Ground Delay Program Decision-making using Multiple Criteria: A Single Airport Case

Yi Liu
Department of Civil and Environmental Engineering
107 McLaughlin Hall
University of California, Berkeley
Berkeley, USA
liuyisha@berkeley.edu

Mark Hansen
Department of Civil and Environmental Engineering
114 McLaughlin Hall
University of California, Berkeley
Berkeley, USA
mhansen@ce.berkeley.edu

Ground Delay Program Decision-making using Multiple Criteria: A Single Airport Case

1. Introduction

On the day-of-operation, airport capacity varies and is often reduced due to poor weather, traffic congestion, or other factors. Ground Delay Programs (GDPs) are usually implemented in this case in order to balance the arrivals with the reduced capacity. This is accomplished by delaying take offs bound for the congested airports. As a result, GDPs transfer expensive and unsafe airborne delays to cheaper and safer ground delays. In 2006, 624 GDPs were called at eight airports with the largest numbers of GDPs of that year and affected 167,584 flights from 11 major airlines and their subcarriers (Xiong, 2010). As one of the most common Traffic Management Initiatives (TMI), GDPs are essential to keeping the air traffic efficient and safe. However, the current GDP planning process is ad-hoc and subjective in several respects.

First, different managers may create different plans for the same situation. Depending on their temperament and experience, a manager may set higher or lower capacity rates, and shorter or longer program durations. Clear evaluation criteria to assist managers in designing GDPs are needed. Although TMI performance categories are described in the literature (Bolczak et al., 1997; Bradford et al., 2000; Sridhar et al., 2008), associated performance criteria and day-of-operation performance metrics are not defined for GDPs.

Second, flight operators influence the GDP decisions through planning telecons with the traffic managers, and frequent interaction with the command center personnel. The inputs from the flight operators focus on the decisions on the GDP parameters, and not the underlying performance objectives. It is unclear to both the FAA and airlines how the performance of the program will be influenced by the GDP decisions. A mechanism linking the GDP performance metrics to decision variables is missing.

Third, the vast majority of the existing studies dealing with GDP decision-making, there is a sole objective-- minimizing the delay cost (Odoni, 1987; Hoffman and Ball, 2000; Ball et al., 2003; Mukherjee and Hansen, 2007; Liu et al., 2008; Mukherjee et al., 2009). Little effort has been made to consider other goals, such as predictability and throughput. We therefore lack the ability to evaluate GDP performance using multiple criteria. While delay is an adequate measure of operational effectiveness in some instances, it does not present a complete picture of the many aspects of performance that determine the quality and level of service that Air Traffic Control (ATC) users receive (Bolczak et al., 1997).

Different flight operators may have different preferences on performance goals. For instance, low-cost carriers may consider efficiency more important, whereas cargo airlines may consider predictability more valuable. Therefore, an improved decision-making process should be able to measure various dimensions of the GDP performance.

In this paper, we propose to address these issues by developing GDP performance criteria and assessing the tradeoffs between multiple performance goals in a manner that can inform air traffic management decision-making. We identify four performance criteria for GDPs: capacity utilization, predictability, efficiency, and equity; and specify the performance metrics for them. We also build performance trade curves between the criteria and associated metrics using proposed GDP models, and illustrate how these curves could be used to assist in GDP decision-making processes when the objective is a linear function of the goal metrics. The research forms a basis for assessing the performance of GDPs using multiple criteria and will ultimately lead to improved GDP decision-making, in which traffic managers and flight operators can make informed tradeoffs based on their assessment of the importance of different performance criteria.

The remainder of the paper is organized as follows. Section 2 introduces our GDP models with and without consideration to early GDP cancellation. Section 3 specifies GDP performance criteria and the associated performance metrics. Section 4 presents the performance trade-offs and discusses how the trade curves can assist in GDP decision-making. Section 5 summarizes the paper and discusses about future research.

2. Ground Delay Program Models

In the literature, Integer Programming (IP) is the predominant technique for GDP performance evaluation (Odoni, 1987; Richetta, 1991; Ball and Lulli, 2004; Mukherjee and Hansen, 2007). IP successfully provides numerical solutions to specific GDP delay cost optimization problems. With this technique, a few efforts have also been made to investigate tradeoffs between performance goal; in particular with respect to equity and efficiency (Ball and Lulli, 2004; Mukherjee and Hansen, 2007). However, there are two disadvantages in the IP technique. First, the computation results are applicable only for a given problem; extra runs are required for a new set of parameters. Therefore, IP fails to characterize the relationship between GDP parameters and multiple dimensions of performance in a generic way. Second, the computational time of IP programs grows with the size of the problem. Considerable effort in improving the IP algorithm is expected before it could be efficient enough to be incorporated in the GDP decision support tool, if multiple resources are involved and multiple performance objectives are considered. In this study, we attempt to address these issues by using the continuum approach based on queueing diagrams (Daganzo, 1997). Most of the notations in this section are also defined in Appendix A.

When the Airport Acceptance Rate (AAR) at the airport is lowered by bad weather, a GDP will be implemented to balance the demand with the reduced capacity. There are three decision variables in the GDP design: the duration of the program, planned airport acceptance rates, and the scope of the area that is subject to GDP. If the duration of the poor weather is underestimated, then there will be airborne delay since more arrivals were planned than could land. Airborne delay could also occur if the AAR is overestimated. On the contrary, capacity may be underutilized if the estimate is too conservative. Compared to the duration of the program, planned airport acceptance levels are more predictable. For example, at SFO, fog could preclude simultaneous arrival operations on its closely spaced parallel runways, reducing the arrival capacity from 60 to 30 flights per hour (Cook and Wood, 2009). However the duration of the fog is hard to predict due to large uncertainty in the weather forecast. In this research, we investigate how the uncertainty in the duration of the program will affect the GDP performance and assume no uncertainty in airport acceptance levels. Different from the duration of the program, the scope of the GDP is a choice independent of the weather forecast. When GDPs are called, the FAA exempts flights from a GDP by limiting the scope of the GDP to a geographical area surrounding the destination airport (Ball and Lulli, 2004). With a small scope, more flights will be exempted from the GDP, but longer delays will be imposed on the affected flights and thus reduce equity. As discussed later, the decision on the scope has substantial impacts on performances.

The queueing diagram of the arrival traffic at a GDP airport is shown in Figure 1. The brown solid line represents the scheduled cumulative demand curve, which is linear based on the assumption of a constant schedule demand rate τ. Mathematically, it can be formulated as:

$$S(t) = \lambda \cdot t$$

$S(t)$ is the basis for Original Time of Arrival (OTA). The piece-wise blue dash line represents the planned cumulative arrival curve under GDP, which is the basis for allocating Controlled Time of Arrivals (CTA) for the delayed flights. Due to poor weather, AAR drops to a low level, C_L, at time zero. In order to meter flow to the airport, a GDP is initiated and the planned AAR is supposed to switch to the high level, C_H, at time T, when the weather is expected to clear up. Delay is planned to develop in the system until time T and then start to vanish. The planned curve intersects the scheduled curve at time T_2, when there is no delay any more and the two curves overlap with each other afterwards. The duration of the program, T_2, is from the time when the queue starts to grow to the time when there is not more delay in the system. With the above assumptions, we can express the duration of the program as

$$T_2 = \frac{C_H - C_L}{C_H - \lambda} T$$

where, T is the expected weather clearance time. We will consider T as a GDP decision variable that determines the planned duration of the program.

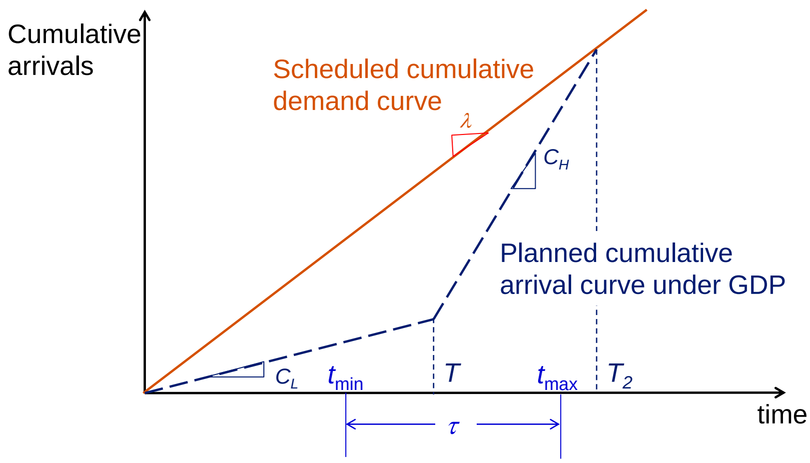

Figure 1: Queueing diagram of the arrival traffic at the GDP affected airport

Given the constant airport acceptance rates, CTA's are determined by the decision on the weather clearance time, T. The formulation of the planned cumulative arrival curve is then

$$N(t|T) = \begin{cases} 0, t \leq 0 \\ C_L t, 0 < t \leq T \\ C_L T + C_H(t - T), T < t \leq T_2 \\ \lambda t, t > T_2 \end{cases}$$

The amount of the planned ground delay in the GDP is

$$D_P(T) = \int_{S(t)>N(t|T)} [S(t) - N(t|T)]dt = \frac{1}{2}\frac{(C_H - C_L)(\lambda - C_L)}{C_H - \lambda} \cdot T^2$$

Due to errors in predication, the actual time when the weather will clear up, τ, may be different from the planned clearance time, T. To increase tractability, we assume that the actual clearance time is uniformly distributed between t_{min} and t_{max}. Consequently, the expected clearance time T will be a value between the same bounds. After decisions are made on T, two possible scenarios could occur during the GDP: early clearance, $\tau < T$;

and late clearance, $\tau > T$. In the case of early clearance, there is unexpected extra capacity, $C_H - C_L$, between τ and T. We could choose not to revise the program and we will have enough capacity to land the planned arrivals as in the original GDP. Alternatively, we could revise the program by taking advantage of the extra part of capacity. In practice, traffic managers usually cancel the GDP earlier in this case to maximize the arrival throughput. Revision will certainly improve capacity utilization and efficiency, but probably reduce predictability since we are making changes to the original GDP plan. There is widespread consensus in the community that predictability is also important. Therefore, in this research we consider early cancellation as an option, but also allow the option of not revising the GDP. In the case of late clearance, there will be a period when high capacity is planned but the actual capacity is still low. GDP extension is assumed in this case so that some additional delay can be converted to ground delay. In the extension, priority will be given to the flights in the air, and flights on the ground will be held and released when all their delay has been absorbed as ground delay. In modeling the extension, we assume that at the time the extension is made, the actual clearance time is known with certainty.

In Sections 2.1 and 2.2, we will present the GDP models for the cases of early clearance and late clearance respectively. In these two sections, we will assume that all the flights bound for the affected airport are involved in the GDP. In practice, only flights within a certain scope will be subject to GDP. Flights that are geographically farther than the scope will be exempted from the program. Impact of scope on the models will be discussed in Section 2.3

2.1 GDP Models with Early Clearance

When high capacity is available earlier than planned, we are able to land the arrivals as planned in the original GDP, or revise the GDP to allow a higher rate. The queueing diagrams for the arrivals are shown below for the cases of no early cancellation and early cancellation.

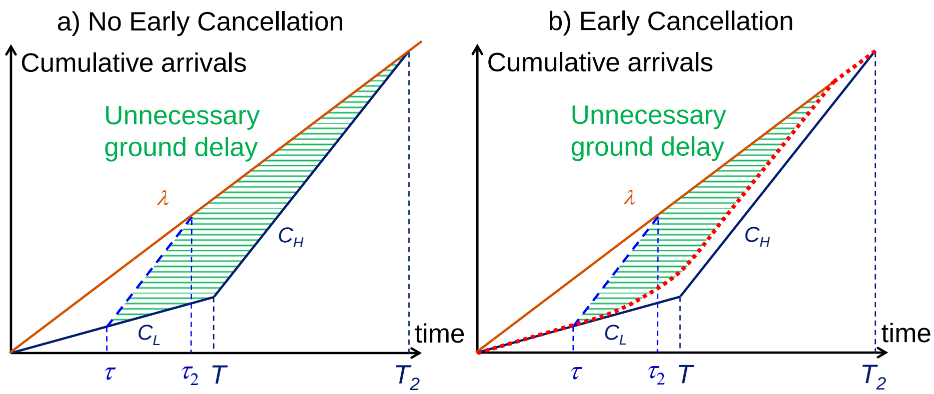

Figure 2: Queueing diagrams for arrivals, early clearance case

If we don't consider GDP cancellation, then the realized cumulative arrival curve will overlap with the planned cumulative arrival curve. The realized throughput, Th, is equal to the planned:

$$Th(t|\tau < T, no\ early\ cancellation) = N(t|T)$$

Realized delay, D_R, is then equal to the planned delay:

$$D_R(\tau < T, no\ early\ cancellation) = D_P(T)$$

If we had perfect information when we were designing the GDP, then we could have allocated the CTA based on the ideal cumulative arrive curve, the slope of which shifts from C_L to C_H at time τ and delay vanishes at time τ_2. More specifically, the ideal cumulative curve is formulated as

$$A(t|\tau) = \begin{cases} 0, t \leq 0 \\ C_L t, 0 < t \leq \tau \\ C_L T + C_H(t - T), \tau < t \leq \tau_2 \\ \lambda t, t > \tau_2 \end{cases}$$

Unnecessary delay, which is defined as the difference between the realized delay and the minimum delay if we had perfect information when designing the GDP, is highlighted by the shaded area in Plot a). If GDP is cancelled earlier to utilize the unexpected part of capacity, then unnecessary delay could be reduced as shown in Plot b). The realized cumulative arrival curve is different from the planned cumulative curve. Conceptually, the realized curve, as depicted as the red dotted curve, should be above the planned curve because we are considering C_H as the acceptance rate starting from τ instead of T in order to accelerate the clearance of delay. This curve begins to deviate from planned cumulative curve not at time τ but sometime after it, because it takes time for flights held on the ground to arrive at the airport.

Without early cancellation, the ideal, planned and realized cumulative arrive curves are just linear piece-wise functions. In the case of early cancellation, if we simply terminate the GDP by releasing flights at the earliest possible take-off time, some flights may encounter airborne delay before landing since we do not have infinite capacity. To avoid this part of airborne delay, we need to release flights by the amount of capacity we have. Here, we assume that GDP is revised at time τ and the new time slots are assigned instantaneously. The model with early cancellation is considerably more complicated because we need to revise the cumulative arrival curve, which will serve as the basis of the new CTA. The revised cumulative curve is jointly determined by the available cumulative arrival demand, D, and the available capacity. Since we are considering early cancellation, the available capacity is $C_L$before τ and C_H after τ. The available demands are obtained by releasing flights at their earliest possible take-off time. Before we derive the mathematical expressions for D, we group the affected flights in the original GDP into three groups. D will be the sum of the available cumulative arrival demands from each group. Specifically, at the time when capacity actually increases, τ, flights heading to the congested airport are either:

- Type I: these flights have already departed. They are scheduled to depart before τ and actually have departed before τ under the original GDP. Assigned ground delay in the original GDP has fully occurred for Type I flights and they will therefore arrive at their allocated CTA. We denote the available cumulative demand curve, which is the same as the planned cumulative arrival curve under the original GDP, for this type of flights as D_- and its rate as D'_-.
- Type II: these flights are being held on the ground at τ. Type II flights would have already departed in the original schedule but are waiting on the ground at time τ due to the initiation of the original GDP. These flights can, in principle, depart immediately if capacity permits. If these flights are all released at time τ, the cumulative arrival curve after this action is then the available cumulative demand curve, which is denoted as D_0 with demand rate D'_0.
- Type III: there flights are scheduled to depart after τ. Ground delay assigned in the original GDP has not occurred yet for flights of this type. Therefore, there would be no delay for these flights if they were allowed to take off as scheduled. Assuming they depart as scheduled, they will arrive earlier than the time slots assigned to them under the original GDP. The available cumulative arrival demand curve is the same as the scheduled cumulative arrival curve. The cumulative demand and its rate for this type are denoted as D_+ and D'_+ respectively.

The total available cumulative arrival demand after revision, D, is then the sum of the cumulative demands of each type after the actions. The difference between D and the planned cumulative arrival demand curve in the original GDP reflects the effect of GDP revision, which is affected by the range of the flight time. At this stage, we assume all the flights that are heading to the affected airport will be subject to the GDP. The maximum flight time of these flights is denoted as F_{max}. When F_{max} is small, the delayed flights are concentrated in the vicinity of the affected airport and they could arrive at the airport earlier under revision, which enables the airport to utilize the expected extra capacity efficiently. If the maximum flight time is increased, delay would be absorbed by more flights but the utilization of the unexpected extra capacity would be less efficient. In this analysis, the flight time is assumed to follow a uniform distribution between F_{min} and F_{max}. If the planned clearance time is between τ and the actual clearance time plus F_{min}, then no modification should be made to the GDP. Because in this case the extra demand from the revision first arrives at time $\tau + F_{min}$, there is little we can do if all the available capacity has already been used at this time under the original GDP. Therefore, GDP revision makes a difference only when $\tau + F_{min}$ is less than T. Depending on F_{max}, the available cumulative arrival demand curve is formulated differently. In total, there are three cases for the available cumulative demand curve. Conditions and formulations for each case are discussed in 2.1.1 to 2.1.3, respectively.

2.1.1 Early GDP Cancellation, $\tau + F_{min} < T < T_2 < \tau + F_{max}$

As shown in Figure 3, flights that have taken off before time τ will arrive between τ and $\tau + F_{max}$, which is later than the planned delay clearance time, T_2. In the analysis, we don't consider flights after T_2, because there is no more delay under the original GDP after this time. It is worth mentioning that delay will not vanish earlier than planned after revision even though the system delay will be reduced. One reason is simply because there will be Type I flights arriving at the affected airport from $\tau + F_{min}$ to T_2, and planned delay of these en-route flights has been fully incurred. To generate the revised cumulative arrival curve, we first need the cumulative available demand curves. As discussed before, the total available demand is just the sum of the available demands for each type of flights. The mechanisms of calculating the demands are different and discussed in 2.1.1.1 to 2.1.1.3.

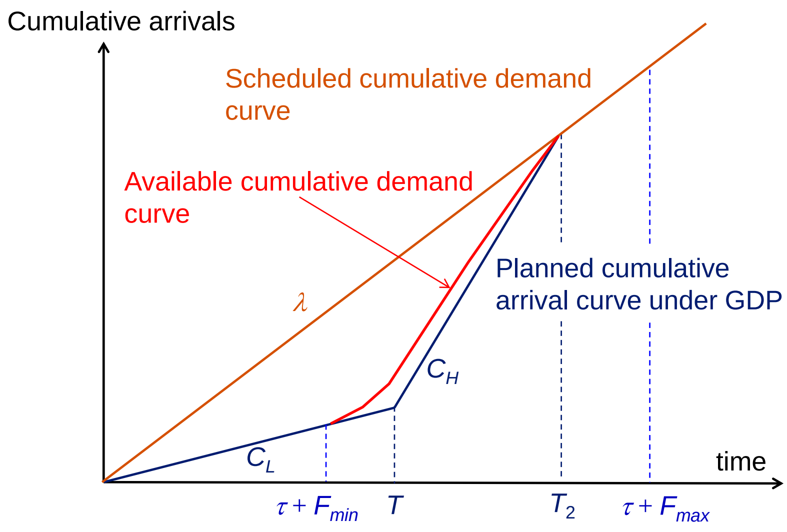

Figure 3. Queueing diagram, $\tau + F_{min} < T < T_2 < \tau + F_{max}$

2.1.1.1 D_-, Type I flights

Type I flights are the flights that have been released from their departure airports when the high capacity is found to be available ahead of time. These flights have arrived at the GDP airport or are in the air. In either case, delay assigned to these flights in the original GDP has already been incurred and they will arrive at their CTA. The principles that are used to derive D_- are:

- Before $\tau + F_{min}$, all planned capacity is utilized for Type I flights. For flights that take off after τ, it is impossible for them to arrive prior to $\tau + F_{min}$ since F_{min} is the minimum flight duration.
- For Type I flights planned to arrive between $\tau + F_{min}$ and T_2, the flight time range at any time t is between $t - \tau$ and F_{max}. If a flight arrives at t with flight time less than $t - \tau$, then this flight must have taken off after τ, so it cannot be a Type I flight. Type I flights are the only flights that have taken off at τ. For a given flight time, type I flights will arrive earlier than other flights which have not taken off yet at τ. Therefore, flights arriving at t with flight time in $[t - \tau, F_{max}]$ will all be Type I flights. The probability that a flights is Type I flight at a given t is then:

$$P_{t,\tau}(D_-) = \frac{F_{max} - (t - \tau)}{F_{max} - F_{min}}$$

Since the flight time distribution for all flights is uniform between F_{min} and F_{max}, flight time for Type I flights will be uniformly distributed between $t - \tau$ and F_{max}. Given the capacity rate is C_L before T and C_H afterwards in the GDP, the available demand rate of Type I flights after revision, which is the same as the planned capacity rate for Type I flights under the original GDP, can be expressed as:

$$D'_{-}(t|\tau,T)=\begin{cases} C_L\cdot\dfrac{F_{max}-(t-\tau)}{F_{max}-F_{min}},\ \ if\ \tau+F_{min}<t<T \\ c_H\cdot\dfrac{F_{max}-(t-\tau)}{F_{max}-F_{min}}, if\ T<t<T_2 \end{cases}$$

With these, we integrate and express the cumulative arrival demand curve for Type I flights as:

$$D_{-}(t|\tau,T)$$
$$=\begin{cases} C_L t, \qquad 0<t<\tau+F_{min} \\ -\dfrac{C_L}{2\cdot\Delta F}[t-(\tau+F_{max})]^2+C_L\left(\tau+F_{min}+\dfrac{\Delta F}{2}\right), \qquad \tau+F_{min}<t<T \\ -\dfrac{C_H}{2\cdot\Delta F}[t-(\tau+F_{max})]^2+C_L\left(\tau+F_{min}+\dfrac{\Delta F}{2}\right)+\dfrac{C_H-C_L}{2\cdot\Delta F}[T-(\tau+F_{max})]^2, \qquad T<t<T_2 \end{cases}$$

2.1.1.2 D_0 , Type II flights

Type II flights are the flights that should have taken off at τ if GDP were not initiated, but have not taken off due to the GDP. All these flights have been delayed to some degree at τ. There is no delay planned for the flights arriving after T_2, and thus these flights cannot be Type II flights. Therefore, under original GDP, Type II flights are planned to arrive before T_2. Flight time range of the flights is then between F_{min} and $T_2-\tau$. Type II flights are held on the ground at τ, and can take off immediately if capacity permits. We can easily plot the cumulative available demands if we know the distribution of flight time since the departure time will be τ for all the Type II flights. Steps for deriving the available demand curve are shown below.

Step 1: $P_\tau(D_0|F)$

$P_\tau(D_0|F)$ is defined as the probability that a flight impacted by the GDP is being held on ground at τ given that its flight time is F. We can write it as

$$P_\tau(D_0|F)=\begin{cases} \dfrac{\lambda\cdot(\tau+F)-N^C(\tau+F)}{\lambda\cdot T_2}, \tau+F\le T_2 \\ 0, \tau+F>T_2 \end{cases}$$

where, $N(t)$ is the planned arrival curve under original GDP.

Substitute the formulation of $N(t)$ in the expression of $P_\tau(D_0|F)$:

$$P_\tau(D_0|F) = \begin{cases} \dfrac{(\lambda - C_L)\cdot(\tau + F)}{\lambda \cdot T_2}, F_{min} \le F \le T - \tau \\ \dfrac{(\lambda - C_H)\cdot(\tau + F) + (C_H - C_L)\cdot T}{\lambda \cdot T_2}, T - \tau \le F \le T_2 - \tau \\ 0, otherwise \end{cases}$$

Step 2: $\boldsymbol{P_\tau(D_0)}$

$P_\tau(D_0)$ is the probability that the flight impacted by GDP is being held on ground at τ. It is also the proportion of Type II flights in all the affected flights. Using the total probability theorem, we get

$$P_\tau(D_0) = \int_{F_{min}}^{F_{max}} P_\tau(D_0|F)\, f(F) dF = \int_{F_{min}}^{T_2-\tau} P_\tau(D_0|F)\, f(F) dF$$

Assume F is uniformly distributed over $[F_{min}, F_{max}]$, we get

$$P_\tau(D_0) = \int_{F_{min}}^{T_2-\tau} P_\tau(D_0|F) \frac{1}{F_{max} - F_{min}} dF$$

$$= \frac{1}{\lambda \cdot T_2 \cdot \Delta F} \cdot \left\{ \int_{F_{min}}^{T-\tau} [\lambda \cdot (\tau + F) - C_L \cdot (\tau + F)]\, dF + \int_{T-\tau}^{T_2-\tau} [\lambda \cdot (\tau + F) - C_L \cdot T - C_H \cdot (\tau + F - T)]\, dF \right\}$$

where, $\Delta F = F_{max} - F_{min}$.

Integrating and multiplying bother sides by $\lambda \cdot T_2 \cdot \Delta F$, we get

$$\lambda \cdot T_2 \cdot \Delta F \cdot P_\tau(D_0) = \frac{(\lambda - C_L)}{2}(T-\tau)^2 - \frac{(\lambda - C_H)}{2}(T-\tau)^2 + \frac{(\lambda - C_H)}{2}(T_2-\tau)^2 - (\lambda - C_L)\cdot\tau^2 - (\lambda - C_L)\cdot\tau\cdot F_{min} - \frac{(\lambda - C_L)}{2}\cdot {F_{min}}^2 + (C_H - C_L)\cdot T \cdot (T_2 - T)$$

Denoting $y_{D_0} = \lambda \cdot T_2 \cdot \Delta F \cdot P_\tau(D_0)$ and simplifying the right hand side, we obtain

$$y_{D_0} = \frac{C_H - C_L}{2}(T-\tau)^2 + \frac{\lambda - C_H}{2}(T_2 - \tau)^2 - \frac{\lambda - C_L}{2}(\tau + F_{min})^2 - \frac{\lambda - C_L}{2}\cdot\tau^2 + \frac{C_H - C_L}{C_H - \lambda}\cdot(\lambda - C_L)\cdot T^2$$

The total number of D_0 flights can then be written as

$$D_{0,total} = \lambda \cdot T_2 \cdot P_\tau(D_0) = \frac{y_{D0}}{\Delta F}$$

Step 3: $f_\tau(F|D_0)$

$f_\tau(F|D_0)$ is defined as the density function of flight time given the flight is Type II flight. Using Bayes' rule, we know

$$f_\tau(F|D_0) = \frac{P_\tau(D_0|F)\cdot f(F)}{P_\tau(D_0)}$$

$$= \begin{cases} \dfrac{(\lambda - C_L)\cdot(\tau + F)}{\lambda\cdot T_2\cdot\Delta F} \Big/ \dfrac{\mathrm{y}_{D_0}}{\lambda\cdot T_2\cdot\Delta F}, F_{min} \le F \le T - \tau \\ \dfrac{(\lambda - C_H)\cdot(\tau + F) + (C_H - C_L)\cdot T}{\lambda\cdot T_2\cdot\Delta F} \Big/ \dfrac{\mathrm{y}_{D_0}}{\lambda\cdot T_2\cdot\Delta F}, T - \tau \le F \le T_2 - \tau \\ 0, otherwise \end{cases}$$

$$= \begin{cases} \dfrac{(\lambda - C_L)\cdot(\tau + F)}{\mathrm{y}_{D_0}}, F_{min} \le F \le T - \tau \\ \dfrac{(\lambda - C_H)\cdot(\tau + F) + (C_H - C_L)\cdot T}{\mathrm{y}_{D_0}}, T - \tau \le F \le T_2 - \tau \\ 0, otherwise \end{cases}$$

Step 4: $F_\tau(F|D_0)$

Integrating the conditional probability, we can get the flight time distribution for Type II flights.

For $F_{min} \le F \le T - \tau$,

$$F_\tau(F|D_0) = \int_{F_{min}}^{F} \frac{(\lambda - C_L)\cdot(\tau + x)}{y_{D_0}} dx = \frac{(\lambda - C_L)}{y_{D_0}} [\left(\tau F + \frac{F^2}{2}\right) - \left(\tau F_{min} + \frac{{F_{min}}^2}{2}\right)]$$

For $T - \tau \le F \le T_2 - \tau$

$$F_\tau(F|D_0) = \int_{F_{min}}^{T-\tau} \frac{(\lambda - C_L)\cdot(\tau + x)}{y_{D_0}} dx + \int_{T-\tau}^{F} \frac{(\lambda - C_H)\cdot(\tau + x) + (C_H - C_L)\cdot T}{y_{D_0}} dx$$

$$= \frac{(\lambda - C_L)}{y_{D_0}}\left[\left(\tau(T-\tau) + \frac{(T-\tau)^2}{2}\right) - \left(\tau F_{min} + \frac{{F_{min}}^2}{2}\right)\right] + \frac{(\lambda - C_H)}{y_{D_0}}\left[\left(\tau F + \frac{F^2}{2}\right) - \left(\tau(T-\tau) + \frac{(T-\tau)^2}{2}\right)\right] + \frac{(C_H - C_L)\cdot T}{y_{D_0}}[F - (T - \tau)]$$

Step 5: D_0

The cumulative available demand of Type II flights, D_0, is obtained by assuming all these flights be taking off immediately at time τ. Therefore, if capacity permits, Type II flights arriving at time t after revision are the Type II flights with flight time as $t-\tau$. The cumulative available demand of Type II flights at time t is then equal to the total number of the total Type II flights multiplying the value of the cumulative flight time distribution function at $t-\tau$:

$$D_0(t|\tau,T) = D_{0,total} \cdot F_\tau(t-\tau|D_0)$$

$$= \begin{cases} 0, t < \tau + F_{min} \\ \frac{(\lambda - C_L)}{\Delta F} \cdot \left[\left(\tau(t-\tau) + \frac{(t-\tau)^2}{2}\right) - \left(\tau F_{min} + \frac{{F_{min}}^2}{2}\right)\right], \tau + F_{min} \le t \le T \\ \frac{(\lambda - C_L)}{\Delta F}\left[\left(\tau(T-\tau) + \frac{(T-\tau)^2}{2}\right) - \left(\tau F_{min} + \frac{{F_{min}}^2}{2}\right)\right] + \frac{(\lambda - C_H)}{\Delta F}\left[\left(\tau(t-\tau) + \frac{(t-\tau)^2}{2}\right) - \left(\tau(T-\tau) + \frac{(T-\tau)^2}{2}\right)\right] + \frac{(C_H - C_L)\cdot T}{\Delta F}(t-T), T \le t \le T_2 \end{cases}$$

It should be emphasized that Type II flights are assigned to arrive before T_2 in the GDP plan. Flights scheduled to arrive after T_2 are not involved in the GDP and there is no delay planned for these flights. However, all the Type II flights have been delayed to some degree at τ. Therefore, flights arriving after T_2 cannot be Type II flights and are excluded in the analysis.

2.1.1.3 D_+, Type III flights

Type III flights are scheduled to take off after τ. Delay has not occurred for these flights, and thus there would be no delay for Type III flights if they could take off as scheduled. In the scheduled, Type III flights arrive gradually after $\tau + F_{min}$, and all the scheduled flights after $\tau + F_{max}$ are Type III flights. So the maximum or say possible demand from this group of flights, which is the same as the scheduled cumulative demand, can be formulated as:

$$D_+(t|\tau,T) = \begin{cases} 0, t \le \tau + F_{min} \\ \frac{\lambda}{2\Delta F}(t-\tau-F_{min})^2, \tau + F_{min} < t \le T_2 \end{cases}$$

2.1.1.4 Revised cumulative arrival demand, D

Sum up the three parts; we get the cumulative available arrival demand as:

$$D(t|\tau,T) = \begin{cases} C_L t, \quad 0 < t < \tau + F_{min} \\ \frac{\lambda - C_L}{\Delta F}\cdot t^2 - \frac{\lambda - C_L}{\Delta F}\cdot(\tau + F_{min})\cdot t + C_L \cdot t, \quad \tau + F_{min} \le t < T \\ \frac{\lambda - C_H}{\Delta F}\cdot t^2 - \frac{\lambda - C_H}{\Delta F}\cdot(\tau + F_{min})\cdot t + C_H\cdot t + \frac{C_H - C_L}{\Delta F}\cdot T\cdot t - \frac{C_H - C_L}{\Delta F}\cdot T\cdot(\tau + F_{max}), \quad T \le t < T_2 \end{cases}$$

The available demand rate can be calculated as the derivate of $D(t|\tau)$:

$$D'(t|\tau,T) = \begin{cases} C_L, & 0 < t < \tau + \mathrm{F_{min}} \\ 2\frac{\lambda - C_L}{\Delta F} \cdot t - \frac{\lambda - C_L}{\Delta F} \cdot (\tau + \mathrm{F_{min}}) + C_L, & \tau + \mathrm{F_{min}} \le t < T \\ 2\frac{\lambda - C_H}{\Delta F} \cdot t - \frac{\lambda - C_H}{\Delta F} \cdot (\tau + \mathrm{F_{min}}) + C_H + \frac{C_H - C_L}{\Delta F} \cdot T, & \mathrm{T} \le t < T_2 \end{cases}$$

If capacity permits, when a GDP is cancelled early then all the held flights can be released immediately and Type III flights can take off as scheduled when the GDP is cancelled earlier. In other words, *D* serves as the basis for allocating new CTA's to flights. In practice, 77% of GDPs are cancelled—without the need to assign new CTA's—in this case because of the natural spread of flight time and schedules (Ball et al., 2010). When capacity is insufficient, the throughput rate will be jointly determined by the demand and the capacity:

$$Th'(t|\tau < T, early\ cancellation) = \min\{D'(t|\tau,T), C(t|\tau)\}$$

Where, $C(t|\tau)$ is the capacity at time t:

$$C(t|\tau) = \begin{cases} C_L, t \le \tau \\ C_H, t > \tau \end{cases}$$

The cumulative throughput is then equal to:

$$Th(t|\tau < T, early\ cancellation) = \int_0^t Th'(s|\tau < T, early\ cancellation)ds$$

And the total realized ground delay is:

$$D_R(\tau < T, early\ cancellation) = \int_{S(t)>Th(t)} [S(t) - Th(t|\tau < T, early\ cancellation)]dt$$

Because there are flights that have taken off before τ arriving after T, delay will not vanish in the system until the planned clearance time T. The field of integration is from zero to T for this case.

The algorithm to calculate the revised cumulative curve and the realized delay is the same regardless of the formulation of D. Therefore, for the other two cases of early cancellation, we will just present how to generate the available cumulative demand curve.

2.1.2 Early GDP Cancellation, $\tau + F_{min} < T < \tau + F_{max} < T_2$

In this case, flights that depart before τ can all arrive at the affected airport before the planned delay clearance time T_2. As shown in Figure 4, there will be no more delay in the system after $\tau + F_{max}$, if capacity permits

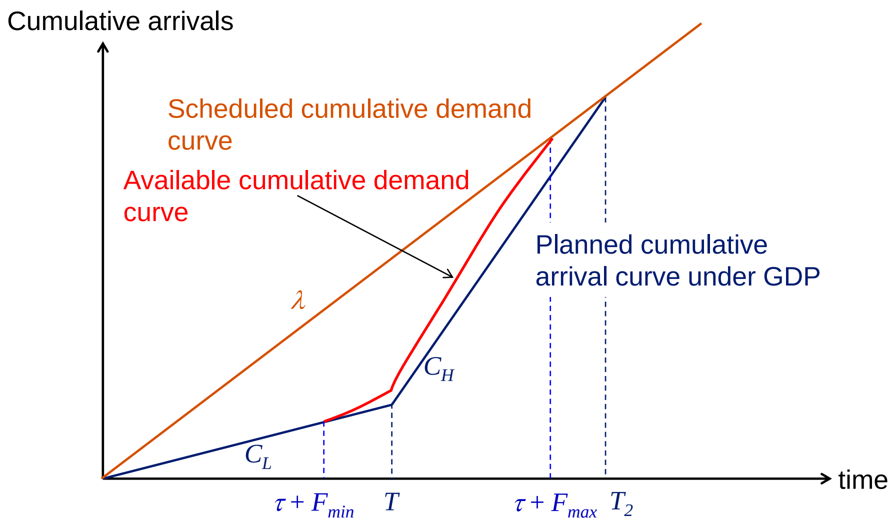

Figure 4. Queueing diagram, $\tau + F_{min} < T < \tau + F_{max} < T_2$

The same methodology is used to generate the available demand curves for the three types of flights. The studied period is still between 0 and T_2. The difference is that the flight time range for Type II flights is $[F_{min}, F_{max}]$ in this case. The available arrival demand curve is found to be

$$D(t|\tau,T) = \begin{cases} C_L t, & 0 < t < \tau + F_{min} \\ \frac{\lambda - C_L}{\Delta F} \cdot t^2 - \frac{\lambda - C_L}{\Delta F} \cdot (\tau + F_{min}) \cdot t + C_L \cdot t, & \tau + F_{min} \le t < T \\ \frac{\lambda - C_H}{\Delta F} \cdot t^2 - \frac{\lambda - C_H}{\Delta F} \cdot (\tau + F_{min}) \cdot t + C_H \cdot t + \frac{C_H - C_L}{\Delta F} \cdot T \cdot t - \frac{C_H - C_L}{\Delta F} \cdot T \cdot (\tau + F_{max}), & T \le t < \tau + F_{max} \\ \lambda t, & \tau + F_{max} \le t \le T_2 \end{cases}$$

2.1.3 Early GDP Cancellation, $\tau + F_{min} < \tau + F_{max} < T$

In this case, the maximum flights time is small and the weather clears up much earlier than the planned time. GDP revision is supposed to be more effective in terms of delay saving. The queueing diagram for arrivals is shown in Figure 5.

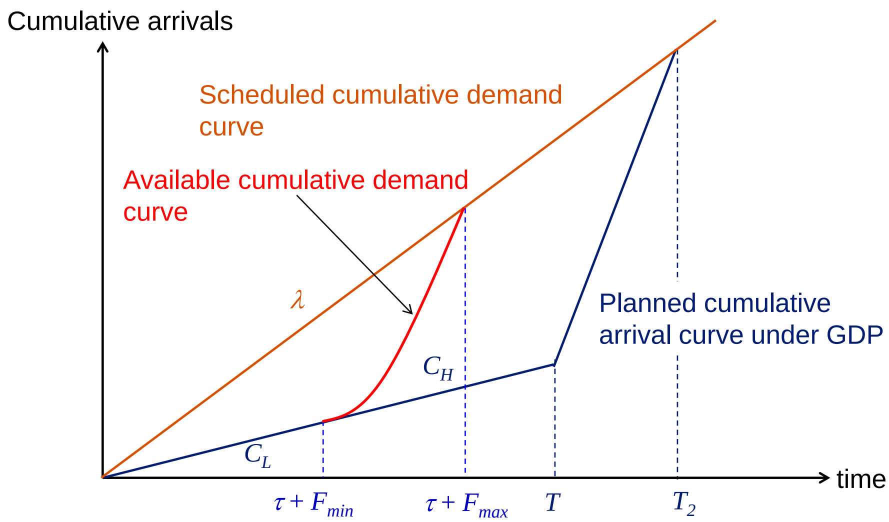

Figure 5. Queueing diagram, $\tau + F_{min} < \tau + F_{max} < T$

The formulation of available demand curve is simple compared to the other two cases:

$$D(t|\tau, T) = \begin{cases} C_L t, & 0 \leq t < \tau + F_{min} \\ \frac{\lambda - C_L}{\Delta F} \cdot t^2 - \frac{\lambda - C_L}{\Delta F} \cdot (\tau + F_{min}) \cdot t + C_L \cdot t, & \tau + F_{min} \leq t < \tau + F_{max} \\ \lambda t, & \tau + F_{max} \leq t \leq T_2 \end{cases}$$

2.2 GDP Extension Models

In the case of late clearance, GDP will be extended. We assume that flight operators are informed of the new CTA at time T. When revising GDP, we assume that τ is known with certainty. Extension is realized by giving priority to flights in the air and further holding flights on the ground if necessary. Plot a) of Figure 6 is the queueing diagram for the late clearance case without GDP extension. The shaded trapezoid represents the total amount of airborne delay due to unexpected late recovery of the capacity. By extending the GDP, we can transform part of this airborne delay to ground delay even though we are not able to reduce the amount of delay in the system, as seen in Plot b).

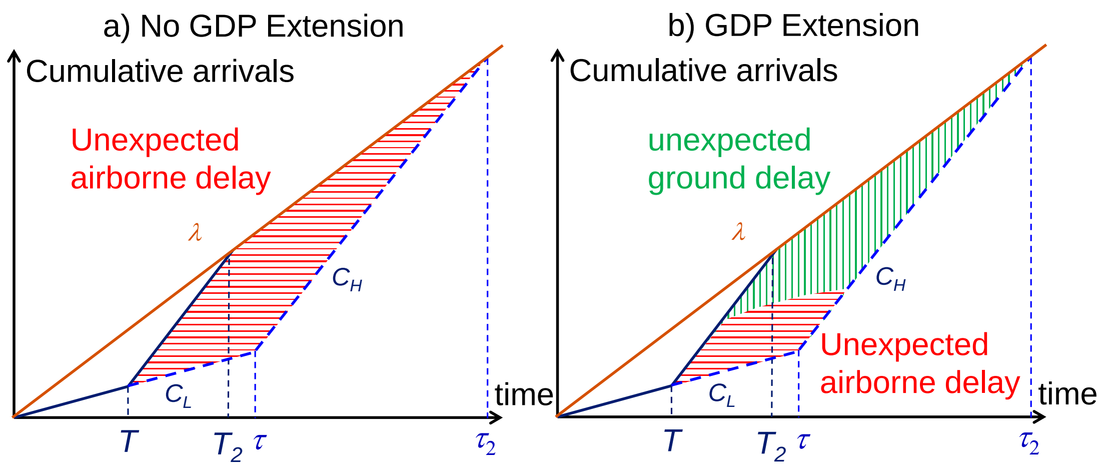

Figure 6. Queueing diagrams for the arrivals, Late Clearance

The cumulative throughput curve is:

$$Th(t|\tau \geq T) = A(t|\tau)$$

where, $A(t|\tau)$ is the cumulative arrival curve if we had perfect information at the beginning of the GDP.

When we extend the GDP, the landing sequence may not be ration-by-schedule anymore because landing priority is given to the en-route flights. An example is illustrated with Table 1, where GDP is implemented due to dense fog. The fog is planned to burn off at 10:00 am in the original plan. At 10:00 am, it does not clear up and the forecast is predicting the weather will be clear at 11:00 am. As a result, there will be another half hour of delay on average for the affected flights. In GDP extension, flights that have not taken off at 10:00 am will be further held on the ground. As seen in Table 1, Controlled Time of Departure (CTD) of Flight 2 is updated to 45 minutes later and it will arrive at the affected airport at 1:45pm instead of 1:00 pm. Delay for Flight 2 increases from 1.5 hours to 2.25 hours, and the extra 0.75 hour is realized as ground delay. On the contrary, flight 1 has already taken off before 10:00 am; therefore, any extra delay will be in the form of airborne delay. Because priority is given to en-route flights in GDP extension, Flight 1 lands at 1:30 pm which is earlier than the actual arrival time of Flight 2. Due to GDP, Flight 1 is delayed for 1.75 hours with 0.25-hour airborne delay. If there were no GDP revision, then Flight 1 would have landed at 1:45 pm with 0.5-hour airborne delay.

Table 1: Example of landing re-sequence

Flight	OTA	Flight Time (hour)	CTD	CTA	New CTD	New CTA
1	11:45 am	5	8:15 am	1:15 pm	8:15 am	1:30 pm
2	11:30 am	1.5	11:30 am	1:00 pm	1:15 pm	1:45 pm

With the GDP extension, we transferred 15-minute airborne delay to ground delay. However, it should be noticed that the total extra delay for these flights is 1 hour, which is the same as if there were no GDP extension. In addition, the arrival sequence is reversed compared to the arrival time in the schedule. Our research focuses the performance at the system level and ignores the impact of GDP extension at the flight level.

Similar to the case of early clearance, we also categorize flights into three groups: Type I, II and III. However, the critical time used to define the groups will be T instead of τ in the previous case. For instance, Type I flights are the flights that have taken off at time T. Denote the planned cumulative arrival curve for Type I flights in this case as C_{-}. We assume that:

- The actual clearance time τ is known at time T, when we extend the GDP.
- All the capacity will be used to land Type I flights first. Type II/III flights will be held on the ground if necessary and released when there are available arrival slots.

To estimate airborne delay in the system, we need to generate the cumulative arrive curve for Type I flights. The difference between this and the actual cumulative arrive curve—with slope shifts to C_H at τ—will be the unexpected airborne delay. The rest of the delay will be the unexpected ground delay. There are four different cases for GDP extension, as illustrated in Figure 7. The unexpected airborne delay is highlighted as in the shaded area.

Plot a) represents the case where all the unexpected delay is airborne delay. This happens because all the unexpected delay is encountered by Type I flights. It is most likely when T is close to τ and C_H is much bigger than C_L. No action should be taken because there is no delay for flights that have not taken off at this time. The airborne delay is

$$AD(\tau \geq T) = \int_{S(t)>A(t|\tau)} [S(t) - A(t|\tau)]dt - D_P = \frac{1}{2}\frac{(C_H - C_L)(\lambda - C_L)}{C_H - \lambda} \cdot (T^2 - \tau^2)$$

There will also be ground delay at the end of the program, which is the same as the planned ground delay in this case. So the total realized delay is

$$D_R(\tau \geq T) = AD(\tau \geq T) + D_P = \frac{1}{2}\frac{(C_H - C_L)(\lambda - C_L)}{C_H - \lambda} \cdot \tau^2$$

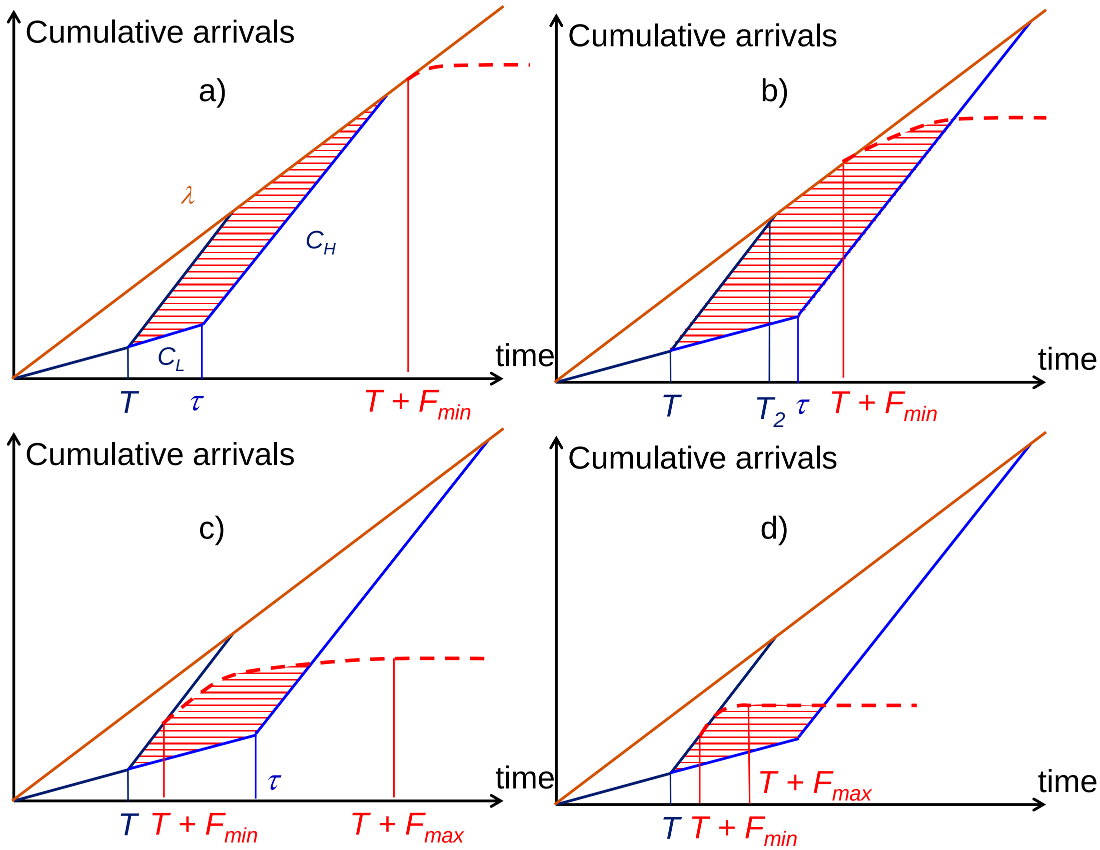

Figure 7. GDP extension models

In the other three plots, GDP extension should be considered and the amount of airborne delay depends on the magnitudes of the minimal flight time, the maximum flight time, and the difference between the planned delay clearance time and the planned capacity recovery time. The expressions of C_{-}, cumulative arrival curve for Type I flights for the three cases are summarized in Appendix B. The algorithm of generating these curves is similar to that shown in Section 2.1.1.1. The area between C_{-} and the actual cumulative curve is the airborne delay for each case. In other words, airborne delay can be calculated as:

$$AD(\tau \geq T) = \int_{C_{-}(t|\tau) > A(t|\tau)} [C_{-}(t|\tau) - A(t|\tau)]dt$$

The total amount of delay is:

$$D_R(\tau \geq T) = \int_{S(t)>A(t|\tau)} [S(t) - A(t|\tau)]dt = \frac{1}{2}\frac{(C_H - C_L)(\lambda - C_L)}{C_H - \lambda} \cdot \tau^2$$

Therefore, the realized ground delay is

$$GD(\tau \geq T) = D_R(\tau \geq T) - AD(\tau \geq T)$$

2.3 Impact of GDP Scope

So far, we have assumed that all the flights heading to the affected airport with arrival time in the constrained period are involved in the GDP. In practice, only flights within a certain region will be subject to the GDP. Flights that are geographically farther than the scope will be exempted from the program. As mentioned, scope is an important design parameter of the GDP. In this analysis, it is reflected by F_{Scope}, the maximum flight time of the GDP affected flights. Flights with flight time between F_{Scope} and F_{max} will exempted from the program. The demand rate of the exempted flights is denoted by λ_e. By assuming a uniform distribution for flight time, we can obtain

$$\lambda_e = \frac{F_{max} - F_{scope}}{F_{max} - F_{min}} \cdot \lambda$$

All the delay will be absorbed by the non-exempted flights whereas the exempted flights will arrive at the airport on time. The queueing diagrams of the GDP arrivals for the non-exemption case and the exemption case are shown in Figure 8. The non-exemption case is represented with dashed lines and the case with exempted flights is represented with solid lines. Compared to the non-exemption case, both the demand rates and the capacity rates in the exemption case are reduced by λ_e, which should be less than C_L. Denote the delay clearance time in the exemption case as $T_{2,e}$. It is found that $T_{2,e}$ is equal to T_2, when delay clears in the non-exemption case. The planned cumulative arrival curve is shaped the same as that in the non-exemption case:

$$N_e(t|T) = \begin{cases} 0, t \leq 0 \\ (C_L - \lambda_e) \cdot t + \lambda_e \cdot t, 0 < t \leq T \\ (C_L - \lambda_e) \cdot T + (C_H - \lambda_e) \cdot (t - T) + \lambda_e \cdot t, T < t \leq T_2 \\ \lambda t, t > T_2 \end{cases} = N^C(t|T)$$

It should be mentioned that this is not the cumulative curve for the exemption case in Figure 8. Since there is no delay for exempted flights, to estimate the delays for the GDP with exemption, we only need to replace $\lambda/C_H/C_L$ in the previous models with $\lambda - \lambda_e/C_H - \lambda_e/C_L - \lambda_e$. The planned delay will be the same for different exemption rates with the same T because the planned cumulative arrival curve remains unchanged.

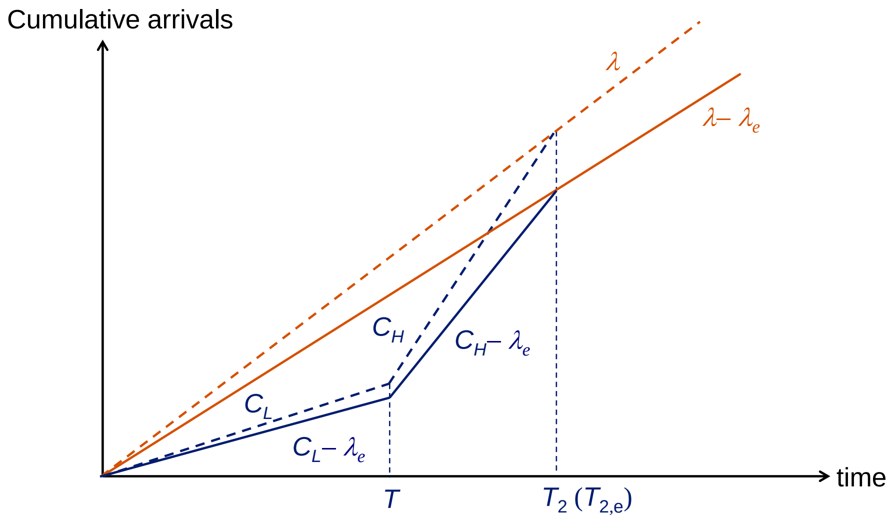

Figure 8. Queuing diagrams of GDP affected arrivals, non-exemption and exemption cases.

3. Performance Metrics

In this Section, we introduce our performance metrics with the proposed GDP models. For each metric, we will first present the general definition based on the performance criteria. After that, we will derive the formulations for the metric under three different situations: early clearance without GDP cancellation; early clearance with GDP cancellation; late clearance with GDP extension. The Section is summarized in 3.5.

3.1 Capacity Utilization

This metric is specified to measure how fully we used our capacity. It is defined as the ratio of throughputs:

$$\alpha_c(\tau, T) = \frac{N_R}{N_I}$$

where,

N_I is ideal throughput under perfect information at the time when queue clears, τ_2;

N_R is realized throughput at this time.

These values are shown in Figure 9, for the cases of early clearance and late clearance respectively. As we see in Plot a), the realized throughputs are less than the idealized throughput at τ_2. Therefore, capacity utilization is less than 1 in the case of early clearance. However, N_R is increased if we consider early GDP cancellation, which benefits capacity utilization. In the case of late clearance, the ideal throughput is the same as the realized throughput since delay could only clear at time τ_2 even if we had perfect information at the beginning. As a result, capacity utilization is equal to 1. In summary, we have

$$\alpha_c(\tau < T|T) = \frac{Th(\tau_2|\tau, T)}{A^C(\tau_2|\tau)} = \begin{cases} \dfrac{N(\tau_2|T)}{\lambda \cdot \tau_2}, \text{no early cancellation} \\ \dfrac{Th(\tau_2|\tau < T, early\ cancellation)}{\lambda \cdot \tau_2}, \text{early cancellation} \end{cases}$$

and

$$\alpha_c(\tau \geq T|T) = 1$$

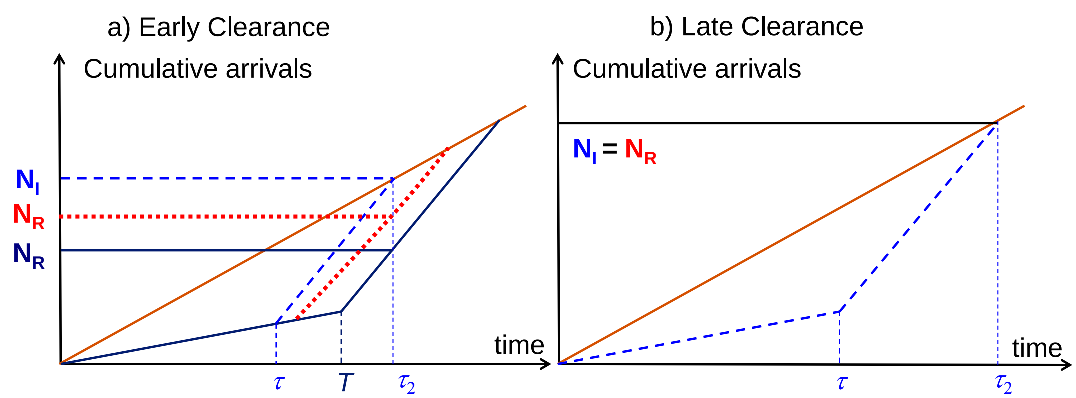

Figure 9. Ideal throughput and realized throughputs

Note: The red dot line represents the cumulative throughput curve with early cancellation

3.2 Predictability

In principle, predictability is defined to measure the amount of information available in advance. In the design of a GDP, a certain amount of delay is planned. Flight operators will be informed of the expected delays before the GDP is implemented. The realized delay will usually be different from the planned delay due to error in prediction.

Predictability is then identified to measure how different the realized delay is from the planned delay:

$$\alpha_P(\tau, T) = \frac{min(D_P, D_R)}{max(D_P, D_R)}$$

where,

D_P is flight delay planned at the beginning of the GDP;

D_R is total realized flight delay.

As shown in Figure 10, D_P is determined by the planned weather clearance time at the beginning of the GDP and does not change with the real clearance time. On the contrary, D_R depends on when the weather will clear up and whether we choose to revise the GDP D_R or not. In the case of early clearance, D_R will be equal to D_P if we choose not to revise the GDP. Flight operators could run their operations relying on the CTA allocated in the original GDP and no further adjustment will be needed. If the GDP is revised, as seen in the bottom-left plot, we will be able to save delay in the system and realized delay will be less than the planned delay. In the case of late weather clearance, realized delay is larger than the planned delay due to unexpected late capacity recovery. We can further write our predictability metric as

$$\alpha_P(\tau < T|T) = \begin{cases} 1, \text{ no early cancellation} \\ \dfrac{D_R(\tau < T, early\ cancellation)}{\frac{1}{2}\frac{(C_H - C_L)(\lambda - C_L)}{C_H - \lambda} \cdot T^2}, \text{ early cancellation} \end{cases}$$

and

$$\alpha_P(\tau \geq T|T) = \frac{\frac{1}{2}\frac{(C_H - C_L)(\lambda - C_L)}{C_H - \lambda} \cdot T^2}{D_R(\tau \geq T)} = \frac{\frac{1}{2}\frac{(C_H - C_L)(\lambda - C_L)}{C_H - \lambda} \cdot T^2}{\frac{1}{2}\frac{(C_H - C_L)(\lambda - C_L)}{C_H - \lambda} \cdot \tau^2} = \frac{T^2}{\tau^2}$$

Predictability will be less than 1 when we make change to the original GDP plan, either by cancelling the program earlier or extending it. Choosing not to revise the GDP when τ is less than T, will cause unnecessary delay in the system but will benefit predictability. By the current practice, GDP is usually cancelled early whenever possible. However, there is widespread consensus in the community that predictability is important. Day-of-operation predictability allows a multitude of benefits, such as reduced communication between command center and airline dispatchers and pilot workload mitigation. In this analysis, we leave early GDP cancellation as an option, which will allow us to examine a GDP design in a more comprehensive way.

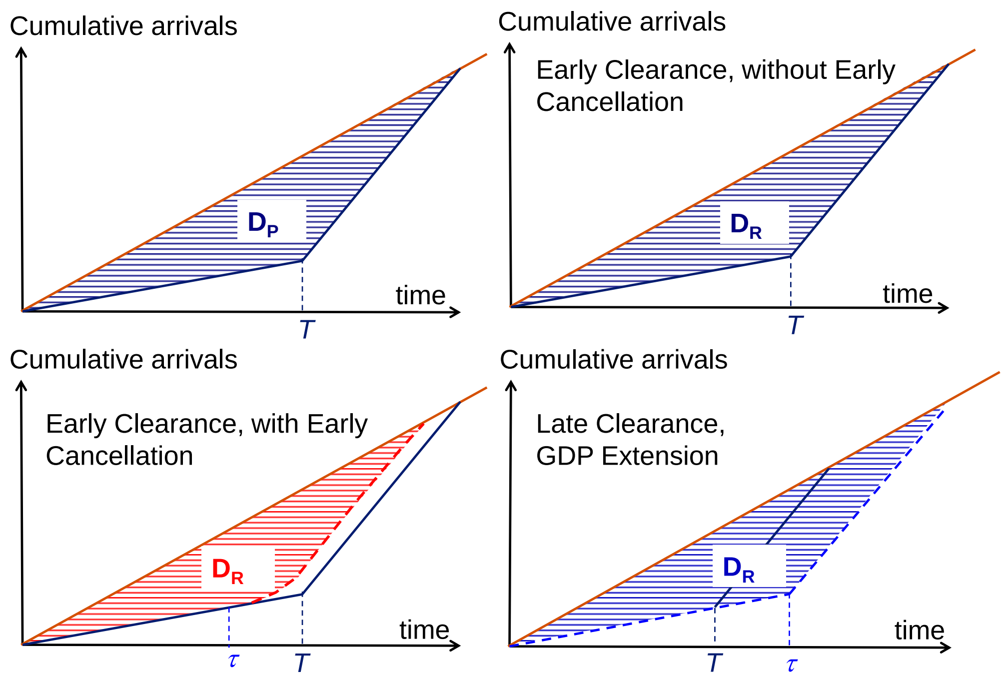

Figure 10. Planned delay and realized delays

3.3 Efficiency

A primary motivation for the GDP is that as long as delay is unavoidable, it is cheaper and safer for flights to absorb delay on the ground before take-off, rather than in the air. The efficiency metric is defined to measure realized delay cost relative to the minimum cost that could be incurred under perfect information. In this metric, we distinguish cost of airborne delay from the cost of ground delay and assume the cost ratio is β (>1). In other words, 1-minute of airborne delay is equivalent to β-minute of ground delay. The efficiency metric is then

$$\alpha_e(\tau, T) = \frac{C_I}{C_R}$$

where,

C_I is minimum cost that would be incurred if perfect information were available about when the capacity will increase:

$$C_I(\tau) = \int_{S(t)>A(t|\tau)} [S(t) - A(t|\tau)]dt = \frac{1}{2}\frac{(C_H - C_L)(\lambda - C_L)}{C_H - \lambda} \cdot \tau^2$$

C_R is total realized cost;

C_I will always be ground delay cost but C_R could include airborne delay cost. The costs are illustrated in Figure 11. All the costs are ground delay cost except the realized cost in the GDP extension case, as shown in the bottom-right plot. As discussed in Section 2.2, there will be airborne delay-highlighted with dots- for flights that have taken off at time T.

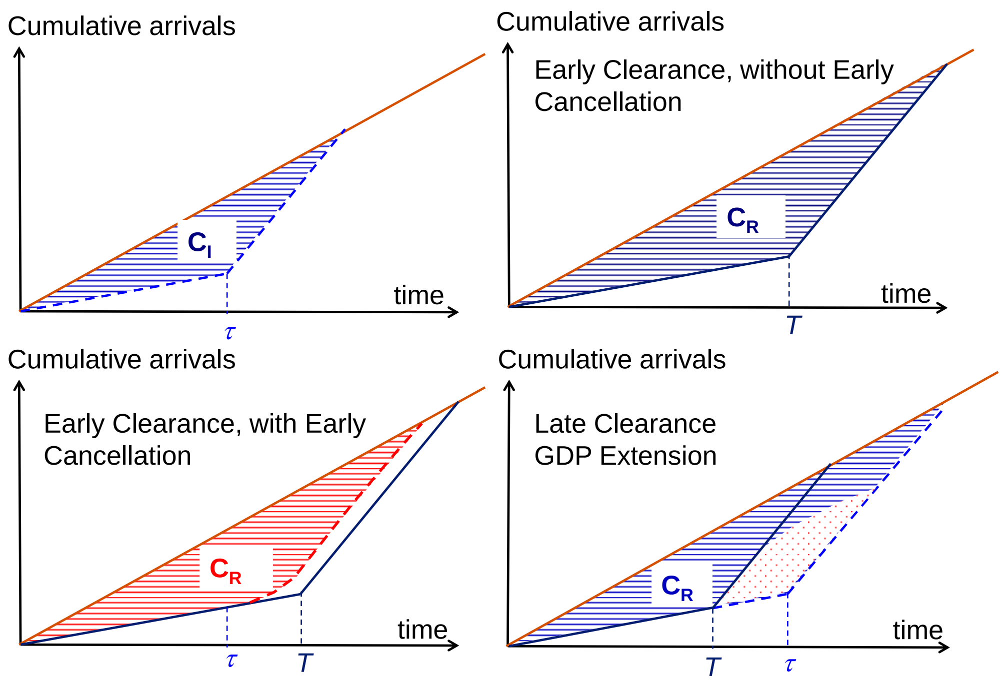

Figure 11. Minimum cost and realized costs

Following the notations in Section 2, we can further write the efficiency metric as:

$$\alpha_e(\tau < T|T) = \begin{cases} \dfrac{\frac{1}{2}\frac{(C_H - C_L)(\lambda - C_L)}{C_H - \lambda} \cdot \tau^2}{\frac{1}{2}\frac{(C_H - C_L)(\lambda - C_L)}{C_H - \lambda} \cdot T^2} = \dfrac{\tau^2}{T^2}, \text{ no early cancellation} \\ \dfrac{\frac{1}{2}\frac{(C_H - C_L)(\lambda - C_L)}{C_H - \lambda} \cdot \tau^2}{D_R(\tau < T, early\ cancellation)}, \text{early cancellation} \end{cases}$$

and

$$\alpha_e(\tau \geq T|T) = \frac{\frac{1}{2}\frac{(C_H - C_L)(\lambda - C_L)}{C_H - \lambda} \cdot \tau^2}{GD(\tau \geq T) + \beta \cdot AD(\tau \geq T)}$$

3.4 Equity

When there are flights exempted from the GDP, total planned delay and delay planned clearance time are still the same as the case without exemption. However, the maximum planned flight delay is different. The maximum planned flight delay in this model can be expressed as:

$$d_{P,max}^{e} = \frac{\lambda - C_L}{\lambda - \lambda_e} T$$

In the non-exemption case, λ_e is zero and the maximum planned delay is minimized. $d_{P,max}^{e}$ is illustrated in Figure 12.

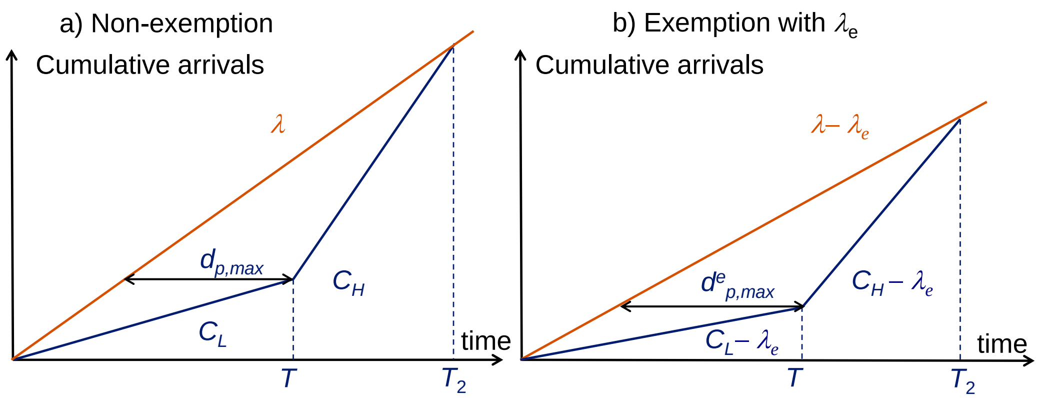

Figure 12. Maximum planned delays, non-exemption and exemption cases

In the exemption case, $d_{P,max}^{e}$ increases with increasing exempted demand rate. With more flights exempted from the program, more delays are allocated to the affected flights whereas the exempted flights are 'free-riders'. This raises the equity issue in the design of GDPs: how much of the demand should be exempted from the program. In practice, the FAA exempts flights from a GDP by limiting the scope of the GDP to a geographical area surrounding the destination airport (Ball and Lulli, 2004). A flight operator, whose flights are mostly long-haul, will prefer a smaller scope so that more of its flights can arrive on time. On the contrary, flight operators with more short-haul flights may prefer a larger scope, in which case total delay will be absorbed by more flights and delay per flight will be reduced. Preserving equity among competing flight operators is an important goal of the FAA. In this study, the equity metric is defined to measure the maximum planned delay, relative to the maximum delay when no flights are exempted from the GDP (Hoffman et al., 2007; Mukherjee and Hansen, 2007; Ball, et al., 2010). Different from the other performance metrics, equity will only be measured when the

GDP is planned. Its value will not be updated upon a GDP revision. The performance metrics of equity is expressed as:

$$\alpha_f = \frac{\min(d_{P,max}^e)}{d_{P,max}^e} = \frac{\lambda - \lambda_e}{\lambda} = \frac{F_{scope} - F_{min}}{F_{max} - F_{min}}$$

As seen in the formula, the equity of a GDP is independent of the decision on the clearance time and only affected by the scope. Specifically, equity is an increasing function of scope.

3.5 Expectations of Performance Metrics

We constructed performance metrics for capacity utilization, predictability, efficiency and equity in the previous sections. All the metrics are dimensionless, and between 0 and 1. The expected value of equity metric is determined once we select the scope of the GDP and independent of the prediction errors. However, the values of the other three metrics depend on τ, the real weather clearance time. We need to integrate over τ to get the expected values of the performances:

$$\alpha(T) = E[\alpha(\tau, T)] = \int_{t_{min}}^{t_{max}} \alpha(\tau, T) \cdot f(\tau) d\tau = \int_{t_{min}}^{t_{max}} \alpha(\tau, T) \cdot \frac{1}{t_{max} - t_{min}} d\tau$$

$$= \frac{1}{t_{max} - t_{min}} \cdot [\int_{t_{min}}^{T} \alpha(\tau < T|T) d\tau + \int_{T}^{t_{max}} \alpha(\tau \geq T|T) d\tau]$$

where, α is any of the three metrics: capacity utilization, predictability and efficiency.

Since GDP decisions are made before the real clearance time is known, the program performance should be assessed using the expected values of the defined metrics.

4 Performance Trade-offs and User Optimization

In this section, through a numerical example, we will first present the influence of the GDP decisions on the performance expectations and the trade-offs among multiple performance goals. Then, we will illustrate how the research results could assist decision-making in the design of GDP under capacity uncertainty.

The set of parameter values in the example is shown in Table 2. Capacity values are chosen referring to the airport capacity benchmark report by the FAA (FAA, 2004). The lower and upper bounds for the τ are estimated after reviewing the air traffic control system command center advisories database, which is available in the FAA website. The cost ratio of airborne delay to ground delay is set as 2 (Mukherjee and Hansen, 2007).

Table 2: Parameter values used in the numerical example

Parameter	Notation	Values	Unit
Scheduled demand rate	λ	60	Arrival per hour
High airport acceptance rate	C_H	80	Arrival per hour
Low airport acceptance rate	C_L	40	Arrival per hour
Lower bound for τ	t_{min}	2	Hour
upper bound for τ	t_{max}	6	Hour
Minimum flight time	F_{min}	0.5	Hour
Maximum flight time	F_{max}	7	Hour
Cost ratio	β	2	-

4.1 Performance Metrics and Their Trade-offs

There are three decisions in the design process of a GDP: scope of the GDP, planned clearance time, and whether to cancel the GDP in the case of early clearance or not. As discussed before, equity is determined by the GDP scope only, and it increases monotonically with the scope. Expectations of the other three performance metrics also depend on the assumed clearance time and the policy on early cancellation. The influence of the decisions on the three performance metrics is shown in Figure 13, where the left three plots demonstrate the non-exemption case and the right plots demonstrate the exemption case, with half of the demand exempted from the GDP.

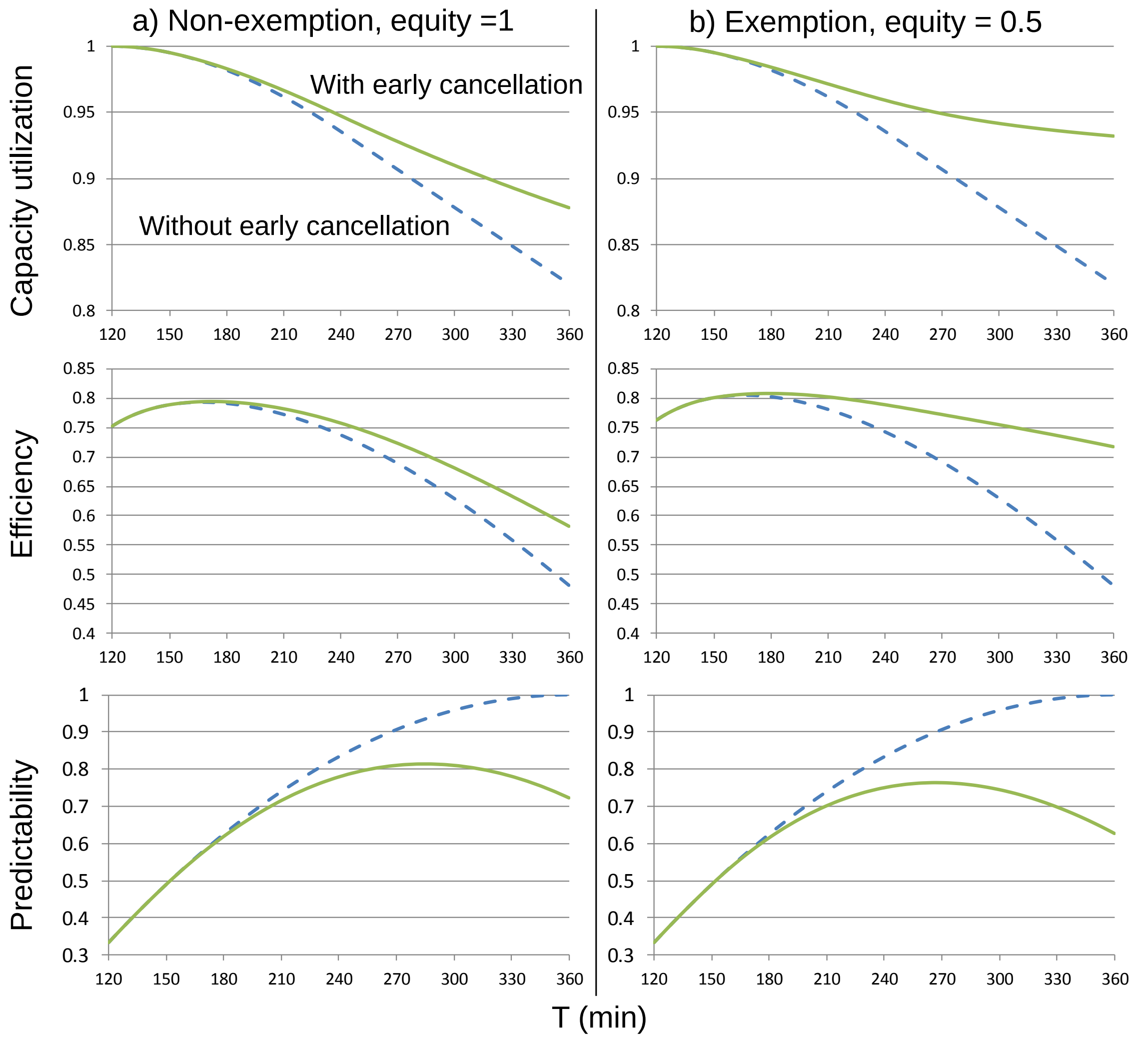

Figure 13. Values of performance metrics as functions of T, with and without early cancellation, with and without GDP exemption

When no flight is exempted from the GDP, equity is equal to one as for the left three plots. Capacity utilization decreases with the planned clearance time because there is a larger chance of early clearance with larger T and part of the high capacity cannot be utilized. Early GDP cancellation will enable us to take advantage of the unexpected high capacity, which benefits capacity utilization. As T increases, efficiency first increases because of reduced chance of expensive airborne delay. After a certain point, efficiency will decrease with T because it is very likely that realized ground delay is much larger than it could be if we had perfect information. Early GDP cancellation saves delay in the system and increases the efficiency. Predictability increases with T without early GDP cancellation. With a larger T, it is very likely that capacity will recover earlier than planned, without early cancellation, and the realized delay will be the same as the planned delay so that predictability approaches 1. In contrast to efficiency and capacity utilization, predictability degrades when we permit early GDP cancellation. Basically, the more adaptable we make the GDP, the less predictability we have. The impact of early cancellation is more obvious with a larger T for all the three metrics.

The conclusions above all also hold when there is exemption, as shown in the right plots. By comparing the early cancellation plots to the non-exemption case (solid lines), we see that exemption will increase capacity utilization and efficiency, but decrease predictability in the system. By exempting long-haul flights from the GDP, the delayed flights are concentrated in the vicinity of the affected airport and they could arrive at the airport earlier if there is early cancellation, which enables the airport to utilize the expected extra capacity earlier. This benefits capacity and efficiency, but reduces predictability. In the case of GDP extension, more flights will still be on the ground if they are closer to the affected airport. As a result, more airborne delay could be transferred to ground delay, which increases efficiency. By comparing the plots without early cancellation for the non-exemption and exemption cases (dashed lines), we find that the plots for capacity and predictability are the same regardless of exemption rate whereas efficiency is slightly improved with exemption. The efficiency gain from reduced scope is because the GDP extension can shift more airborne delay to ground delay.

Performance trade-off curves are shown in Figure 15. Movement toward the right along these curves is associated with earlier planned clearance times. Equity is equal to 1 for the left two plots. The bottom-left plot presents the trade-offs between efficiency and capacity utilization. The dashed blue line is for the case without early cancellation and the solid green line is for the case with early cancellation. Both plots have internal optima, and the points located on the left of the internal peaks are inferior because we can increase efficiency and capacity utilization simultaneously by decreasing T. On the right of the peaks, the line for early cancellation is above that for no early cancellation. Therefore, if only efficiency and capacity utilization are concerned, then we will always choose to terminate the GDP earlier if possible and we tend to pick an earlier planned clearance time. The situation changes when predictability is also taken in to account. Comparing the two dashed plots on the left, we see that a choice of larger Tdegrades efficiency and capacity utilization but benefits predictability. As a result, flight operators who value predictability more may prefer a larger T. Additionally, early cancellation is not necessarily to be a better choice when predictability is important. For instance, at capacity utilization equal to 0.894, both predictability and efficiency are higher if we choose not to terminate the GDP earlier. It should be pointed out that the planned clearance time of the case with no early cancellation case is smaller than that of the early cancellation case here. The trade-off relationship is similar when equity is reduced to 0.5, as shown in the right plots. Since early cancellation is more effective with long-haul flights exempted, the differences between two cases are more pronounced.

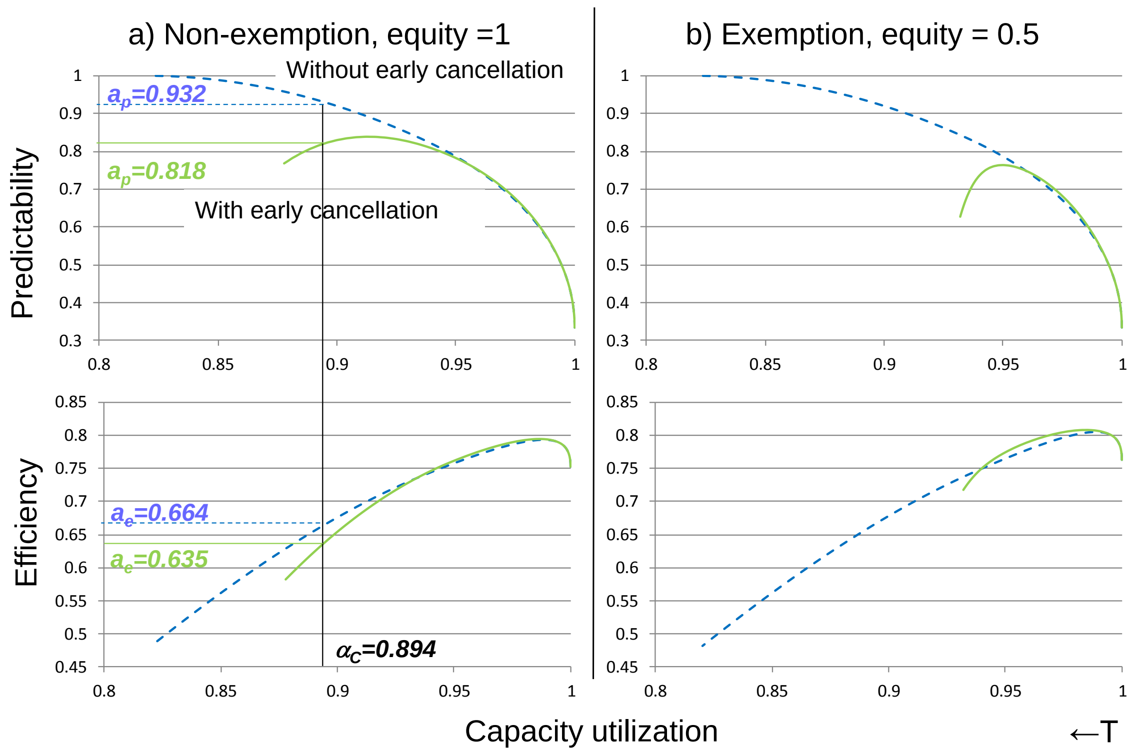

Figure 14. Performance trade-off curves, with *T* increase from the right to the left

4.2 User Optimization

Different flight operators may have different preferences regarding performance goals. Each flight operator may prefer a different point on the trade-off curves, and correspondingly opt for different GDP plans. Rationally, each user will prefer the point that maximizes their utility. The decision-making process could then be formed as a utility optimization problem. In 4.2.1 and 4.2.2, we will present two utility functions. In 4.2.1, the FAA will predetermine the level of equity and the flight operators will only consider the other three metrics in the utility function. In 4.2.2, equity is votable and all the four metrics are involved in the flight operator utility maximization process. We assume linear utility functions to illustrate how the trade-off curves could be used by flight operators to select their preferred GDP decisions. Using the same set up, optimal solutions can always be found for users with concave utility functions.

4.2.1 Predetermined Equity

The constrained optimization problem could be set up as the following. Each system user will choose a set of the performance vectors, $[\alpha_c, \alpha_p, \alpha_e]$, to

maximize: $U(\alpha_c, \alpha_p, \alpha_e)$, subject to: $F_{early\_cancellation}(\alpha_c, \alpha_p, \alpha_e) = 0$ or $F_{no\_early\_cancellation}(\alpha_c, \alpha_p, \alpha_e) = 0$

where, the constraints are limiting the feasible region to the points on the trade-off curves.

Here, we assume a linear utility function:

$$U(\alpha_c, \alpha_p, \alpha_e) = C_c \cdot \alpha_c + C_p \cdot \alpha_p + C_e \cdot \alpha_e$$

where, the coefficients are the weights of the performance goals.

Implied ideal plans of three users with different preferences on performance goals are compared in Table 3. User 1 is concerned most with capacity utilization and weights predictability and efficiency equally. Predictability has double the importance to User 2 compared to the other metrics, and is an even more of a critical performance goal to User 3. If User 1 is the only user in the system, then the GDP should be planned for 3.8 hours and early cancellation should be performed if possible, as long as all the flights are involved in the system. If half demand is exempted, then the GDP should be planned for a little longer. T increases with exemption rate if early cancellation happens, because early cancellation is more effective with more short-haul flights. With predictability as the most critical performance goal, both User 2 and User 3 choose not to revise the program in the case of early clearance. The preferred planned clearance times of the two users are not affected by the value of equity or say the choices on the GDP scopes. As discussed before, GDP extension only benefits efficiency but has no effect on predictability and capacity utilization. Without early cancellation, exemption makes no difference to the performance metrics in the case of early clearance. This is because realized throughput and realized delay are the same with different GDP exemption rates when no early cancellation is considered, as discussed in Section 2.3. Overall, the preferred T remains unchanged for different values of equity when predictability is dominating.

Table 3: Preferred GDP decisions by different system users, predetermined equity

	Weights			Equity =1		Equity =0.5	
User	C_c	C_P	C_e	T (Hour)	Early cancellation?	T (Hour)	Early cancellation?
1	0.5	0.25	0.25	3.8	Yes	3.88	Yes
2	0.25	0.5	0.25	4.88	No	4.88	No
3	0	0.75	0.25	5.4	No	5.4	No

4.2.1 Endogenous Equity

In this mechanism, equity like the other performance goals, is treated as an argument in the utility function. Thus the problem is:

maximize: $U(\alpha_c, \alpha_p, \alpha_e, \alpha_f)$,

subject to: $F_{no\_early\_cancellation}(\alpha_c, \alpha_p, \alpha_e, \alpha_f) = 0$ or $F_{early\_cancellation}(\alpha_c, \alpha_p, \alpha_e, \alpha_f) = 0$

where, the constraints are limiting the feasible region to the points on the trade-off curves. We consider 6 equity levels, from 0.5 to 1 (most equitable) with increment 0.1. We have different trade-off curves at each equity level. In total, we have 6 pairs of 3-d trade-off curves.

Again, we assume a linear utility function:

$$U(\alpha_c, \alpha_p, \alpha_e, \alpha_f) = C_c \cdot \alpha_c + C_p \cdot \alpha_p + C_e \cdot \alpha_e + C_f \cdot \alpha_f$$

where, the coefficients are the weights of the performance goals.

The optimization results and corresponding GDP decisions are shown in Table 4. The italicized rows assume that equity is unimportant. As seen from the italicized rows, the largest exemption rate, 0.5, is always better regardless of the preferences on the other three performance goals. The unexpected part of capacity can be utilized more effectively and earlier with a larger exemption rate when the GDP is cancelled earlier. Therefore, when capacity utilization is the dominating metric as in 1*, early cancellation should be allowed, but not when predictability is dominating, as in 2* and 3*.. GDP extension is not affected the performances in terms of capacity utilization and predictability, but benefits efficiency. GDP extension is more efficient when only short-haul flights are involved. Overall, small scopes are always preferred in the design of GDPs when equity is not an issue. It should be mentioned that the scope cannot be smaller than the lower bound, C_L.

Table 4: Preferred GDP decisions by different system users, votable equity

	Weights				Implied ideal plan		
User	C_c	C_p	C_e	C_f	Equity (Scope)	T (Hour)	Early cancellation?
1	**0.5**	**0.25**	**0.25**	**0.001**	**0.5**	**3.88**	**Yes**
*1**	*0.5*	*0.25*	*0.25*	*0*	*0.5*	*3.88*	*Yes*
2	**0.25**	**0.5**	**0.25**	**0.001**	**1**	**4.88**	**No**
*2**	*0.25*	*0.5*	*0.25*	*0*	*0.5*	*4.88*	*No*
3	**0**	**0.75**	**0.15**	**0.1**	**1**	**5.64**	**No**
*3**	*0*	*0.75*	*0.25*	*0*	*0.5*	*5.4*	*No*

The bolded rows highlight the situations where equity is an argument in the utility function. The preference of User 1 remains unchanged whereas User 2 and 3 update their implied ideal plans. A small weight of equity makes sense since the magnitude of change in equity is big. Comparing 2 to 2*, we find that the preferred planned clearance time is independent of the scope when predictability dominates and early cancellation is not considered. This happens because exemption is not benefitting predictability without early cancellation. Comparing 3 to 3*, the choice on the planned clearance time is more conservative since predictability is more valuable. With a larger T, there is a larger chance that the high capacity level could be available earlier and the realized delay will be equal to the planned delay since early cancellation is not considered.

5. Summary and Conclusions

In this paper, we model the GDPs using continuum approximation. Two models are developed, based on the policy on early GDP cancellation. In the model with no early cancellation, flights arrive at their planned arrival time as allocated in the GDP. No further change is expected and realized delay is equal to the planned delay at the beginning of the GDP. In the case of early cancellation, we revise the cumulative arrive curve by taking advantage of the unexpected extra capacity. The revised cumulative arrive curve is determined by the available cumulative arrival demand and the available capacities together. Analytical solutions are presented for the available cumulative arrival demands for all the possible situations, and the available capacities. By utilizing the unexpected extra capacity, we save delay in the system and delay may vanish earlier. In the case of late clearance, GDP extension is assumed to further transfer some airborne delay to ground delay. When extending the GDP, landing priority is given to the en-route flights and flights that have not taken off yet are further delayed on the ground if necessary.

In Section 3, we identify criteria and develop day-of-operation metrics in the GDP design for four performance goals: capacity utilization, predictability, efficiency, and equity. All the metrics are dimensionless and have values between 0 and 1. To evaluate service level expectation, we integrate the performance metrics over the actual clearance time to calculate the expected values of the defined metrics. Using the expectations of the performance goals, we represent trade-offs in the design of GDPS and relate these to the GDP decisions on: the GDP planned clearance time, the GDP scope and the choice on GDP early cancellation in Section 4.1. It is found that the equity of GDPs is determined by the choice on the scope, whereas the other metrics are also affected by the other two decisions.

Capacity utilization decreases with the planned clearance time, T, because there is a larger chance of early clearance with larger T and part of the high capacity cannot be utilized. Early GDP cancellation will enable us to take advantage of the unexpected high capacity, which benefits capacity utilization. As T increases, efficiency first increases because of reduced chance of expensive airborne delay. After a certain point, efficiency will decrease with T because it is very likely that realized ground delay is much larger than it could be if we had perfect information. Early GDP cancellation saves delay in the system and increases the efficiency. Predictability increases with T without early GDP cancellation. With a larger T, it is very likely that capacity will recover earlier than planned and the realized delay will be the same as the planned delay without early cancellation so that predictability approaches 1. Different from the other two metrics, predictability degrades when we permit early GDP cancellation. The impact of early cancellation is more obvious with a larger T for all the three metrics. The conclusions above also hold when there is exemption. With exemption, expectations of the capacity utilization and efficiency increase, whereas it decreases predictability since it makes the program more adaptable. In the case of GDP extension, capacity and predictability remain unchanged regardless of exemption rate whereas efficiency is slightly improved with exemption. This is because GDP extension can only transfer expensive airborne delay to cheaper ground delay but cannot reduce the amount of delay or improve throughputs.

If only efficiency and capacity utilization are concerned, then we will always choose to terminate the GDP earlier if possible and we tend to pick a small planned clearance time. The situation changes when predictability is also taken in to account. A choice of larger T

or larger scope degrades efficiency and capacity utilization but benefits predictability. Different flight operators may have different preferences on performance goals and prefer different points on the trade-off curves that maximize their utilities.

In Section 4.2, we illustrate how the trades could assist the system users in GDP decision-making use a linear function of the performance metrics as the objective. Equity choice is considered in two ways. In one case it is predetermined, while in the other it is considered an argument in the utility function. When capacity utilization is the dominant factor in the utility function, we cancel the case of early clearance. On the contrary, if predictability is the dominating metric, then we choose not to take advantage of the unexpected high capacity and operate as planned in the original GDP. When equity is not considered, smaller GDP scopes are always preferred for flight operators with difference preferences on performance goals. When equity matters, we may choose a larger GDP depending on the weights assigned by the flight operators to equity.

The work enables us to make GDP decisions using multiple criteria. This capacity will lead to improved decision-making, in which traffic managers and flight operators can make informed trade-offs based on their assessment of the importance of different performance criteria. An obvious problem is that different flight operators may have different utility functions. In that case, there must be a process for taking conflicting inputs and arriving at an acceptable compromising plan. Work on this subject, by other researchers, is currently underway.

References

Andreatta, G., Brunetta, L., Guastalla, G., 2000. From ground holding to free flight: an exact approach, Transportation Science 34(4), 394-401.

Ball, M.O., Hoffman, R., Mukherjee, A., 2010. Ground delay program planning under uncertainty based on the ration-by-distance principle, Transportation Science 44 (1), 1-14.

Ball, M.O., Hoffman, R., Odoni, A., Rifkin, R., 2003. A stochastic integer program with dual network structure and its application to the ground-holding problem, Operations Research 51(1), 167-171.

Ball, M.O., Lulli, G., 2004. Ground delay program: optimizing over the included flight set based on distance, Air Traffic Control Quarterly 12, 1-25.

Ball, M.O., Vossen, T., Hoffman, R., 2001. Analysis of demand uncertainty effects in ground delay programs, 4th US/Europe Air Traffic Management R&D Seminar, Santa Fe, New Mexico.

Bertsimas, D., Patterson, S.S., 2000. The traffic flow management rerouting problem in air traffic control: a dynamic network flow approach, Transportation Science 34(3), 239-255.

Bertsimas, D., Lulli, G., Odoni, A., 2011. An integer optimization approach to large-scale air traffic flow management, Operations Research 59(1), 211-227.

Bolczak, C.N., Hoffman, J.H., Jensen, A.J., Trigeiro, W.W., 1997. National airspace system performance measurement: overview, MITRE Technical Report MTR 97W0000035, Center for Advanced Aviation System Development, McLean, Virginia.

Bradford, S., Knorr, D., Liang, D., 2000. Performance measures for future architecture, Proceedings of 3rd USA/Europe Air Traffic Management R&D Seminar, Napoli, Italy.

Churchill, A.M., 2010. Coordinated and robust aviation network resource allocation, PhD Thesis, University of Maryland, College Park, Maryland.

Clarke, J.B., Solak, S., Chang., Y.H., Ren, L. L., Vela, A.E., 2009. Air traffic flow management in the presence of uncertainty, Proceedings of 8th USA/Europe Air Traffic Management R&D Seminar, Napa, California.

Cook, L.S., Wood, B., 2009. A model for determining ground delay program parameters using a probabilistic forecast of stratus clearing, 8th USA/Europe Air Traffic Management R&D Seminar.

Daganzo, C.F., 1997. Fundamentals of Transportation and Traffic Operations, Elsevier Science Inc., New York.

FAA, 2004. Airport Capacity Benchmarking Report 2004. Washington, D.C..

Gosling, G.D., 1999. Aviation system performance measures, Nextor Working Paper UCB-ITS-WP-99-1.

Grabbe, S., Sridhar, B., Mukherjee, A., 2009. Integrated traffic flow management decision making, AIAA Guidance, Navigation and Control Conference, Chicago.

Hoffman, R., Ball, M.O., 2000. A comparison of formulations for the single-airport ground-holding problem with banking constraints, Operations Research, 48 (4), 578-590.

Kim, A.M., 2011. Collaborative resource allocation strategies for air traffic flow management, PhD Thesis, University of California, Berkeley, California.

Kotnyek, B., Richetta, O., 2006. Equitable models for the stochastic ground holding problem under collaborative decision making, Transportation Science 40 (2), 133-146.

Liu, P.B., Hansen, M., Mukherjee, A., 2008. Scenario-based air traffic flow management: from theory to practice, Transportation Research Part B 42, 685-702

Mukherjee, A., Hansen, M., 2007. A dynamic stochastic model for the single airport ground holding problem, Transportation Science 41(4), 444-456.

Mukherjee, A., Hansen, M., Grabbe, S., 2009. Ground delay program planning under uncertainty in airport capacity, AIAA Guidance, Navigation, and Control Conference Chicago.

Odoni, A.R., 1987. The flow management problem in air traffic control. In: Odoni, A.R., Bianco, L., Szego. G. (Eds.), Flow Control of Congested Networks, Springer-Verlag, Berlin, 269-288.

Richetta, O., 1991. Ground holding strategies for air traffic control under uncertainty, Ph.D. thesis, Massachusetts Institute of Technology, Cambridge, Mass.

Sridhar. B., Grabbe. S.R., Mukherjee, A., 2008. Modeling and optimization in traffic flow management, Proceedings of the IEEE 96(12), 2060-2080.

Vranas, P., Bertsimas, D., 1994a. The multi-airport ground holding problem for air traffic control, Operations Research 42(2), 249-261.

Vranas, P., Bertsimas, D., Odoni, A.R., 1994b. Dynamic ground-holding policies for a network of airports, Transportation Science 28, 275-291

Xiong, J., 2010. Revealed preference of airline’s behavior under air traffic management initiatives, PhD Thesis, University of California, Berkeley, California.

Appendix A: Notations

λ: scheduled arrival rate.

λ_e: exempted demand rate.

$C_H(C_L)$: planned high (low) airport acceptance rate.

T: expected weather clearance time.

T_2: planned delay clearance time.

τ: actual weather clearance time.

τ_2: ideal delay clearance time if perfect information were available at the decision time.

$t_{max}(t_{min})$: upper (lower) bound of the actual clearance time.

$F_{max}(F_{min})$: upper (lower) bound of the flight time.

F_{Scope}: maximum flight time of the GDP affected flights.

$d_{P,max}^{e}$: maximum planned flight delay with exemption rate as λ_e.

S: scheduled cumulative arrival curve.

N: planned cumulative arrival curve.

N_e: planned cumulative arrival curve with exemption rate as λ_e.

Th: realized cumulative arrival curve.

A: ideal cumulative arrival curve if perfect information were available at the decision time.

$D(D')$: available cumulative arrival demand (demand rate) if capacity permits and GDP is cancelled earlier.

> $D_-(D'_-)$: available cumulative arrival demand (demand rate) of Type I flights.
>
> $D_0(D'_0)$: available cumulative arrival demand (demand rate) of Type II flights.
>
> $D_+(D'_+)$: available cumulative arrival demand (demand rate) of Type III flights.

C_-: planned cumulative arrival curve for Type I flights in the case of GDP extension.

D_P: planned delay in the original GDP.

D_R: realized delay at the end of the GDP.

AD: realized airborne delay in the case of GDP extension at the end of the GDP.

GD: realized ground delay at the end of the GDP.

Appendix B: The Cumulative Arrival Curves for Type I flights, Late Clearance

1. For Plot b) in Figure 7:

Condition:

$$T_2 \le T + F_{min} \le \tau_2$$

Formulation:

$$C_- = \begin{cases} C_L \cdot t, t \le T \\ C_L \cdot T + C_H \cdot (t - T), T < t \le T_2 \\ \lambda \cdot t, T_2 < t \le T + F_{min} \\ -\frac{\lambda}{2\Delta F} \cdot [t - (T + F_{max})]^2 + \lambda \cdot (T + F_{min}) + \frac{\lambda}{2} \cdot \Delta F, T + F_{min} \le t \le T + F_{max} \\ \lambda \cdot (T + F_{min}) + \frac{\lambda}{2} \cdot \Delta F, t > T + F_{max} \end{cases}$$

2. For Plot c) in Figure 7:

Condition:

$$T + F_{min} \le T_2 \le T + F_{max}$$

Formulation:

$$C_- = \begin{cases} C_L \cdot t, t \le T \\ C_L \cdot T + C_H \cdot (t - T), T < t \le T + F_{min} \\ -\frac{C_H}{2\Delta F} \cdot [t - (T + F_{max})]^2 + C_H \cdot \left(F_{min} + \frac{\Delta F}{2}\right) + C_L \cdot T, T + F_{min} \le t \le T_2 \\ -\frac{\lambda}{2\Delta F} \cdot [t - (T + F_{max})]^2 + \frac{\lambda - C_H}{2\Delta F} \cdot [T_2 - (T + F_{max})]^2 + C_H \cdot \left(F_{min} + \frac{\Delta F}{2}\right) + C_L \cdot T, T_2 < t \le T + F_{max} \\ \frac{\lambda - C_H}{2\Delta F} \cdot [T_2 - (T + F_{max})]^2 + C_H \cdot \left(F_{min} + \frac{\Delta F}{2}\right) + C_L \cdot T, t > T + F_{max} \end{cases}$$

3. For Plot d) in Figure 7:

Condition:

$$T + F_{max} \leq T_2$$

Formulation:

$$C_{-} = \begin{cases} C_L \cdot t, t \leq T \\ C_L \cdot T + C_H \cdot (t - T), T < t \leq T + F_{min} \\ -\frac{C_H}{2\Delta F} \cdot [t - (T + F_{max})]^2 + C_H \cdot \left(F_{min} + \frac{\Delta F}{2}\right) + C_L \cdot T, T + F_{min} \leq t \leq T + F_{max} \\ C_H \cdot \left(F_{min} + \frac{\Delta F}{2}\right) + C_L \cdot T, t > T + F_{max} \end{cases}$$