COMPARISON OF MAINTENANCE STRATEGIES FOR TRANSPORTATION INFRASTRUCTURE LIFE-CYCLE MANAGEMENT

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ABSTRACT

Threshold-based maintenance is one of the most widely used strategies in transportation life-cycle management. Threshold-based maintenance means that facilities receive maintenance when their performances reach the thresholds. This study adopts hybrid dynamic modeling (HDM) to simulate and optimize threshold-based pavement maintenance. HDMs are capable of incorporating multiple deterioration modes and switching between them automatically. As a result, they are appropriate for the life-cycle optimization using multiple maintenance actions with heterogeneous effects. The results show that hybrid dynamic models simulate the life cycles of transportation infrastructure more realistically and support maintenance decision-making effectively. To evaluate the quality of the optimized threshold-based maintenance, benchmark plans are generated by relaxing the threshold-based strategy. The proposed methodology is demonstrated with a network of 20 highway pavements. Based on the numerical results, some guidelines for selecting maintenance strategies in different scenarios such as budget availability and variable thresholds are proposed.

Keywords: pavement management, hybrid dynamic modeling, maintenance threshold, optimization

INTRODUCTION

A country's pavement infrastructure is very important because it has a direct impact on the economic development and the quality of people's life. In order to improve the performance of pavement infrastructure, the concept of life-cycle management has been introduced to maintenance decision-making processes. However, most of the life-cycle management optimization methods generate “optimal” maintenance plans (not to be confused with optimal *thresholds* targeted in this study), which are detailed future schedules for the types and timings of maintenance actions. As discussed in Task Force on Pavements and the AASHTO (2001), the detailed, pre-determined plans are not well accepted by transportation agencies because they are difficult to interpret and incompatible with the workflow in practice. In practice, transportation agencies inspect conditions of pavements and then select maintenance actions based on maintenance thresholds. One of the drawbacks of the current practice is that pavement maintenance thresholds are often determined based on engineering judgment (Khurshid et al., 2011). Without a systematic approach, the quality of these thresholds is difficult to evaluate and could be far from their optimal values.

METHODOLOGY

To address the problem, a method of finding optimal maintenance thresholds is proposed in Chu and Chen (2012). In that research, hybrid dynamic modeling (HDM) initially proposed in Torrisi and Bemporad (2004) for control engineering is adopted. HDM includes logic conditions and discrete states in traditional dynamic models. The implication for pavement management is that HDM is capable of modeling maintenance thresholds and describing multiple deterioration mechanisms and maintenance effects simultaneously in the same model. Using this approach, the generated maintenance thresholds are expected to be more realistic than using other methodologies or engineering judgment. This study follows hybrid dynamic modeling to describe the life cycles of pavement infrastructure under threshold-based management and focuses on the evaluation of the quality of threshold-based strategies. Fig. 1 shows the concept of HDM, which includes four sub-models: event generator (EG), finite state machine (FSM), mode selector (MS) and switched systems (SS). These four sub-models and other major components of the proposed methodology are briefly explained in this section.

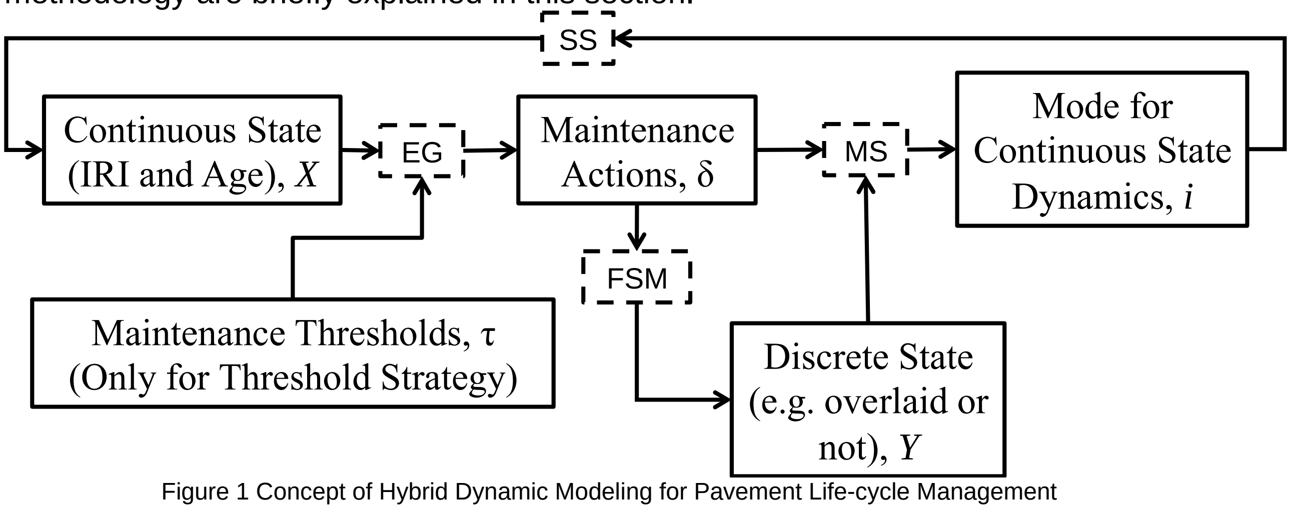

Figure 1 Concept of Hybrid Dynamic Modeling for Pavement Life-cycle Management

Initialization

Pavement conditions are represented by three state variables in this study. The continuous state variables include roughness (International Roughness Index, IRI) and age. The IRI and age of pavement *n* at time *t* are denoted as $X_{IRI}(n,t)$ and $X_{AGE}(n,t)$ respectively in the model formulation. The discrete state variable $Y(n,t)$ is used to indicate if a pavement is overlaid or not. If pavement *n* has been overlaid at time *t*, $Y(n,t)=1$; otherwise, $Y(n,t)=0$. To initialize the pavement life cycles, Eqs. 1-3 set the initial values of the three state variables for each pavement, where $\Xi_{IRI}(n)$, $\Xi_{AGE}(n)$, and $\Pi(n)$ are the initial values for pavement *n*'s IRI, age, and overlaid or not.

$$X_{IRI}(n,1)=\Xi_{IRI}(n), \forall n=1,\cdots,N \tag{1}$$
$$X_{AGE}(n,1)=\Xi_{AGE}(n), \forall n=1,\cdots,N \tag{2}$$
$$Y(n,1)=\Pi(n), \forall n=1,\cdots,N \tag{3}$$

Event Generator

In this study, we consider four maintenance actions: “no action”, “fog seal”, “overlay”, and “reconstruction”. To demonstrate the flexibility of HDM, two sets of threshold-based maintenance rules are listed. *Strategy 1* is described in Eqs. 4-7, where τ_1 is the threshold for overlay, τ_2 is the threshold for fog seal, and τ_3 is the threshold for reconstruction. Eq. 4 states that when the age of pavement *n* reaches τ_3 at time *t*, reconstruction will be adopted in that time period ($\delta_{Reconst}(n,t)=1$). If a reconstruction is not warranted, then the other three actions can be considered. Eq. 5 states that no action is required for pavement *n* at time *t* ($\delta_{NoAction}(n,t)=1$) if the IRI of pavement *n* is lower than both τ_1 and τ_2 . Fog seal should be selected ($\delta_{FogSeal}(n,t)=1$) if the IRI of pavement *n* is lower than τ_1 and higher or equal to τ_2 at time *t* (Eq. 6). Overlay is warranted ($\delta_{Overlay}(n,t)=1$) if the IRI of pavement *n* reaches both τ_1 and τ_2 at time *t*, which is described in Eq. 7.

$$\delta_{Reconst}(n,t)=\begin{cases}1, & \text{if } X_{AGE}(n,t)\geq\tau_3 \\ 0, & \text{otherwise}\end{cases}, \quad \forall n=1,\cdots,N, \forall t=1,\cdots,T \tag{4}$$

$$\delta_{NoAction}(n,t) = \begin{cases} 1, & \text{if } X_{IRI}(n,t) < \tau_1 \text{ and } X_{IRI}(n,t) < \tau_2 \text{ and } \delta_{Reconst}(n,t) = 0 \\ 0, & \text{otherwise} \end{cases}, \quad (5)$$

$$\forall n = 1,\cdots,N, \forall t = 1,\cdots,T$$

$$\delta_{FogSeal}(n,t) = \begin{cases} 1, & \text{if } X_{IRI}(n,t) < \tau_1 \text{ and } X_{IRI}(n,t) \geq \tau_2 \text{ and } \delta_{Reconst}(n,t) = 0 \\ 0, & \text{otherwise} \end{cases}, \quad (6)$$

$$\forall n = 1,\cdots,N, \forall t = 1,\cdots,T$$

$$\delta_{Overlay}(n,t) = \begin{cases} 1, & \text{if } X_{IRI}(n,t) \geq \tau_1 \text{ and } X_{IRI}(n,t) \geq \tau_2 \text{ and } \delta_{Reconst}(n,t) = 0 \\ 0, & \text{otherwise} \end{cases}, \quad (7)$$

$$\forall n = 1,\cdots,N, \forall t = 1,\cdots,T$$

We note that a pavement is reconstructed even when its IRI is still low under Strategy 1, which could be a waste of resource. To address this issue, another IRI threshold can be introduced for the selection rules for reconstruction. Thus, the second set of maintenance rules (*Strategy 2*) that adopt 4 thresholds are described as follows. To avoid the construction of pavements that IRI values are still relatively low, reconstruction is only applied when the age of a pavement reaches τ_3 and its IRI reaches threshold τ_4. In this strategy, the rule for applying reconstruction follows Eq. 8 and the rules for other maintenance actions still follow Eqs. 5-7.

$$\delta_{Reconst}(n,t) = \begin{cases} 1, & \text{if } X_{AGE}(n,t) \geq \tau_3 \text{ and } X_{IRI}(n,t) \geq \tau_4 \\ 0, & \text{otherwise} \end{cases}, \quad (8)$$

$$\forall n = 1,\cdots,N, \forall t = 1,\cdots,T$$

Based on the above rules and the physical properties of pavements, the thresholds must also follow Eqs. 9-11. Eq. 9 states that fog seal cannot be applied before overlay. Because a perfectly smooth pavement (zero IRI) is only theoretically possible, the lower bound for IRI thresholds is set to 2.0 (Eq. 10). Eq. 11 is introduced because pavement ages cannot be negative.

$$\tau_1 \geq \tau_2 \quad (9)$$

$$\tau_1, \tau_2, \tau_4 \geq 2.0 \quad (10)$$

$$\tau_3 \geq 0 \quad (11)$$

In order to generate benchmark plans to evaluate the threshold-based strategy, *Threshold-free Strategy* is also considered. In this strategy, the maintenance actions,

$\delta_a(n,t), \forall a,n,t$, are not regulated by the thresholds and can be determined freely. As a result, the "optimal" maintenance plans given the pavement conditions and budgets can be obtained. However, we mentioned earlier that the "optimal" plans are not intuitive to the engineers and thus not well accepted. In this study, the "optimal" plans are only used as a benchmark for the quality of threshold-based maintenance. Under this strategy, Eqs. 4-11 are discarded and Eq. 12 that ensures exactly one action can be selected for a pavement in the same time period is added:

$$\sum_a \delta_a(n,t), \forall n = 1,\cdots,N, \forall t = 1,\cdots,T \tag{12}$$

Finite State Machine

As defined earlier, $Y(n,t) = 1$ indicates pavement n has been overlaid at time t; otherwise, $Y(n,t) = 0$. Chu and Durango-Cohen (2008), Livneh (1998), and Zhang et al. (2010) observed that pavement overlay replaces the weaker pavement surface and/or increases pavement thickness, which causes the deterioration rate to decrease. To capture this phenomenon, Eq. 13 is introduced to govern the transition of the discrete state variable. The equation states the two conditions that a pavement has been overlaid at time *t+1*. First, the pavement has not been overlaid and the action chosen is overlay at time t. Second, the pavement has been overlaid and the selected action is not reconstruction at time *t*.

$$Y(n,t+1) = \begin{cases} 1, & \text{if } (Y(n,t)=0 \text{ and } \delta_{Overlay}(n,t)=1) \text{ or } (Y(n,t)=1 \text{ and } \delta_{Reconst}(n,t)\neq 1), \\ 0, & \text{otherwise} \end{cases} \tag{13}$$
$$\forall n = 1,\cdots,N, \forall t = 1,\cdots,T$$

Mode Selector

This sub-model determines the appropriate deterioration mode for a pavement according to the selected action and whether the pavement has been overlaid. Because the deterioration rates are different for a pavement with or without overlay, each of no action and fog seal actions has two related modes. If no action is chosen and the pavement has not been overlaid, mode 1 should be selected (Eq. 14); if it has been overlaid, mode 5 is selected (Eq. 18). If fog seal is chosen and the pavement has not been overlaid, mode 2 is chosen (Eq. 15); if it has been overlaid, mode 6 should be selected (Eq. 19). Because the application of overlay causes significant change in IRI and thus the slight difference in IRI progression caused by overlay is negligible, mode 3 is always adopted for overlay regardless of its discrete state (Eq. 16). Similarly, mode 4 will be selected to represent the deterioration mechanism of reconstruction with or without overlaid (Eq. 17).

$$i^1(n,t)=\begin{cases}1, & \text{if } Y(n,t)=0 \text{ and } \delta_{NoAction}(n,t)=1\\ 0, & \text{otherwise}\end{cases}, \forall n=1,\cdots,N, \forall t=1,\cdots,T \quad (14)$$

$$i^2(n,t)=\begin{cases}1, & \text{if } Y(n,t)=0 \text{ and } \delta_{FogSeal}(n,t)=1\\ 0, & \text{otherwise}\end{cases}, \forall n=1,\cdots,N, \forall t=1,\cdots,T \quad (15)$$

$$i^3(n,t)=\begin{cases}1, & \text{if } \delta_{Overlay}(n,t)=1\\ 0, & \text{otherwise}\end{cases}, \forall n=1,\cdots,N, \forall t=1,\cdots,T \quad (16)$$

$$i^4(n,t)=\begin{cases}1, & \text{if } \delta_{Reconst}(n,t)=1\\ 0, & \text{otherwise}\end{cases}, \forall n=1,\cdots,N, \forall t=1,\cdots,T \quad (17)$$

$$i^5(n,t)=\begin{cases}1, & \text{if } Y(n,t)=1 \text{ and } \delta_{NoAction}(n,t)=1\\ 0, & \text{otherwise}\end{cases}, \forall n=1,\cdots,N, \forall t=1,\cdots,T \quad (18)$$

$$i^6(n,t)=\begin{cases}1, & \text{if } Y(n,t)=1 \text{ and } \delta_{FogSeal}(n,t)=1\\ 0, & \text{otherwise}\end{cases}, \forall n=1,\cdots,N, \forall t=1,\cdots,T \quad (19)$$

Switched System.

The progression of pavement age and IRI is governed by the following two equations. Eq. 20 defines the deterioration mode for pavement age. The equation indicates that the age of pavement increases by 1 in the next time period unless the action selected currently is reconstruction. If reconstruction is selected now, the age is reset to 0 in the next time period. Eq. 21 describes the deterioration modes for IRI, where $V(n,t)$ is the annual traffic per lane. The relationships in the equation are extracted from N.D. Lea International Ltd. (1995) and Paterson and Attoh-Okine (1992). Note that some of the parameter values in modes 1, 2, 5, and 6 in Eq. 21 are unavailable to this study and must be assumed according to the weather conditions in Taiwan; see more details in Paterson and Attoh-Okine (1992) and Chu and Chen (2012).

$$X_{AGE}(n,t+1)=\begin{cases}X_{AGE}(n,t)+1, & \text{if } i^4(n,t)\neq 1\\ 0, & \text{if } i^4(n,t)=1\end{cases}, \forall n=1,\cdots,N, \forall t=1,\cdots,T \quad (20)$$

$$X_{IRI}(n,t+1)=\begin{cases} 1.023X_{IRI}(n,t)+\left(1.023^{X_{AGE}(n,t)+1}\right)0.026V(n,t), & \text{if } i^1(n,t)=1 \\ 1.015X_{IRI}(n,t)+\left(1.015^{X_{AGE}(n,t)+1}\right)0.026V(n,t), & \text{if } i^2(n,t)=1 \\ 1.400+0.300X_{IRI}(n,t), & \text{if } i^3(n,t)=1 \\ 2.000 & \text{if } i^4(n,t)=1 \\ 1.023X_{IRI}(n,t)+\left(1.023^{X_{AGE}(n,t)+1}\right)0.035V(n,t), & \text{if } i^5(n,t)=1 \\ 1.015X_{IRI}(n,t)+\left(1.015^{X_{AGE}(n,t)+1}\right)0.035V(n,t), & \text{if } i^6(n,t)=1 \end{cases}, \tag{21}$$

$$\forall n=1,\cdots,N,\forall t=1,\cdots,T$$

Objective function and Budget Constraint

Constrained by the hybrid dynamic modeling listed above, the threshold-based life-cycle management problem is essentially the search for the thresholds that minimize the traffic weighted IRI (Eq. 22) under the constraint of the annual budget (Eq. 23).

$$\min\sum_{n=1}^{N}\sum_{t=2}^{T+1}V(n,t)X_{IRI}\left(n,t\right) \tag{22}$$

$$\sum_{a\in A}\sum_{n=1}^{N}C_a(n)\delta_a(n,t)\le B(t),\forall t=1,\cdots,T \tag{23}$$

Reformulation as a Mixed-integer Programming Problem.

Most of the above equations are logical statements. These equations are straightforward to describe the life cycles of pavements under the threshold-based maintenance strategies; however, they are difficult to optimize directly using existing optimization tools. To optimize the thresholds efficiently, the model can be converted into a mixed-integer programming problem, which can be solved with commercial tools efficiently (Chu and Chen, 2012). It should be noted that Eq. 21 is nonlinear due to the exponential functions of the pavement age. To avoid the optimization of a nonlinear mixed-linear programming problem, the function is approximated with a piece-wise linear function.

NUMERICAL EXAMPLE

The environment for the numerical example is a desktop computer with Intel Core i7-2600 CPU (3.40GHz) and 4G RAM. The optimization software is AMPL with Gurobi 4.6. The example we used in this study is the 20 highways sections of Dasi Township, Taoyuan County in Taiwan. The area is selected because all roads in the township are managed by the township government, which is consistent with the assumption in this study. Based on the history of budget availability of Dasi Township, the annual budgets of 7, 9, 11, 13, and 15 million NT$ are considered. The planning horizon is arbitrarily set to 10 years. The attributes of the highways such as length, number of lanes, and traffic volume, are extracted from Chang (2001). The maintenance costs for fog seal, overlay, and reconstruction are set to NT$18, 221, and 1,566 per square meter, respectively (THI Consultants, Inc., 2006).

Results for Strategy 1

The results for adopting Strategy 1 are summarized in Table 1. Note that the objective value and threshold for overlay (τ_1) decrease as the budget increases under both strategies, which are expected results. The trend for the thresholds for fog seal (τ_2) has no clear trend; the reason might be that the impact of fog seal on the objective value is negligible compared with overlay and reconstruction. The age thresholds for reconstruction (τ_3) are almost identical, which are between 36-37 years. Finally, we note that the computational time for solving a problem increases as the budget increases. The possible reason is that the feasible ranges for maintenance thresholds are larger when a bigger budget is available, which requires more time to solve.

Table 1 Result Summary for Strategy 1

Budget (million NT$)	τ_1	τ_2	τ_3	Objective Value	CPU Time (s)
7	3.800	2.000	36	8490510	19.1
9	3.237	2.100	36	7914660	30.0
11	2.900	2.856	36	7621580	52.9
13	2.701	2.100	36	7220320	55.8
15	2.500	2.461	37	6884850	153.8

Comparison between Threshold-free Strategy and Threshold-based Strategy

In order to understand the quality of threshold-based strategies, we use Threshold-free Strategy to generate the best possible maintenance plan as the benchmark. Under Threshold-free Strategy, the maintenance actions for each pavement at each time period can be decided freely without the constraints of maintenance thresholds (i.e., event generator). The results of Threshold-free Strategy are summarized in Table 2. The table shows that the objective value decreases as the budget decreases, which is reasonable. Contrary to the threshold-based strategy, the computational time using Threshold-free Strategy decreases as the budget increases. The reason might be that when the budget is generous, the most effective maintenance actions such as overlay and reconstruction can be applied as desired, which greatly simplifies the optimization procedure and reduces the solution time. The comparisons between the two strategies are listed in Table 3. As expected, Threshold-free Strategy generates lower objective values than Threshold-based strategy. However, we note that gap between Threshold-free Strategy and the threshold-based strategy reduces from 28% to 10% as the budget increases from 7 to 15 million NT$. This result indicates the threshold-based maintenance is more effective when the budget is larger.

Table 2 Result Summary for Threshold-free Strategy

Budget (million NT$)	Objective Value	CPU Time (s)
7	6643600	335.0
9	6453900	79.4
11	6366150	15.2
13	6289700	6.4
15	6261650	6.4

Table 3 Comparison of Objective Values

Budget (million NT$)	Threshold-free	Strategy 1	Gap between Strategy 1 and Threshold-free
7	6643600	8490510	28%
9	6453900	7914660	23%
11	6366150	7621580	20%
13	6289700	7220320	15%
15	6261650	6884850	10%

Variable Thresholds

In this analysis, we relax the assumption that the thresholds are identical in all years and consider two additional strategies. First, we allow the thresholds to change once after the first year. Second, we allow the thresholds to change twice after the first year and the second year. The results are summarized in Table 4. It is clear that adjusting the thresholds provides more flexibility for the maintenance planning and thus can further improve the objective values. Allowing one change reduces the objective values by 6-13% and allowing two changes reduces the objective values by 8-16%. We also observe that more computation time is required when more sets of thresholds are considered, which is reasonable.

CONCLUSIONS

A road network of 20 pavements is used as the numerical example to demonstrate the proposed methodology. The results show that hybrid dynamic models simulate the life cycles of transportation infrastructure more realistically and support maintenance decision-making effectively. To evaluate the quality of the optimized threshold-based maintenance, benchmark plans are generated and compared with the threshold-based plans. We find that the threshold-based strategy is more effective when the budget is larger. When the maintenance budget is very limited, the quality of threshold-maintenance is not ideal and more flexible rules should be considered. We also find that adjusting the thresholds provides more flexibility and improves the objective values, and variable thresholds should be considered if possible. The direction for the future research is the solution method. The numerical example contains only 20 facilities because the mixed-integer programming formulation requires significant time to solve for large-scale problems. More efficient solution algorithm should be developed in the future for larger problems.

Table 4 Comparison for Number of Threshold Sets

Budget (million NT$)		1 Set			2 Sets			3 Sets		
		τ_1	τ_2	τ_3	τ_1	τ_2	τ_3	τ_1	τ_2	τ_3
7	Yr. 1	3.800	2.000	36	3.800	2.700	27	3.800	2.700	27
	Yr. 2	3.800	2.000	36	2.767	2.000	36	2.946	2.150	28
	Yr. 3-10	3.800	2.000	36	2.767	2.000	36	2.615	2.184	36
	CPU (s)		19.1			47.5			88.9	
	Obj. Val.		8490510			7402540			7164050	
9	Yr. 1	3.237	2.100	36	3.600	3.200	27	3.600	2.200	27
	Yr. 2	3.237	2.100	36	2.570	2.570	36	2.570	2.570	28
	Yr. 3-10	3.237	2.100	36	2.570	2.570	36	2.380	2.341	36
	CPU (s)		30.0			54.3			136.8	
	Obj. Val.		7914660			7096980			6802070	
11	Yr. 1	2.900	2.856	36	3.000	2.100	27	3.200	2.100	27
	Yr. 2	2.900	2.856	36	2.544	2.165	36	2.540	2.133	28
	Yr. 3-10	2.900	2.856	36	2.544	2.165	36	2.282	2.000	36
	CPU (s)		52.9			90.0			214.0	
	Obj. Val.		7621580			6981080			6593910	
13	Yr. 1	2.701	2.100	36	2.900	2.100	27	2.900	2.100	27
	Yr. 2	2.701	2.100	36	2.438	2.360	36	2.438	2.133	28
	Yr. 3-10	2.701	2.100	36	2.438	2.360	36	2.180	2.055	36
	CPU (s)		55.8			130.3			36.3	
	Obj. Val.		7220320			6803230			6402150	
15	Yr. 1	2.500	2.461	37	2.700	2.200	27	2.700	2.200	27
	Yr. 2	2.500	2.461	37	2.260	2.219	37	2.300	2.150	29
	Yr. 3-10	2.500	2.461	37	2.260	2.219	37	2.162	2.049	36
	CPU (s)		153.8			845.1			407.6	
	Obj. Val.		6884850			6496690			6325010	

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