also to be useful in the future applications.

Externalities and optimal level of social interactions

Solving the equilibrium model under internalizing the externalities derives the social optimal outcome $\mathbf{v}^O$(the subsctipt O refers to the "social optimum" while a star refers to the "Nash equilibrium" outcome). There are two types of externalities: the negative traffic congestion and the positive externality in social interactions. The latter is assumed as a fruit of strategic complementarities. To maximize the social welfare, the planner chooses $v_{ij}, \forall ij = 11, 12, ..., nn$

that maximize total welfare, that is:

$$\begin{aligned}\max_{\mathbf{v}} W &= \max_{v_{11},...,v_{nn}} \sum_{i=1}^{n} U_i \\ &= \max_{v_{11},...,v_{nn}} \sum_{i=1}^{n} \Bigg\{ y - p_f \sum_{k=1}^{n} v_{ik} - \sum_{k=1}^{n} \Bigg(v_{ik} \sum_{\xi=a}^{\Xi} \mathrm{p}_{ik\xi} t_\xi \sum_{k'=1}^{n} \sum_{k''=1}^{n} \mathrm{p}_{\xi k' k''} v_{k'k''} \Bigg) \\ &\qquad + \alpha \sum_{k=1}^{n} g_{ik} v_{ik} - \frac{1}{2} \sum_{k=1}^{n} g_{ik} v_{ik}^2 + \theta \sum_{k=1}^{n} g_{ik} v_{ik} \sum_{l=1}^{n} g_{kl} v_{kl} \Bigg\}. \quad (25)\end{aligned}$$

Analogously to the derivation of (22), the socially optimal level of travel demand in matrix form is given by:

$$\mathbf{v}^O = [2\mathbf{\Lambda} + \mathbf{I} - \theta(\hat{\mathbf{G}} + \hat{\mathbf{G}}^T)]^{-1} \cdot (\alpha - p_f)\mathbf{1}. \quad (26)$$

By internalizing only the traffic congestion, the travel demands $\mathbf{v}^{(1)}$ are achieved:

$$\mathbf{v}^{(1)} = [2\mathbf{\Lambda} + \mathbf{I} - \theta\hat{\mathbf{G}}]^{-1} \cdot (\alpha - p_f)\mathbf{1}, \quad (27)$$

which results from the choice of each agent, who counts the marginal social cost of congestion but neglects the marginal social benefit of interactions.
On the other hand, internalizing only the positive externality of social interactions gives the travel demands $\mathbf{v}^{(2)}$:

$$\mathbf{v}^{(2)} = [\mathbf{\Lambda}' + \mathbf{\Lambda} + \mathbf{I} - \theta(\hat{\mathbf{G}} + \hat{\mathbf{G}}^T)]^{-1} \cdot (\alpha - p_f)\mathbf{1}. \quad (28)$$

This is the results of the choice of each agent counting the positive impact of others' effort on her but ignoring the negative impact on the traffic congestion. The above solutions are all analytically solvable and thus applicable to arbitrary network. In the following section, we show case studies to examine what those results suggest in detail.

NUMERICAL ANALYSIS—CASE STUDIES

We present a number of numerical example to illustrate the theoretical results. Consider the following social networks (SN1- SN6) each of which consists of 5 agents(i.e. $n = 5$) and the geography, that is, the transportation network (TN), which consists of 5 zones (i.e. $m = 5$) and links between them. In the experiments, we try combinations of each of these 6 social networks and the transportation network. We set the parameters as $y = 5, \alpha = 1, p_f = 0.1, t_\xi = 0.01$ ($\forall \xi = a, ..., e$), $\theta = 0.1$. We now show two numerical analysis as follows.

Topology of social networks and equilibrium interactions

Consider six cases with respective social network SN1- SN6 and the common transportation network TN in Figure 4. SN1 is tandem; SN2 is a kind of tree networks; SN3 is a star network; SN4 is circle; SN5 is a combination of star and circle; and SN6 is the complete network. Table 1 and Table 2 declare the relation between topology of social network and equilibrium outcomes.

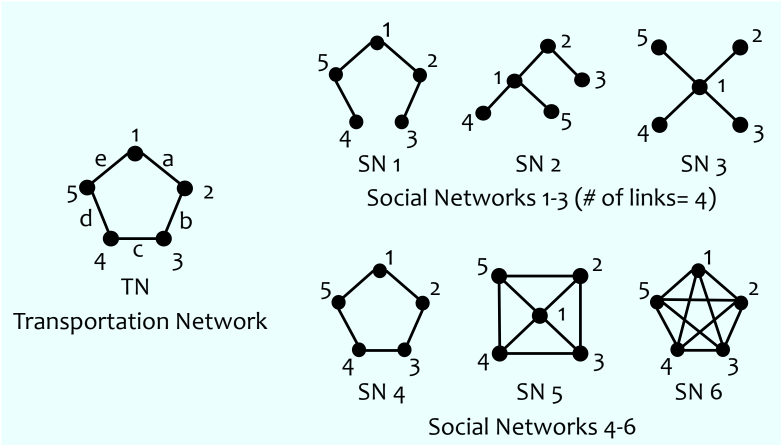


Figure 4: Social networks and transportation networks

Table 1: Social Network and Travel Demand (SN 1-3)

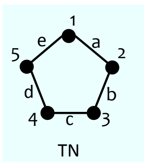
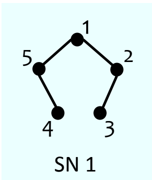
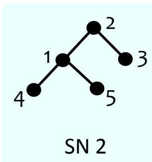
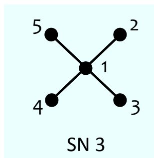
							
i	j	v_{ij}	$\sum_j v_{ij}$ (U_i)	v_{ij}	$\sum_j v_{ij}$ (U_i)	v_{ij}	$\sum_j v_{ij}$ (U_i)
1	2	1.087	2.173 (6.181)	1.095	3.032 (6.527)	0.987	3.906 (6.881)
	3	0		0		0.966	
	4	0		0.958		0.966	
	5	1.087		0.979		0.987	
2	1	1.095	2.084 (6.088)	1.18	2.169 (6.185)	1.265	1.265 (5.776)
	3	0.989		0.99		0	
	4	0		0		0	
	5	0		0		0	
3	1	0	1.087 (5.592)	0	1.095 (5.601)	1.241	1.241 (5.749)
	2	1.087		1.095		0	
	4	0		0		0	
	5	0		0		0	
4	1	0	1.087 (5.592)	1.157	1.157 (5.649)	1.241	1.241 (5.749)
	2	0		0		0	
	3	0		0		0	
	5	1.087		0		0	
5	1	1.095	2.084 (6.088)	1.18	1.18 (5.673)	1.265	1.265 (5.776)
	2	0		0		0	
	3	0		0		0	
	4	0.989		0		0	
$V_i = \sum_i v_i$ ($W = \sum U_i$)			8.515 (29.54)		8.633 (29.64)		8.919 (29.93)

Table 1 can compare the differences among outcomes resulting from topological variations of social networks which have respectively four social links within while Table 2 let us consider the growth of a social network from SN4 to SN6, which is mentioned in Proposition 2.
Table 1 suggests the influences of positions in social networks upon the travel demand. Agent 1 is the most central player in each social network and achieves the highest travel demand level in every case. The level of an agent's travel demand is largely dependent of the agent's position in the social network. Agent 1 in SN3 who has the largest number of direct links ($= 4$) achieves the highest aggregate travel demand in each case of Table 1. We can say that under the values of those parameters the concentration of the interaction toward agent 1 enhances the effect of strategic complementarity of face-to-face communication. Agent 1 also achieves the highest level of utility in each case.
Relative positions in geography also affects on the levels of interactivity. Interestingly, in the case with SN3 all agents except for agent 1 have the same position in the social network, but travel demand level of agents 2 and 5 is higher than that of agents 3 and 4. The agents who are geographically closer to the socially central agent are more interactive than the others even if they are socially in the symmetric positions.The locational advantage results in a higher centrality and thus causes higher level of interaction demand.
Let us turn to the cases with SN4, SN5, and SN6. SN4 has five links and it is the sparsest social network; SN5 has eight links; SN6 has ten links and the densest one. Table 2 indicates that agents in denser social network provide more travel to their friends and derive more benefits. Individual travel demand levels v_i also increase with the dense of social links for all $i, \forall i = 1, ..., 5$. Furthermore, for instance, agent 1 interacts more with agent 2 in SN5 than in SN4 and much more in SN6 than in SN5. These results suggest that agents within social network g', previously mentioned in Proposition 2, such that $g \subset g'$ travel to interact more and experience higher utility than those in g even if there is congestion in this case.

First-best analysis and social welfare

Next, we consider the case with SN5 and TN2 where agent 1 holds central position and agents $2, ..., 5$ are peripherals in the social network while all the agents are symmetric in the geography as Figure 5 indicates.

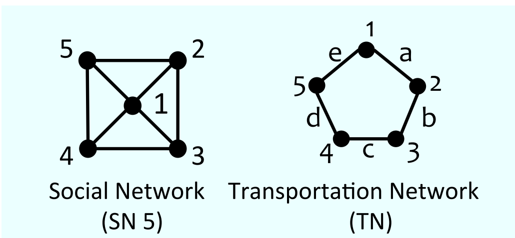
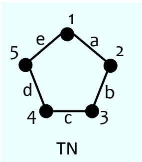
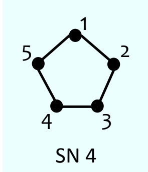
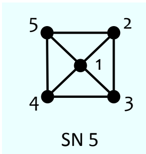
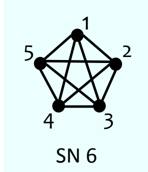


Figure 5: Example of Social and Transportation Network

We here would like to see whether the initial equilibrium outcomes are efficient. The social optimal outcome v_i^O is more than v_i^*, $\forall i = 1, ..., 5$ and thus there are less travel demand at the Nash equilibrium as compared to the social optimum outcome. Equilibrium interaction level differs from the social optimal level because each agent ignores the positive impact of an

Table 2: Social Network and Travel Demand (SN 4-6)

							
i	j	v_{ij}	$\sum_j v_{ij}$ (U_i)	v_{ij}	$\sum_j v_{ij}$ (U_i)	v_{ij}	$\sum_j v_{ij}$ (U_i)
1	2	1.098		1.231		1.399	
	3	0	2.195	1.206	4.875	1.371	5.539
	4	0	(6.205)	1.206	(8.532)	1.371	(8.532)
	5	1.098		1.231		1.399	
2	1	1.098		1.337		1.399	
	3	1.098	2.195	1.254	3.796	1.399	5.539
	4	0	(6.205)	0	(7.252)	1.371	(8.532)
	5	0		1.205		1.371	
3	1	0		1.31		1.371	
	2	1.098	2.195	1.229	3.793	1.399	5.539
	4	1.098	(6.205)	1.254	(7.338)	1.399	(8.532)
	5	0		0		1.371	
4	1	0		1.31		1.371	
	2	0	2.195	0	3.793	1.371	5.539
	3	1.098	(6.205)	1.254	(7.338)	1.399	(8.532)
	5	1.098		1.229		1.399	
5	1	1.098		1.337		1.399	
	2	0	2.195	1.205	3.796	1.371	5.539
	3	0	(6.205)	0	(7.252)	1.371	(8.532)
	4	1.098		1.254		1.399	
$V_i = \sum_i v_i$ ($W = \sum U_i$)			10.98 (31.02)		20.05 (36.61)		27.7 (42.66)

interaction on the interaction choices of others and the negative effect of a travel on the travel demand choices of others. In other words, each agent ignores both the positive externality arising from complementarity in interaction choices and the negative externality resulting from traffic congestion. As a result, the market equilibrium is not efficient. It is necessary for the planner to suggest the first-best congestion tax and subsidy for interactions policy in order to restore the optimum outcome W^O.
Consider the alternative policies: (1) the marginal congestion tax policy to internalize the negative externality caused by the traffic congestion and (2) the subsidy for interactions policy. Comparing (2) (col.3) to the equilibrium (col.1) shows the increase in the total travel demand and the improvement both in each agent's utility and social welfare while (1) (col.2) indicates the downturn both in the total travel demand and in the social welfare than Equilibrium. This implies that internalizing only one kind of externalities, it is the congestion externality here, would cause the worsening in the social welfare.

Table 3: Travel Demand and Social Welfare

		Equilibrium		(1)		(2)		Social Optimum	
i	j	v_{ij}^*	$\sum_j v_{ij}^*$ (U_i^*)	$v_{ij}^{(1)}$	$\sum_j v_{ij}^{(1)}$ ($U_i^{(1)}$)	$v_{ij}^{(2)}$	$\sum_j v_{ij}^{(2)}$ ($U_i^{(2)}$)	v_{ij}^O	$\sum_j v_{ij}^O$ (U_i^O)
1	2	1.231	4.875 (7.78)	1.163	4.548 (7.715)	2.202	8.717 (7.808)	1.977	7.753 (8.054)
	3	1.206		1.111		2.157		1.899	
	4	1.206		1.111		2.157		1.899	
	5	1.231		1.163		2.202		1.977	
2	1	1.337	3.796 (7.252)	1.262	3.547 (7.186)	2.549	6.944 (7.299)	2.289	6.182 (7.463)
	3	1.254		1.202		2.243		2.057	
	4	0		0		0		0	
	5	1.205		1.082		2.152		1.836	
3	1	1.31	3.793 (7.338)	1.203	3.611 (7.26)	2.5	6.937 (7.585)	2.185	6.294 (7.62)
	2	1.229		1.184		2.195		2.013	
	4	1.254		1.224		2.243		2.096	
	5	0		0		0		0	
4	1	1.31	3.793 (7.338)	1.203	3.611 (7.26)	2.5	6.937 (7.585)	2.185	6.294 (7.62)
	2	0		0		0		0	
	3	1.254		1.224		2.243		2.096	
	5	1.229		1.184		2.195		2.013	
5	1	1.337	3.796 (7.252)	1.262	3.547 (7.186)	2.549	6.944 (7.299)	2.289	6.182 (7.463)
	2	1.205		1.082		2.152		1.836	
	3	0		0		0		0	
	4	1.254		1.202		2.243		2.057	
$V_i = \sum_i v_i$ ($W = \sum U_i$)			20.05 (36.96)		18.86 (36.61)		36.48 (37.57)		32.71 (38.22)

CONCLUDING REMARKS

This paper provides a novel framework to estimate origin-destination trip demand which originated with a social network. In this paper, we have incorporated both social and transportation networks into an equilibrium model of social interactions and examined how activity-travel behavior depends on topology of these networks. Our model describes the spatial propagation patterns of social interactions with two kinds of externalities: the positive externality arising from complementarity in interaction choices and the negative externality resulting from traffic congestion. Through the numerical analysis, we show that internalizing only one of these externalities may cause the worsening in the social welfare. We have applied to the analysis of firms' location choice and, furthermore, that of spatial structure of cities.[9)] Our model can make up for traditional demand forecast models that cannot explicitly take into account social linkages between people, firms or cities. Empirical analysis is an issue in a future research.

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APPENDIX

Appendix I. The Bonacich Network Centrality Measure

There are many ways to measure the importance or centrality of an agent in a social network. For example, degree centrality measures importance by the number of direct connections that an agent has with all others, while closeness centrality measures importance by the average distance (in terms of links in the network)between an agent and all others. Before analyzing our game, we introduce a useful measure of an agent's importance in the social network, a Bonacich network centrality measure.[7)] This has proven to be extremely useful in game theoretic applications[4)] and presumes that the power or prestige of a node is simply a weighted sum of the walks emanate from it.
The n-square adjacency matrix $\mathbf{G}$ of a network g keeps track of the direct connections in this network. By definition, i and j are directly connected in g if and only if $g_{ij} > 0$, in which case $0 \leq g_{ij} \leq 1$ measures the weight associated to this direct connection. Let $\mathbf{G}^k$ be the kth power of $\mathbf{G}$, where is k is some integer. The matrix $\mathbf{G}^k$ keeps track of the indirect connections in the network: $g_{ij}^{[k]} \geq 0$ measures the number of paths of length $k \geq 1$ in g from i to j. In particular, $\mathbf{G}^0 = \mathbf{I}$ (I is the identity matrix).

Given a scalar $\theta \geq 0$ and a network g, we define the matrix

$$\mathbf{M}(g,\theta) = [\mathbf{I} - \theta\mathbf{G}]^{-1} = \sum_{k=0}^{+\infty} \theta^k \mathbf{G}^k.$$

These expressions are all well defined for small enough values of θ.
The matrix power series $\sum_{k=0}^{+\infty} \theta^k \mathbf{G}^k$ converges if and only if

$$\|\mathbf{G}\| < r = \liminf_{k\to\infty} |\theta^k|^{-1/k} = \frac{1}{\theta}$$

where r is the radius of convergence and $\|\mathbf{G}\|$ is the "norm" of the matrix $\mathbf{G}$. This norm is generally taken to be the "spectral radius" of $\mathbf{G}$, written $\rho(\mathbf{G}) = \max_i |\lambda_i|$, where λ_i is the eigenvalue of $\mathbf{G}$. Thus the matrix power series converges, and $\mathbf{M}$ is well-defined, for $\theta\rho(\mathbf{G}) < 1$.
The parameter θ is a decay factor that scales down the relative weight of longer paths. If $\mathbf{M}(g,\theta)$ is a nonnegative matrix, its coefficients $m_{ij}(g,\theta) = \sum_{k=0}^{+\infty} \theta^k g_{ij}^{[k]}$ count the number of paths in g that start at i and end at j, where paths of length k are weighted by θ^k.

> DEFINITION: Consider a network g with adjacency n-square matrix $\mathbf{G}$ and a scalar θ such that $\mathbf{M}(g,\theta) = [\mathbf{I} - \theta\mathbf{G}]^{-1}$ is well defined and nonnegative. Let $\mathbf{1}$ be the vector of ones. The vector of Bonacich centralities of parameter θ in g is
>
> $$\mathbf{b}(g,\theta) = [\mathbf{I} - \theta\mathbf{G}]^{-1} \cdot \mathbf{1} \tag{29}$$
>
> and the Bonacich centrality of agent i is
>
> $$b_i(g,\theta) = \sum_{j=1}^{n} m_{ij} = \sum_{j=1}^{n}\sum_{k=0}^{+\infty} \theta^k g_{ij}^{[k]} \tag{30}$$

The Bonacich centrality of node i is $b_i(g,\theta) = \sum_{j=1}^{n} m_{ij}(g,\theta)$ and it counts the total number of paths in g that start at i. It is the sum of all loops $m_{ii}(g,\theta)$ from i to i itself and of all the outer paths $\sum_{j\neq i} m_{ij}(g,\theta)$ from i to every other player $j \neq i$, that is,

$$b_i(g,\theta) = m_{ii}(g,\theta) + \sum_{j\neq i} m_{ij}(g,\theta).$$

By definition, $m_{ii}(g,\theta) \geq 1$ and thus $b_i(g,\theta) \geq 1$, with equality when $\theta = 0$.

Appendix II. Derivation of (13)

Using (6), we obtain

$$\theta g_{ij} \sum_{l=1}^{n} g_{jl} v_{jl} = -(\alpha - v_{ij}) g_{ij} + p_f + \tau_{ij}.$$

We multiply v_{ij} on the both sides and take summation with respect to j to get

$$\theta \sum_{j=1}^{n} g_{ij} v_{ij} \sum_{l=1}^{n} g_{jl} v_{jl} = -\alpha \sum_{j=1}^{n} g_{ij} v_{ij} + \sum_{j=1}^{n} g_{ij} v_{ij}^2 + \sum_{j=1}^{n} (p_f + \tau_{ij}) v_{ij}.$$

Using this expression to substitute in (4) gives (13).

Appendix III. Matrices of the Geography in Figure 3

The path-link incidence matrix $\mathbf{R}$ represents the transportation network r independently from the location of agents. Each row of $\mathbf{R}$ corresponds to the pairs of zones (h, h') $(h, h' = 1, 2, ..., m)$ and each column does to the transport links ξ $(\xi = a, b, ...\Xi)$. In the case of Figure 3, link a connects zones 1 and 2 and zones 2 and 3 are connected by link b while there is no link which connects zones 1 and 3 directly. Therefore, $\mathbf{R}$ is given by:

$$\mathbf{R} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}. \tag{31}$$

$\mathbf{t}$ is the column vector with coefficients t_ξ $(\forall \xi = a, ..., \Xi)$. In the transportation network of Figure 3, it consists of the marginal costs to use link a, t_a and link b, t_b.
The matrix $\mathbf{X}$ represents the geography, which combines the zones of the transportation network and the locations of agents. The i-th row corresponds to agent i (in this case, $i = 1, 2, 3$). The (i, h) element is of value 1 in $\mathbf{X}$ if and only if agent i resides in zone h ($h = 1, 2, 3$).

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{32}$$

$\mathbf{L}$ indicates each pair of agents of the social network and their residental locations. Each row of $\mathbf{L}$ corresponds to the pairs of directly connected agents (i, j), and each column does to different two zones (h, h'). For instance, agents 1 and 3 are directlr connected in the social network of this case and $\tilde{x}_1^1 = \tilde{x}_3^3 = 1$ is given in $\mathbf{X}$, therefore the second row, corresponding to the pair of agents $(1, 3)$, $l_{13}^{13} = 1$ and the other factors in this row are equal to 0.

$$\mathbf{L} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{33}$$

The transportation cost τ_{ij} for all i, j are computed by $\boldsymbol{\tau} = \mathbf{LRt}$.