INDICES FOR IMPROVEMENT OF HIGHWAY NETWORK RELIABILITY:
COMPARATIVE STUDY OF IMPORTANCE INDICES IN NETWORK RELIABILITY IMPROVEMENT

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ABSTRACT

It is very important to keep highly reliable traffic networks for both normal periods and abnormal periods. Network reliability can be improved effectively by improving the key links in the network. Once such key links have been identified, network reliability can be efficiently improved and maintained. To find the key links, several importance indices such as Birnbaum's importance (*RI*) and criticality importance (*CI*) have been proposed. However, *RI* does not reflect that to improve a higher reliable link is more difficult than to improve a less reliable link. In a parallel network, in addition, use of *RI* and *CI* leads to unreasonable result that the more reliable links should be improved, and the less reliable links will be ignored. Thus, an advanced index of the criticality importance (*CIW*) has been proposed. However, the degree of importance is the same for the links in a parallel network. Thus, none of these indices can obtain a good solution for improving network reliability by the theoretical analysis. In order to identify the efficiency of these importance indices for improving network reliability, a cost-benefit analysis combined with these importance indices is made into the point paying our attention in this paper.
This paper firstly proposes the importance indices of *RI*, *CI*, and *CIW*. Secondly, since the calculation work for network reliability increases exponentially with the number of network links, an efficient calculation algorithm with a partial differential will be proposed for calculating these importance indices on the basis of the calculation algorithm for Boolean absorption (CABA). It enables us to automatically calculate the terminal reliability and the values of *RI*, *CI*, and *CIW* of all links, even for complex networks. Thirdly, a method of cost-benefit analysis combined with the previously proposed indices will be presented in order to compare the efficiency of these importance indices. Series network, parallel network, a simple bridge network and a field-shape network will be discussed. Depending on the cost-reliability function, the behavior of the network improvement process will differ, and the

importance index of CIW is the best when the network is large. Lastly, concluding remarks of our method for effective and efficient network improvement are discussed.

Keywords: Reliability importance; Network reliability improvement; Criticality importance; Cost-benefit analysis.

1. INTRODUCTION

It is important to keep a traffic network highly reliable for both normal periods and abnormal periods, such as during a disaster. However, at such a time, the traffic system may be seriously damaged, and it is difficult to determine which damaged roads take priority in order to maintain or improve the traffic network reliability. According to the current researches on network reliability improvement, network reliability can be improved effectively by improving the key links in the network. Thus, once the key links for improving the traffic network reliability is found, the network reliability can be improved and maintained efficiently by improving the reliability of the key links. Some importance indices, such as those for reliability importance (RI) and criticality importance (CI), have been proposed for finding the key links of the traffic network. However, these indices have their own shortcomings for finding the key links in the network. The objective of which importance index is the most effective index for finding the key links for improving the network reliability is made into the point paying our attention in this research. In addition, the available capital for repairing the traffic system is also important for improving the network reliability. Thus, a cost-benefit analysis combined with these importance indices for improving the traffic network reliability should be developed. Base on this cost-benefit analysis, the most effective importance index can be identified. In addition, it is very difficult to calculate the terminal reliability of the traffic network and the values of importance indices of all links when the network becomes huge. Thus, the calculation algorithm combined with path sets and path cuts for the terminal reliability and the values of importance indices of the complex network should be developed.

2. CURRENT IMPORTANCE INDICES FOR NETWORK RELIABILITY ANALYSIS

The concept of importance has long been proposed in the field of systems engineering, but has appeared in only a few papers in the transportation field (Barlow and Proschan, 1975). Importance is defined as the degree of magnitude that improvement in the reliability of a link contributes for system reliability. The indices of importance proposed in this paper are on the basis of the connectivity reliability.

2.1 Terminal reliability

The connectivity reliability (also referred to as terminal reliability) of a highway network is defined as the probability that two given nodes over the network are connected with a certain service level of traffic for a given time period. Similarly, link reliability in the network is defined as the probability that the traffic reaches a certain service level for a given time period. Terminal reliability, R, is given by an expression using minimal-path sets, as follows:

$$R(r) = E[1 - \prod_{S=1}^{p} (1 - \prod_{a \in P_s} X_a)] \tag{1}$$

where P_S is the S-th minimal-path set, and p is the total number of minimal-path sets. This calculation method is on the basis of the Boolean absorption method (Wakabayashi and Iida, 1992). Here, X_a is the binary indicator variable for link a:

$$X_a = \begin{cases} 1, & \text{if link } a \text{ provides a certain service level,} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Link reliability, r_a, is defined as

$$r_a = E[X_a] \tag{3}$$

The connectivity reliability of a traffic network depends on the network structure and the link reliabilities. Therefore, two basic approaches have been taken to improve network reliability: to improve the network structure or to improve the reliability of the links. The focus here is on identifying which links should be improved to maximize the improvement in network reliability.

2.2 Reliability importance

2.2.1 Definition of reliability importance

To find the key link for improving the terminal reliability efficiently, the reliability importance index was proposed (Birnbaum, 1969). The reliability importance (also referred to as Birnbaum structural importance) is defined as

$$RI_a = \frac{\partial R(r)}{\partial r_a}, \quad 0 \le RI_a \le 1 \tag{4}$$

Reliability importance (RI) indicates the impact of a link, such that the increase or decrease in the reliability of the link affects the increase or decrease in terminal reliability.

2.2.2 Advantages and shortcomings of reliability importance

Although reliability importance has the potential to improve network reliability, it has a disadvantage, which will be discussed in this section.

For case of two links in a series network shown as Fig.1.a, terminal reliability R_{AB} is shown in Eq. (5):

$$R_{AB} = r_1 r_2 \tag{5}$$

where r_1, r_2 are the reliabilities for link 1 and link 2, respectively.
For the case of two links in a parallel network shown as Fig.1.b, the terminal reliability R_{AB} is shown in Eq. (6):

$$R_{AB} = 1-(1-r_1)(1-r_2) \tag{6}$$

The two values of reliability importance for a series network, RI_1 and RI_2, are obtained from Eq. (4) and Eq. (5) as

$$RI_1 = r_2 \quad \text{and} \quad RI_2 = r_1 \tag{7}$$

It follows that

$$RI_1 > RI_2, \text{ if } \quad r_1 < r_2 \tag{8}$$

Equation (8) indicates that improving the least reliable link in a series-type network is most effective for improving network reliability. This fact is easily expanded for large series-type networks. This result for improving, managing and reconstructing a network is the expected result.
RI_1 and RI_2 for these two links in a parallel network are obtained from Eq. (4) and (6) as

$$RI_1 = 1 - r_2 \text{ and } RI_2 = 1 - r_1 \tag{9}$$

It follows that

$$RI_1 < RI_2, \text{ if } r_1 < r_2 \tag{10}$$

The result from Eq.(10) indicates that improving the most reliable link in a parallel-type network is more effective for improving terminal reliability. Usually, however, it is difficult to improve a more reliable link, whereas it is rather easy to improve a less reliable link (Barlow and Proschan, 1975). This result is counter to what one would expect for improving, managing, and reconstructing a network.

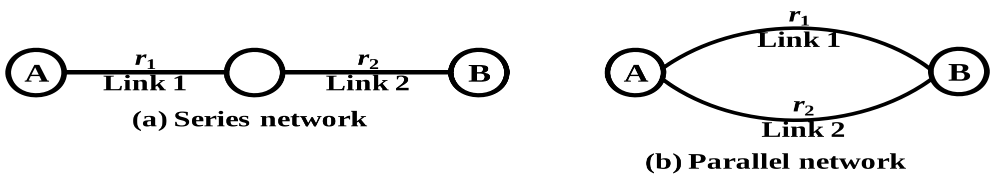

Fig.5.1 A simple series network including two links

2.3 Criticality importance

2.3.1 Definition of criticality importance

Because of the shortcoming of reliability importance for the parallel network, the criticality importance index was proposed. Criticality Importance (CI) is the ratio of the proportional improvement in network reliability to the proportional improvement in link reliability (Henley and Kumamoto, 1981):

$$CI_a = \frac{\partial R / R}{\partial r_a / r_a} = RI_a \frac{r_a}{R} \tag{11}$$

2.3.2 Advantages and shortcomings of criticality importance

Criticality importance also has shortcomings, which is discussed in this section.
For the case of two links in a series network shown as Fig.1.a, it follows from Eq. (4), (5), (7) and Eq.(11) that

$$CI_1 = \frac{r_1 r_2}{R} = CI_2 \tag{12}$$

The results from Eq. (11) and Eq. (12) suggest that the criticality importance index is the same for both links in a series network. However, in a series network, it is reasonable to strengthen a less reliable link, and thus this is a shortcoming of the criticality importance index. In addition, it does not provide information to distinguish between the two links in terms of improving network reliability.
For the case of two links in a parallel network shown as Fig.1.b, it follows from Eq.(4), (6), (9) and Eq.(11) that

$$CI_1 = \frac{r_1 - r_1 r_2}{R} \tag{13}$$

$$CI_2 = \frac{r_2 - r_1 r_2}{R} \tag{14}$$

It follows that

$$CI_1 < CI_2, \text{ if } r_1 < r_2 \tag{15}$$

Therefore, from Eq. (5.18), the criticality importance index also indicates that in the case of a parallel-type network, improving a more reliable link further increases the terminal reliability of the network. The results for a parallel network provided by both reliability importance (RI) and criticality importance (CI) suggest that a less reliable link should be ignored in a parallel system. In other words, people who live along a less reliable link would be neglected after a disaster. This is not reasonable planning for disaster prevention and reduction. Thus, this result is not expected.

2.4 Advanced criticality importance

2.4.1 Definition of advanced criticality importance

On the basis of the shortcomings of Eq. (10) and Eq. (15), reliability importance and criticality importance do not reflect the fact that it is more difficult to improve a more reliable link than to improve a less reliable link. Thus, it is convenient to define importance as the proportion of the marginal change in terminal reliability against the marginal change in link reliability. Changing the definition of the equation in reliability engineering, the advanced criticality importance index (CIW) proposed by Wakabayashi is defined as Eq.(16) (Wakabayashi, 2004)

$$CIW_a = \lim_{\Delta q_a \to 0} \left\{ -\frac{\Delta R(r) / R(r)}{\Delta q_a / q_a} \right\} = RI_a \frac{1-r_a}{R} \tag{16}$$

where q_a=1-r_a is the unreliability of link a.

2.4.2 Advantages and shortcomings of advanced criticality importance

For the case of two links in a series network shown as Fig.1.a, it follows from Eq. (4), (5), (7) and Eq.(16) that

$$CIW_1 = \frac{1}{r_1} - 1 \tag{17}$$

$$CIW_2 = \frac{1}{r_2} - 1 \tag{18}$$

It also follows that

$$CIW_1 > CIW_2, \text{ if } r_1 < r_2 \tag{19}$$

Thus, in a series-type network, advanced criticality importance has the same property as reliability importance, and this property from Eq. (19) is exactly as one would expect.
For the case of two links in a parallel network shown as Fig.1.b, it follows from Eq. (4), (6), (9) and Eq. (16) that

$$CIW_1 = \frac{(1-r_1)(1-r_2)}{r_1 + r_2 - r_1 r_2} = CIW_2 \tag{20}$$

From Eq. (19) and Eq. (20), although the advanced criticality importance index is better than CI proposed by Henley and Kumamoto (1981), this index is the same for both links in a parallel network, so it does not provide information to distinguish between them in terms of improving network reliability.
The importance indices, RI, CI, and CIW discussed above, because of their own shortcomings, cannot be directly used to select the most important key link of a traffic network. Therefore, a good solution cannot be obtained by these indices for evaluating the

improvement of network reliability. In addition, although the cost-benefit ratio is also important (Nicholson, 2007), these indices cannot predict the increase in cost for improving link reliability when link reliability increases. Thus, the traffic network reliability increase in accordance with a different investment strategy is discussed.

3. A METHOD FOR COST-BENEFIT ANALYSIS FOR IMPROVEMENT OF TRAFFIC NETWORK RELIABILITY

According to the criticality importance proposed by Wakabayashi described in Section 2, the less reliable link in a series network should be improved in accordance with Eq.(19). However, the result from Eq.(20) does not provide distinguishable information as to which link should be improved first in a parallel network. Thus, a method to determine the investment of the improvement of the reliability of the traffic network will be proposed in this section.

We will assume three cases of the invest strategies needed to improve the link reliability (cost-reliability function):

Case 1: The cost to improve a link of higher reliability is more than to improve a link of lower reliability, and the investment to increase the same degree of the link reliability varies according to the link reliability. *Case* 1 of the investment strategy is shown as Eq. (21).

$$\frac{\partial Cost_a}{\partial r_a} = C_1 * r_a + C_{10} \tag{21}$$

where the initial value of C_1=50,000 and C_{10}=2500.

Case 2: The investment to increase the same degree of the link reliability is cumulative and varies according to a quadratic function of the link reliability. *Case 2* of the investment strategy is shown as Eq. (22).

$$\frac{\partial Cost_a}{\partial r_a} = C_2 * r_a^2 + C_3 * r_a + C_{20} \tag{22}$$

where the initial value of C_2=250,000, C_3=50,000, and C_{20}=5000/3.

Case 3: The investment to increase the link reliability is proportional to the increase in reliability. *Case 3* of the investment strategy is shown as Eq. (23).

$$\frac{\partial Cost_a}{\partial r_a} = C_{30} \tag{23}$$

where the initial value of C_{30}=5000.

The improvement to the network reliability due to investment may not be obvious in the short term; thus, a simple cost-benefit function that shows the improvement of the network reliability against the cost increase over a long time is defined in Eq.(24).

$$Eff(Y) = \sum_{n=1}^{Y} \frac{F_n}{(1+d)^n} * \frac{(R_{AB} - R_{AB0})}{Cost_{AB}} \tag{24}$$

Y: Number of years to invest;

d: Cost discount, which is assumed to be a constant value;

F_n: The conversion cost benefit of the increased traffic volume obtained by the reliability improvement in the n-th unit time (In this paper, the unit time is one year);

R_{AB0} : Original network reliability;

$Cost_{AB}$: Cost increase to improve the network reliability from R_{AB0} to R_{AB} .

$Eff\,(Y)$: The efficiency of cost benefit obtained by the reliability improvement of traffic systems in Y years.

To simplify the calculation, d is assumed to be 0, $F_1 = F_2 = \cdots = F_Y = F$ is assumed. Thus, Eq. (24) evolves into Eq. (25). The original data is shown as r_1=0.4, r_2=0.5, R_{AB0}=0.7, $Y = 50$ and F =10,000 units /year (unit: 1,000 Japanese yen). There, the value of F is only a virtual data to discuss the efficiency of cost benefit obtained by the reliability improvement and it should be tested and verified in the actual application.

$$Eff(Y) = \frac{(R_{AB} - R_{AB0})}{Cost_{AB}} * Y * F \tag{25}$$

4. A CALCULATION ALGORITHM FOR BOOLEAN ABSORPTION OF TERMINAL RELIABILITY, *RI*, *CI* AND *CIW*

The main point of this algorithm is to expand Eq.(1) directly (Wakabayashi and Iida, 1992). Only one bit of memory is used to store each random variable of every link of the network and a minimal-path can be stored as a decimal number in a memory unit with 32-bits. In addition, reliability importance (*RI*) can be obtained by Eq.(4), criticality importance (*CI*) can be obtained by Eq.(11) and the advanced criticality importance can be obtained by Eq.(16). Thus, the calculation for the importance indices of link *a* can also be obtained by this algorithm. The algorithm is as follows:

Step 1: Let p be the number of minimal-path sets to be used in this calculation. Store these minimal-path sets. Here, every minimal-path set that is composed of links, expressed as binary numbers, is stored as a decimal number. For example, minimal path set $\gamma = X_1X_2X_5X_{10}$, that is, $\{1, 2, 5, 10\}$, is expressed as the binary number 0000010000010011 (read this figure from the right). At this step, the number is translated into a decimal number then stored; the binary number 0000010000010011 is stored as the decimal number 531 ($=2^0+2^1+2^4+2^9$). This procedure permits reduction in the size of the memory region used in the computer.

Step 2: Let $g = 1$, where g is the number of minimal-path sets in iteration.

Step 3: Any product composed of g minimal-path sets (obtained in the expansion of Eq.(1) into $2^p - 1$ terms) is expressed as $(-1)^m \cdot \alpha_{S_1} \cdot \alpha_{S_2} \cdot \cdots \cdot \alpha_{S_n}$.

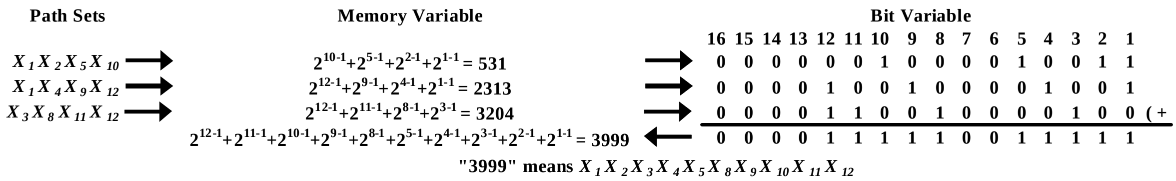

Fig.2 Example of the process of Boolean absorption for terminal reliability: $X_1X_2X_5X_{10}*X_1X_4X_9X_{12}*X_3X_8X_{10}X_{11}X_{12}=X_1X_2X_3X_4X_5X_8X_9X_{10}X_{11}X_{12}$

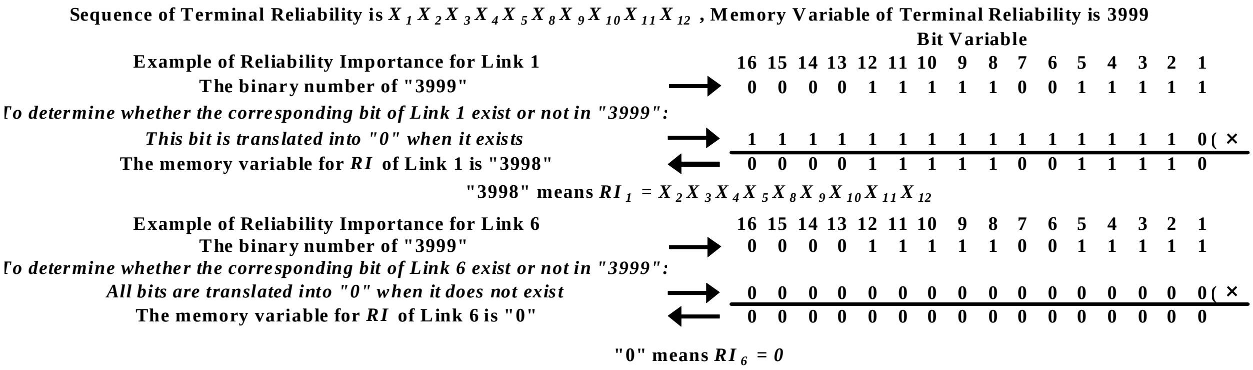

Fig.3 Examples of the process of CABA for reliability importance (*RI*)

Arrange this product by Boolean absorption in terms of links. For example, the product of the minimal-path sets {1, 2, 5, 10}, {1, 4, 9, 12}, and {3, 8, 11, 12} is translated into the memory variable 3999, which indicates $X_1X_2X_3X_4X_5X_8X_9X_{10}X_{11}X_{12}$. This procedure is demonstrated in Fig.2.

On the basis of the memory variable of the product for terminal reliability, the memory variable for the *RI* of all links can be calculated and stored in other locations. If the corresponding bit of X_a does not exist in the memory variable of the product for terminal reliability, the memory variable for the *RI* of link *a* translates into 0, otherwise, the corresponding bit of X_a in the memory variable of the product for terminal reliability is translated into 0, and the new memory variable is stored in other locations as the memory variable for the *RI* of link *a*. For example, the product of RI_1 is $X_2X_3X_4X_5X_8X_9X_{10}X_{11}X_{12}$ according to the memory variable 3999 of terminal reliability, thus the memory variable of RI_1 is 3998. However, the memory variable of RI_6 is 0 because link 6 does not exist in the memory variable 3999. This procedure is demonstrated in Fig.2.

When reliability importance of link *a* has been calculated, criticality importance of link *a* can be calculated by using Eq.(11), at the same time, the advanced Criticality Importance (*CIW*) of link *a* can be calculated by using Eq.(16).

Step 4: Combine like terms. The products generated in *Step 3* are checked as to whether the same product has been generated in the preceding process. For the above examples, the numbers 3999 and 3998 are checked as to whether the same number exists in the same locations. When the same product exists, the coefficient of the product is updated; when not, it is newly stored.

Step 5: Iterate *Steps* 3 and 4 for all combinations of $(-1)^m \cdot \alpha_{S_1} \cdot \alpha_{S_2} \cdot \cdots \cdot \alpha_{S_n}$. The number of iterations is $\binom{p}{g}$.

Step 6: Iterate *Steps* 3 through 5 for g =2, 3,…, p.

Step 7: Each number in the storage region corresponds to each term in the polynomial expression of X_a, for which Boolean absorption has already been carried out. If the number

3999 remains in the storage region for terminal reliability, the corresponding term, $X_1X_2X_3X_4X_5X_8X_9X_{10}X_{11}X_{12}$, exists in the polynomial expression for terminal reliability. Similarly, if the number 3998 remains in the storage region for reliability importance, the corresponding term, $X_2X_3X_4X_5X_8X_9X_{10}X_{11}X_{12}$, exists in the polynomial expression for reliability importance. Therefore, the value of the terminal reliability and reliability importance are obtained by substituting the value for the link reliability into the corresponding terms. On the basis of terminal reliability of the object network and reliability importance of every link in the object network, criticality importance and the advanced criticality importance are calculated and stored into the corresponding storage regions.

5. NUMERICAL EXAMPLES

A method for the cost-benefit analysis for the improvement of traffic network reliability was proposed in Section 3, and three investment strategies for improving the link reliability were assumed to find the most important key link for improving the network reliability. According to the index of *CIW* proposed by Wakabayashi, the least-reliable link in a series-type network should be selected as the most important key link for improving the network reliability. Thus, in this section, a simple parallel network, a simple bridge network and a field-shape network will be selected to carry out the cost-benefit analysis for the improvement of network reliability.

In this section, three strategies for selecting the most important key link for improving the network reliability are discussed according to *RI*, *CI* and *CIW*, respectively.

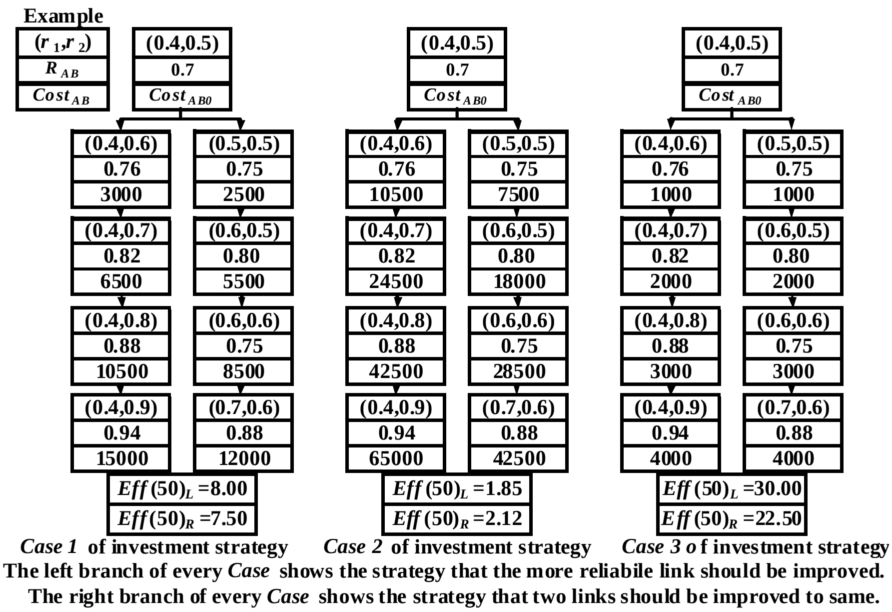

Fig.4. Efficiency of improving network reliability for a simple parallel network on the basis of the three investment strategies

5.1 Cost-benefit analysis for a simple parallel network

A simple parallel network (only two links) as shown in Fig.1.b will be discussed in this section, and the original reliability of two links is r_1=0.4 and r_2=0.5, the initial value of those parameters is Y= 50 and F=10,000,000 yen/year.
Figure 4 shows three investment strategies of the link reliability improvement for the simple parallel network, and the left branch of every *Case* in Fig.4 shows that the more reliable link should be improved according to *RI* and *CI*, and the right branch shows that the less reliable link should be improved according to *CIW*.

*5.1.1 The cost-benefit analysis on the basis of **Case 1** of the investment strategy*

The cost to improve the link reliability varies as the link reliability in *Case 1*. The result $Eff(50,1)_L$ from the left branch of *Case 1* is **8.00** by using Eq.(25), and the result $Eff(50,1)_R$ from the right branch of *Case 1* is **7.50.** Thus, $Eff(50,1)_L > Eff(50,1)_R$. These results suggest that the more reliable link should be selected as the most important key link, on the basis of *Case 1* of the investment strategy.

*5.1.2 The cost-benefit analysis on the basis of **Case 2** of the investment strategy*

The cost to improve the link reliability is cumulative with a quadratic function in *Case 2*. The result $Eff(50,1)_L$ is **1.85** from the left branch of *Case 2* by using Eq. (25), and the result $Eff(50,1)_R$ from the right branch of *Case 2* is **2.12.** Thus, $Eff(50,1)_L < Eff(50,1)_R$. These results suggest that the less reliable link should be selected as the most important key link, on the basis of *Case 2* of the investment strategy.

*5.1.3 The cost-benefit analysis on the basis of **Case 3** of the investment strategy*

The cost is fixed when the link reliability is improved with the same degree in *Case 3*. The result $Eff(50,1)_L$ from the left branch of *Case 3* is **30.0** by using Eq. (25), and the result $Eff(50,1)_R$ from the right branch of *Case 3* is **22.5.** Thus, $Eff(50,1)_L > Eff(50,1)_R$. These results suggest that the more reliable link should be selected as the most important key link, on the basis of *Case 3* of the investment strategy.
On the basis of both *Case 1* and *Case 3* of the investment strategies, the more reliable link should be selected as the most important key link; on the contrary, the less reliable link should be selected as the most important key link to be improved, on the basis of *Case 2* of the investment strategy. Therefore, the different link should be selected as the most important key link according to the different investment strategies by using Eq. (25) in a parallel-type network.

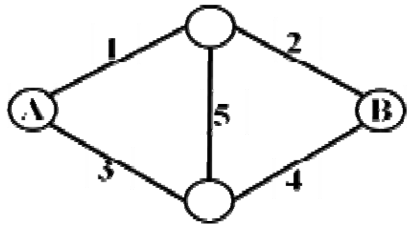

Fig.5 A simple bridge network

5.2 Cost-benefit analysis for a simple bridge network

It is easy to calculate the exact value of terminal reliability and reliability importance of the above-mentioned parallel network. However, it is very complicated and impractical to calculate the exact value of the terminal reliability as the size of the network expands.
In this section, we consider a simple bridge network that has four nodes and five links, as shown in Fig.5. The minimal-path sets of this network are $P_1= \{1, 2\}$, $P_2= \{3, 4\}$, $P_3= \{1, 5, 4\}$, and $P_4= \{3, 5, 2\}$. The independent minimal-path set is a series network system (Wakabayashi and Iida, 1991), thus, the reliability of the minimal-path set is shown as following, where r_1, r_2, r_3 and r_4 are the reliability of link 1, link 2, link 3 and link 4 respectively:

$$R(P_1)=r_1r_2,\ R(P_2)=r_3r_4,\ R(P_3)=r_1r_5r_4,\ R(P_4)=r_3r_5r_2 \quad (26)$$

The exact value of the network reliability for the bridge network is shown in Eq. (27) by using CABA.

$$R(r)=r_1r_2+r_3r_4+r_1r_5r_4+r_3r_5r_2-r_1r_2r_3r_4-r_1r_2r_4r_5-r_1r_3r_4r_5-r_1r_2r_3r_5-r_2r_3r_4r_5+2r_1r_2r_3r_4r_5 \quad (27)$$

Three strategies for selecting the most important key link for maximizing the improvement of network reliability on the basis of the same investment strategy are presented:

a) Some link should be selected as the most important key link according to *RI*;
b) Some link should be selected as the most important key link according to *CI*;
c) Some link should be selected as the most important key link according to *CIW*.

Although the combinations of the original reliability of five links are infinite to find the most important key link, only the following three cases will be discussed due to page limits:

Case BA) The difference of the original reliability between link 1 and link 2 is great and the terminal reliability of the primary minimal-path {1, 2} is the same as the terminal reliability of the primary minimal-path {3, 4}. In this case, r_{10}=0.3, r_{20}=0.8, r_{30}=0.6, r_{40}=0.4 and r_{50}=0.5 are assigned.

Case BB) The difference of the original reliability between link 1 and link 2 is small and the difference of the original reliability between link 3 and link 4 is also small, at the same time, the terminal reliability of the primary minimal-path {1, 2} is almost the same as the terminal reliability of the primary minimal-path {3, 4}. In this case, r_{10}=0.5, r_{20}=0.3, r_{30}=0.4, r_{40}=0.4 and r_{50}=0.9 are assigned.

Case BC) The terminal reliability of the primary minimal-path {1, 2} is different greatly from the terminal reliability of the primary minimal-path {3, 4}. And r_{10}=0.7, r_{20}=0.6, r_{30}=0.1, r_{40}=0.3 and r_{50}=0.2 are assigned.

In order to simplify the calculation of the cost-benefit analysis, the investment strategies of *Case1* and *Case 2* will be used for this simple bridge network. The parameters of Eq. (25) are assigned as Y=50 (months), F= 100,000,000 in order to carry out the cost-benefit analysis of the improvement of network reliability.
The results for improving network reliability according to the importance indices of *RI*, *CI*, and *CIW* are shown in Table 1. Because the investment strategy of *Case 3* is not used for maintaining or improving traffic network reliability in actual situation of transportation, the investment strategy of *Case 3* is not discussed in this section. The results of cost-benefit analysis of three Cases of *Case BA*, *Case BB* and *Case BC* according to these importance indices are shown in Table 2.

Table 1 The variety of reliability of all links in the simple bridge network according to three importance indices

Cases	Original reliability		*RI*	*CIW*	*CI*
Case BA	r_1	0.3	0.7	0.8	0.3
	r_2	0.8	0.9	0.8	0.9
	r_3	0.6	0.6	0.6	1.0
	r_4	0.4	0.4	0.4	0.4
	r_5	0.5	0.5	0.5	0.5
	R_{AB}	0.528	0.773	0.768	0.751
	Total cost of *Case 1*		15500	15000	21500
	Total cost of *Case 2*		59500	55000	104500
	Times of improving link reliability		5 times		
Case BB	r_1	0.5	0.6	0.7	0.5
	r_2	0.3	0.3	0.7	0.3
	r_3	0.4	0.4	0.4	0.4
	r_4	0.4	0.9	0.4	1.0
	r_5	0.9	0.9	0.9	0.9
	R_{AB}	0.394	0.684	0.662	0.679
	Total cost of *Case 1*		20500	17500	22500
	Total cost of *Case 2*		83000	61500	100000
	Times of improving link reliability		6 times		
Case BC	r_1	0.7	0.9	0.9	1.0
	r_2	0.6	0.8	0.8	0.7
	r_3	0.1	0.1	0.1	0.1
	r_4	0.3	0.3	0.3	0.3
	r_5	0.2	0.2	0.2	0.2
	R_{AB}	0.455	0.739	0.739	0.725
	Total cost of *Case 1*		16000	16000	17000
	Total cost of *Case 2*		72500	72500	82000
	Times of improving link reliability		4 times		

Table 2 The results of cost-benefit analysis for network reliability improvement of the simple hybrid network

Cases	Items		Results		
Case BA	Terminal reliability of network	Importance indices	*RI*	*CI*	*CIW*
		R_{AB}	0.773	0.751	0.768
	Total cost of *Case 1*	*Efficiency*	7.903	5.186	8.000
		Ranking	2	3	1
	Total cost of *Case 2*	*Efficiency*	2.059	1.067	2.182
		Ranking	2	3	1
Case BB	Terminal reliability of network	Importance indices	*RI*	*CI*	*CIW*
		R_{AB}	0.684	0.679	0.662
	Total cost of *Case 1*	*Efficiency*	7.064	6.333	7.666
		Ranking	2	3	1
	Total cost of *Case 2*	*Efficiency*	1.745	1.425	2.181
		Ranking	2	3	1
Case BC	Terminal reliability of network	Importance indices	*RI*	*CI*	*CIW*
		R_{AB}	0.739	0.725	0.739
	Total cost of *Case 1*	*Efficiency*	8.881	7.946	8.881
		Ranking	1	3	1
	Total cost of *Case 2*	*Efficiency*	1.960	1.647	1.960
		Ranking	1	3	1

From Table 1, these results can be got as following:

(1) The improvement of network reliability according to *RI* is the highest when the improving times are same and the improved degree of link reliability is the same every time; therefore, reliability importance is the best if the attention of improvement in this simple bridge traffic system is paid only to the improvement of network reliability and the cost of improvement shouldn't be considered.

(2) The cost for improving network reliability according to *CI* is most costly.

(3) Comparing the difference of the original link reliability among all links, the difference of the improved link reliability is larger after improving network reliability according to *RI* and *CI*, whereas the difference of the improved link reliability is smaller after improving network reliability according to *CIW*. Therefore, importance indices of *CIW* are relatively more acceptable than the importance indices of *RI* and *CI*.

From Table 2, these results can be got as following:

(4) The Efficiency of network reliability improvement according to *CIW* is the best in *Case BA, Case BB.*

(5) The Efficiency of network reliability improvement according to *CIW* and *RI* is the same in *Case BC.*

(6) The Efficiency of network reliability improvement according to *CI* is almost the worst in all cases.

5.3 Cost-benefit analysis for a field-shape network

A simple field-shaple network includes nine nodes and twelve links shown in Fig.6. The minimal-path sets of this filed-shape network are shown in Table 3, and the primary minimal-path sets are P_1, P_2, P_3, P_4, P_5 and P_6.

The independent path set is a set of links in a series system, the reliability of one path set is a combination of the link reliability. The network can be considered as a parallel system composed by all the independent path sets. Once the key path set is found, the most important key link belonging to the key path set can be found according to the importance indices. However, it is complicate to lay out the expression of terminal reliability of this field-shape network. It is also difficult to calculate the exact value of terminal reliability of this field-shape network.

In this section, three strategies according to the importance indices of *RI*, *CI* and *CIW* are presented for improving network reliability on the basis of the same investment strategy. The investment strategies of *Case 1* and *Case 2* will be used for the field-shape network and the parameters of Eq. (25) are assigned as Y=50 (months), F= 100,000,000.

Although the combinations of the original reliability of twelve links are infinite to find the most important key link, only the following three cases will be discussed due to page limits:

Case FA) The original reliability of all links is the same and the original terminal reliability of the primary minimal-path sets is the same. In this case, the link reliability of all links is assigned as 0.5.

Case FB) The original terminal reliability of two primary minimal-path sets (P_1 and P_2) is relatively greater than the original terminal reliability of other primary minimal-path sets. In addition, the difference of the original terminal reliability between these two primary minimal-path sets is little. Therefore, in this case, r_{10}=0.6, r_{20}=0.6, r_{30}=0.4, r_{40}=0.7, r_{50}=0.5, r_{60}=0.7, r_{70}=0.4, r_{80}=0.4, r_{90}=0.3, r_{100}=0.5, r_{110}=0.5, and r_{120}=0.4 are assigned.

Case FC) The original terminal reliability of someone primary minimal-path set (P_1) is very greater than the original terminal reliability of other primary minimal-path sets. In addition, the original link reliability of links located in this primary minimal-path set is different. In this case, r_{10}=0.4, r_{20}=0.6, r_{30}=0.2, r_{40}=0.1, r_{50}=0.8, r_{60}=0.1, r_{70}=0.1, r_{80}=0.3, r_{90}=0.1, r_{100}=0.5, r_{110}=0.3 and r_{120}=0.2 are assigned.

Before selecting the key link from this field-shape network, the values of importance indices of every link should be calculated by using CABA algorithm. And then, the efficiency of cost-benefit analysis according to the importance indices of *RI*, *CI* and *CIW* is compared. The results of network reliability improvement of all links and the whole network in the three Cases of *Case FA*, *Case FB* and *Case FC* are shown in Table 4. The results of cost-benefit analysis of three Cases of *Case FA*, *Case FB* and *Case FC* according to these importance indices are shown in Table 5.

From Table 4, these results can be got as following:

(1) The improvement of network reliability according to *RI* is the highest when the improving times are same and the improved degree of link reliability is the same every time; therefore, reliability importance *RI* is the best if the attention of improvement in the field-shape traffic system is paid only to an improvement of network reliability and the cost of improvement shouldn't be considered.

(2) The cost for improving network reliability according to *CI* is most costly.

(3) From *Case FC*, the results of selecting the most important key links according to *RI* and *CIW* are the same when the original terminal reliability of someone primary minimal-path set is very greater than the original terminal reliability of other primary minimal-path sets.

From Table 5, these results can be got as following:

(4) The Efficiency of network reliability improvement according to *CIW* is the best in *Case FA*, *Case FB* and *Case FC* by using the investment strategy of *Case 1* and *Case 2*. It means that the advanced criticality importance is the best by using cost-benefit analysis for improving network reliability. This result also can be certificated by any example of the original reliability of all links in this field-shape network.

(5) The Efficiency of network reliability improvement according to *CI* is almost the worst in all cases.

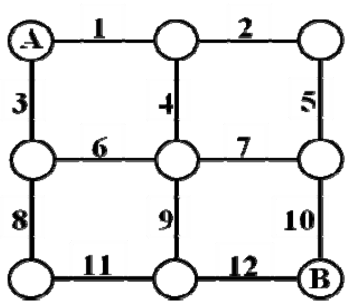

Fig.6 A field-shape network

Table 3 The minimal-path sets of the field-shape network

Path name	Elements of path	Number of links	Path name	Elements of path	Number of links
P_1	{1, 2, 5, 10}	4	P_7	{3,6,4,2,5,10}	6
P_2	{1, 4, 7, 10}	4	P_8	{3,8,11,9,7,10}	6
P_3	{3, 6, 7, 10}	4	P_9	{1,2,5,7,9,12}	6
P_4	{1, 4, 9, 12}	4	P_{10}	{1,4,6,8,11,12}	6
P_5	{3, 6, 9, 12}	4	P_{11}	{3,8,11,9,4,2,5,10}	8
P_6	{3, 8, 11, 12}	4	P_{12}	{1,2,5,7,6,8,11,12}	8

Table 4 The variety of reliability of all links in the field-shape network according to three importance indices

Cases	Original reliability		*RI*	*CI*	*CIW*
Case FA	r_1	0.500	0.800	1.000	0.700
	r_2	0.500	0.500	0.500	0.500
	r_3	0.500	0.500	0.500	0.600
	r_4	0.500	0.500	0.500	0.500
	r_5	0.500	0.500	0.500	0.500
	r_6	0.500	0.500	0.500	0.500
	r_7	0.500	0.500	0.500	0.500
	r_8	0.500	0.500	0.500	0.500
	r_9	0.500	0.500	0.500	0.500
	r_{10}	0.500	0.700	0.500	0.600
	r_{11}	0.500	0.500	0.500	0.500
	r_{12}	0.500	0.500	0.500	0.600
	R_{AB}	0.2771	0.4070	0.3921	0.3966
	Total cost of *Case 1*		17000	20000	15500
	Total cost of *Case 2*		67000	92500	56000
	Times of improving link reliability		5 times		
Case FB	r_1	0.600	0.700	0.600	0.700
	r_2	0.600	0.600	0.600	0.600
	r_3	0.400	0.400	0.400	0.400
	r_4	0.700	0.700	0.700	0.700
	r_5	0.500	0.500	0.500	0.500
	r_6	0.700	0.700	0.700	0.700
	r_7	0.400	0.400	0.400	0.600
	r_8	0.400	0.400	0.400	0.400
	r_9	0.300	0.300	0.300	0.300
	r_{10}	0.500	0.900	1.000	0.700
	r_{11}	0.500	0.500	0.500	0.500
	r_{12}	0.400	0.400	0.400	0.400
	R_{AB}	0.2626	0.4980	0.4191	0.4083
	Total cost of *Case 1*		18500	20000	15500
	Total cost of *Case 2*		79000	92500	56500
	Times of improving link reliability		5 times		
Case FC	r_1	0.400	0.700	0.400	0.700
	r_2	0.600	0.700	0.600	0.700
	r_3	0.200	0.200	0.200	0.200
	r_4	0.100	0.100	0.100	0.100
	r_5	0.800	0.800	0.800	0.800
	r_6	0.100	0.100	0.100	0.100
	r_7	0.100	0.100	0.100	0.100
	r_8	0.300	0.300	0.300	0.300
	r_9	0.100	0.100	0.100	0.100
	r_{10}	0.500	0.700	1.000	0.700
	r_{11}	0.300	0.300	0.300	0.300
	r_{12}	0.200	0.200	0.200	0.200
	R_{AB}	0.1025	0.2430	0.1998	0.2430
	Total cost of *Case 1*		15500	20000	15500
	Total cost of *Case 2*		56500	92500	56500
	Times of improving link reliability		5 times		

Table 5 The results of cost-benefit analysis for network reliability improvement of the field-shape network

Cases	Items		Results		
Case FA	Terminal reliability of network	Importance indices	*RI*	*CI*	*CIW*
		R_{AB}	0.4070	0.3921	0.3966
	Total cost of *Case 1*	*Efficiency*	3.8215	2.8748	3.8549
		Ranking	2	3	1
	Total cost of *Case 2*	*Efficiency*	0.9696	0.6216	1.0670
		Ranking	2	3	1
Case FB	Terminal reliability of network	Importance indices	*RI*	*CI*	*CIW*
		R_{AB}	0.4980	0.4191	0.4083
	Total cost of *Case 1*	*Efficiency*	4.3643	3.9124	4.6987
		Ranking	2	3	1
	Total cost of *Case 2*	*Efficiency*	1.0220	0.8459	1.2890
		Ranking	2	3	1
Case FC	Terminal reliability of network	Importance indices	*RI*	*CI*	*CIW*
		R_{AB}	0.2430	0.1998	0.2430
	Total cost of *Case 1*	*Efficiency*	4.5314	2.4334	4.5314
		Ranking	1	3	1
	Total cost of *Case 2*	*Efficiency*	1.2431	0.5261	1.2431
		Ranking	1	3	1

(6) From Table 4 and Table 5, *RI* is the best of importance index for finding the most important key link when the cost to improve the whole network reliability should not be considered. However, if the cost to improve the whole network reliability is limited, the importance index of *CIW* should be used to find the most important key link on the basis of overall consideration of the cost and the network reliability improvement.

6. CONCLUSION

In this paper, firstly, in order to discuss the improvement of the reliability of a traffic network, the current indices of reliability, including *RI*, *CI* and *CIW*, were introduced and the faults of these indices were pointed out.

Secondly, a method for a cost-benefit analysis, on the basis of the cost-reliability function, was proposed for improvement of the reliability of the traffic network.

Thirdly, the calculation algorithm for Boolean absorption for terminal reliability and reliability importance were developed.

Finally, three numerical examples for the simple parallel network, the simple bridge network and the field-shape network were simulated on the basis of the calculation algorithm for Boolean absorption and the cost-reliability function. From these simulations, general conclusions can be obtained as follows:

(1) In a very simple network, the most important key link should be selected according to the investment strategy for improvement of the traffic network reliability.

(2) In a general network, *RI* is the best of importance index for finding the most important key link when the cost to improve the whole network reliability should not be considered. However, if the cost for improving the whole network reliability is limited, the importance index of *CIW* should be used for finding the most important key link on the basis of overall consideration of the cost and the network reliability improvement.

However, these conclusions are on the basis of only limited types of traffic networks. As an area of future study, more types of traffic network should be used for finding the most important key link in some typical networks.

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