OPTIMAL CONDITION SAMPLING OF INFRASTRUCTURE NETWORKS: OVERVIEW AND SENSITIVITY ANALYSIS

Rabi G. Mishalani (corresponding author), The Ohio State University, Dept. of Civil, Environmental & Geodetic Engineering, 2070 Neil Ave., Room 470, Columbus, OH 43210, USA, mishalani@osu.edu, +614-292-5949

Prem K. Goel, The Ohio State University, Dept. of Statistics, Columbus, OH 43210, USA

Cheng Chen, The Ohio State University, Dept. of Civil, Environmental & Geodetic Engineering, Columbus, OH 43210, USA

Dunke Zhou, Amazon.com Inc, Seattle, WA, 98121, USA

ABSTRACT

In response to the developments in inspection technologies, infrastructure decision-making methods consider the optimum combination of inspection decisions on the one hand and maintenance and rehabilitation decisions on the other based on an economic evaluation that captures the long-term costs and benefits. Sample size has been included in inspection, maintenance, and rehabilitation (IM&R) decision-making as a decision variable when considering a single facility. While, the question of dealing with a network of facilities in making IM&R decisions has been addressed in the literature, this treatment does not consider condition sampling whereby each facility could require a different set of sample sizes over time. This paper presents an overview of a methodology developed to address the network level problem. The uncertainty due to condition sampling is captured and the related decision variables are incorporated in the IM&R decision making process. A sensitivity analysis based on a realistic literature- and practice-derived hypothetical network of facilities is conducted to explore the effects various factors have on the optimal solution.

Keywords: Infrastructure management, condition inspection, maintenance, decision-making under uncertainty.

1. INTRODUCTION

Transportation infrastructure systems consist of spatially extensive and long-lived sets of facilities. Over the past two decades, several non-destructive inspection technologies have been developed and applied in collecting raw condition data and processing them to produce useful condition input to infrastructure inspection, maintenance, and rehabilitation (IM&R) decision-making aimed at minimizing total expected life-cycle cost. Inspection deals with the gathering of data on the extent of facility damage. The data may be collected by visual inspection, through manual measurements, or by automated sensors. An average of collected damage measurements over a facility (defined as a homogeneous section) is an estimate of the current condition of that facility and, in turn, is one primary input to maintenance and rehabilitation (M&R) decision-making.

The developments in nondestructive inspection technologies make it possible to estimate facilities' conditions using large quantities of data. The quality of measurements, the sampling frequency and sample size, and the nature of correlation among condition variables at different locations determine the accuracy of condition estimates. Naturally, more accurate estimates have the potential to lead to more effective maintenance and rehabilitation decisions. Consequently, the expected combined user costs and maintenance and rehabilitation costs are reduced over the planning horizon. However, more accurate information requires more resources such as increased inspection frequency, advanced inspection sensor technologies, larger sample sizes, or possibly less correlated observations, as well as data processing methods that appropriately combine all this information.

In response to the developments in inspection technologies, decision-making methods consider the optimum combination of inspection decisions on the one hand and M&R decisions on the other based on an economic evaluation that captures the long-term costs and benefits. Madanat (1993), Madanat and Ben-Akiva (1994), and Ellis *et al*. (1995) extended the Markov Decision Process (MDP) based infrastructure management decision-making framework (Golabi *et al.* 1982, Garnahan *et al.* 1987), which captures forecasting uncertainty, to the Latent Markov Decision Process (LMDP) framework by incorporating measurement errors associated with condition inspection. In addition, inspection technology and timing were introduced as decision variables. The LMDP framework has since been extended to include condition sample size as a decision variable in IM&R decision-making (Mishalani and Gong 2009). However, several of the aforementioned studies including this latest extension only considered decisions for a single facility.

The question of dealing with a network of facilities in making M&R decisions has been addressed in the literature through a variety of formulations (*e.g.*, Golabi *et al.*1982, Golabi and Shepard 1997, Murakami and Turnquist 1995, Smilowitz and Madanat 2000, Durango-Cohen and Sarutipand 2007, Kuhn 2010), however, these studies, while addressing several important issues, do not consider condition sampling whereby each facility could require different and time-varying sample sizes. Doing so optimally is valuable given the network nature of facilities that most infrastructure agencies are responsible for, the increasing number of inspection technology choices with possible varying degrees of accuracy and cost, and budget constraints agencies have to work within. In light of the importance of considering condition sampling, it is imperative to recognize the correlation that is expected among observations of condition of the same facility.

In this paper, a methodology that captures the uncertainty due to condition sampling and includes sampling related decision variables in the IM&R decision-making process at the network level is presented. In addition, a sensitivity analysis based on a realistic literature- and practice-derived hypothetical network of facilities is conducted and the resulting derived insights are discussed.

2. METHODOLOGY

Some network-level IM&R decision-making methods in the literature adopt randomized policies (Golabi *et al.*1982, Golabi and Shepard 1997, Murakami and Turnquist 1995, Smilowitz and Madanat 2000, Harper *et al.* 1990, Gopal and Majidzadeh 1991). Smilowitz and Madanat (2000) proposed a linear programming formulation for solving the infrastructure IM&R optimization problem at the network level considering inspection technology and timing as decision variables. Given the advances that study achieved in capturing these inspection decisions, it provides a natural basis for the methodology presented in this paper.

In the latent condition framework (Madanat 1993, Madanat and Ben-Akiva 1994, Ellis *et al.* 1995) Smilowitz and Madanat (2000) built upon, assessed facility condition is assumed to fall into one of a finite number of discrete condition states. Considering that inspection is not perfect, it is assumed that the measurement of condition states must not produce the true condition state. In order to infer the true condition states based on measurements, the nature of measurement error has to be considered. To do so, the concept of the information vector is introduced. This vector is a probability mass function on all possible condition states conditional on prior information. This prior information consists of the initial condition state before any decisions are made (*i.e.*, at time 0), the M&R actions applied up to the current point in time, and all the condition measurements taken including those resulting from the most recent inspection.

The formulation developed by Smilowitz and Madanat (2000) generalizes the facility level framework developed by Madanat and Ben-Akiva (1994) to a network level one by introducing randomized decisions and solving a linear programming problem. The assessed facility condition is assumed to fall into one of a finite number of discrete condition states. Given the current condition measurement along with historical information including previous measurements and IM&R actions, the posterior probability mass function of the true condition state of each facility (*i.e.*, the information vector) is determined. In addition, transition probabilities are adopted to specify how facility condition evolves during the next time period, given the current condition state and the M&R action applied. A transition probability represents the likelihood of a facility to transition to a certain condition state in one period given the condition state it is currently in. Therefore, transition probabilities are organized into a matrix representing all combinations of transitions from state to state. While in the numerical application presented in Smilowitz and Madanat (2000) the transition probabilities are assumed invariant with respect to the age of facilities (*i.e.*, the transition probabilities are stationary), the formulation is not restricted in this regard and could capture transition probabilities that depend on age. The formulation includes many realistic elements. However, an important decision variable and an associated modeling element are not captured in this formulation, namely, the sample size and spatial correlation among measurements of condition taken at different locations along the same facility.

The role of sampling is to increase the accuracy of the information regarding facility condition state. On the one hand, more samples result in higher accuracy. On the other hand, more samples will introduce more cost. Thus, sample sizes for each facility over time are important decision variables. Effectively, in the formulation developed by Smilowitz and Madanat (2000), for a facility either one sample is taken or not, which is quite limiting. Therefore, the extended formulation developed in this paper includes sample sizes as decision variables. In addition, non-stationary transition probabilities are assumed to depend on the age of a facility, where age is defined as the number of years since the most recent rehabilitation action was applied. That is, two facilities in the same condition state to which the same M&R action is applied will have different transition probabilities if their age is different. Specifically, the facility with lower age is assumed to have a smaller probability of deteriorating to a poorer state during the next period. Moreover, in the sensitivity analysis presented in section 3 of this paper two condition inspection technologies are considered rendering the set of inspection decisions more flexible. Once multiple samples are taken from the same facility, the spatial correlation among these condition observations must be considered in quantifying the combined measurement and sampling uncertainty. Therefore, a spatial correction function, representing the correlation between two observations as a function of the distance between them (Mishalani and Koutsopoulos 2002), is adopted in determining this uncertainty where the function is positive and monotonically decreasing.

The variance of the assessed condition is a critical element of the developed formulation and constitutes a major departure from the Smilowitz and Madanat (2000) formulation. This variance is determined as a function of the measurement technology, sample size, and the characteristics of the facility in terms of its intrinsic variability in condition and the spatial correlation. The determination of this variance is based on the formulation developed by Mishalani and Gong (2009) for a single facility. Another important departure from the formulation discussed above is the introduction of facility length, h, in representing the network. This variable influences the value of the determined variance of the assessed condition depending on the sample size, intrinsic variability, and spatial correlation. Finally, in addition to the user and IM&R costs discussed above, the hypothetical terminal cost incurred at the end of the time horizon represents the cost of bringing the facility back to the best condition state for the purpose of equalizing the service life from that point onward.

In light of the elements described above, the problem of determining the best decision can be formulated as a linear program that extends that of Smilowitz and Madanat (2000). More specifically, the components of the formation are the following.

- *Decision variables*: $W_t(\mathrm{I},h,y,a,r,n)$ denotes the number of facilities at time t whose information vector is I, their length is h, and are of age y on which M&R action a will be performed, and inspection technology r will be used with n samples. As in the case of the formulation developed by Smilowitz and Madanat (2000), the nature of the decision variables when solving the network problem is that of a randomized policy. That is, facilities associated with the same information vector could receive different actions where the numbers of facilities to receive each set of actions are optimally determined. However, the optimal set of actions specific facilities associated with the same information vector are to receive is not part of the solution outcome. Such decisions would be made based on practical field considerations once the optimal policy is determined.

- *Objective function*: For each feasible realization of the decision variables, the expected total discounted cost includes the user, IM&R, and terminal costs. Of course, the decision variables include the sample size n and the measurement and sampling uncertainty takes into account the sample size, facility length, intrinsic variability, and spatial correlation. The objective function is a linear function of the decision variables.

- *Constraints*: The constraints are similar to those of the formulation developed by Smilowitz and Madanat (2000) and are as follows. (i) Non-negativity constraints guarantee that each decision variable is non-negative. (ii) Conservation constraints ensure the conservation of facilities over time. That is, the information vectors must transition from one period to the next in a manner consistent with the condition state transition probabilities. (iii) The initial distribution of facilities as defined by a set of information vectors is assumed known and takes the form of an initial state constraint. (iv) Condition state constraints require that the proportion or number of facilities in the condition states considered to be poor is bounded by a maximum value each year. (v) Budget constraints require that the IM&R cost is bounded by a maximum and possibly a minimum value each year.

Based on the above, the problem is reduced to finding the values of the decision variables that satisfy all the constraints and achieve the minimal objective function value. Since the objective function and all the constraints are linear with respect to the decision variables, the problem can be solved using linear programming.

3. SENSITIVITY ANALYSIS

A sensitivity analysis is conducted to explore the effect of various factors on the optimal solution. First, a base scenario, represented by one set of parameter value levels, is developed. More scenarios are then constructed by introducing additional parameter value levels that capture realistic ranges the various parameters could take. The parameters of the base scenario are determined by drawing upon various cases reported in the literature to arrive at a realistic representation. The specification is for the most part based on a realistic example developed by Gong (2006). The resulting parameter values are shown in Table 1.

The factors considered in the sensitivity analysis and their corresponding values at different levels are shown in Table 2. The levels and parameter values corresponding to the base scenario presented above are shown in bold. In addition, budget constrain levels are introduced. Note that routine and rehabilitation maintenance cost, the intrinsic variance, and the variance of the inspection technologies are assumed fixed across all the scenarios.

Table 1 – Base scenario parameter values (all costs in $\$/m^2$)

Condition State	4	3	2	1
Routine maintenance cost	0.34	1.63	4.4	14.79
User cost	18.47	56.735	75.155	113.43
Terminal cost	2.675	22.485	55.695	64.87
Rehabilitation cost	64.87			
Additional user cost due to M&R	1.45 for routine maintenance and 10.13 for rehabilitation			
Additional user cost due to insp.	0.09 for insp. tech. 1 and 0.0015 for inspection tech. 2			
Fixed inspection cost	0.0119 for insp. tech. 1 and 0.0093 for inspection tech. 2			
Unit inspection cost	0.00023 for insp. tech. 1 and 0.000085 for insp. tech. 2			
Intrinsic variance	Function of the true condition state			
Variance of inspection technology	Technology 1: 7.99; technology 2: 17.95			

In total, 1,458 scenarios are considered. The average expected total cost at optimality is calculated for each factor level, one factor at a time. The results are shown in Figure 1. User cost has the largest effect on the expected total cost. The annual budget constraint is the second most influential factor. The terminal cost and spatial correlation have similar effects on the expected total cost, in terms of magnitude. The effects of fixed inspection cost and unit inspection cost appear to be negligible.

Table 2 – Factor levels and values (base scenario values are indicated in bold)

Factor	Level	Pavement Condition State			
		4	3	2	1
Correlation function	**0**	**independent observation**			
	1	$\rho(s)=\exp(-0.054271\times s), \forall s>0$			
	2	$\rho(s)=\exp(-0.026348\times s), \forall s>0$			
Fixed inspection cost ($\$/m^2$)	0	0.0045 for 1 and 0.0042for 2			
	1	**0.0119 for 1 and 0.0093 for 2**			
	2	0. 0291 for 1 and 0.0152 for 2			
Unit cost ratio: tech 2 to tech 1	0	5.75			
	1	**2.71**			
	2	1.62			
User cost ($\$/m^2$)	0	13.67	42	55.64	83.97
	1	**18.47**	**56.74**	**75.16**	**113.4**
	2	34.72	106.7	81.29	213.2
Terminal cost ($\$/m^2$)	**0**	**2.675**	**22.49**	**48.14**	**64.87**
	1	0	0	0	0
Annual Budget Constraint ($\$/m^2$)	**0**	**100**			
	1	20			
	2	15			
	3	10			
	4	8			
	5	6			
	6	4			
	7	2			
	8	1			

Based on these results, not surprisingly, user cost is a key driver of IM&R decisions. Higher user costs result in more M&R actions to be taken to avoid increased used costs at poorer condition states. Therefore, it is critical for agencies to assess and represent user costs accurately to avoid either over-spending on M&R actions (in the case where user costs are overestimated) or under-spending on M&R actions (in the case where use costs are underestimated) resulting in large user costs. Again not surprisingly, the annual budget constraint is another key aspect of the problem. The lower the constraint, the more restricted the agency is in applying expensive M&R actions and the higher the user costs are due to the resulting poorer condition states. In light of the quantification of the effect of the budget constraint on the expected total cost at optimality, it is worthwhile for agencies to determine such quantifications and present them to budget developers as an important input to setting budgets with a clear understanding of their implications on user costs.

It is valuable to note that the spatial correlation among adjacent observations along a facility, which has been shown to be present (Mishalani and Koutsopoulos 2002), does have an impact on the optimal solution and, thus, it is important for agencies to have a good understanding of the nature of this correlation. More specifically, stronger spatial correlation among adjacent observations along a facility increases the average expected total cost at optimality as a result of the reduction in the information gained from observations reflecting higher positive correlation. As for terminal cost, the magnitude of the impact of ignoring such adjustments is not trivial and, therefore, it is crucial from a practical perspective for agencies not to overlook the equalization of service life concept, which has been theoretically established. While the effects of inspection costs are negligible when it comes to the actual expected total cost at optimality, as is expected given the much larger magnitudes associated with M&R and user costs, it is important not to misinterpret these result as an indication of the lack of importance of considering inspection and sampling as part of the decision-making framework. The uncertainty associated with inspection and sampling is fully captured in the sensitivity analysis along the lines discussed in section 2 and, if the evaluation results of the single facility case (Mishalani and Gong 2008) extend to the network level case, this uncertainty has important implications on the optimal solution.

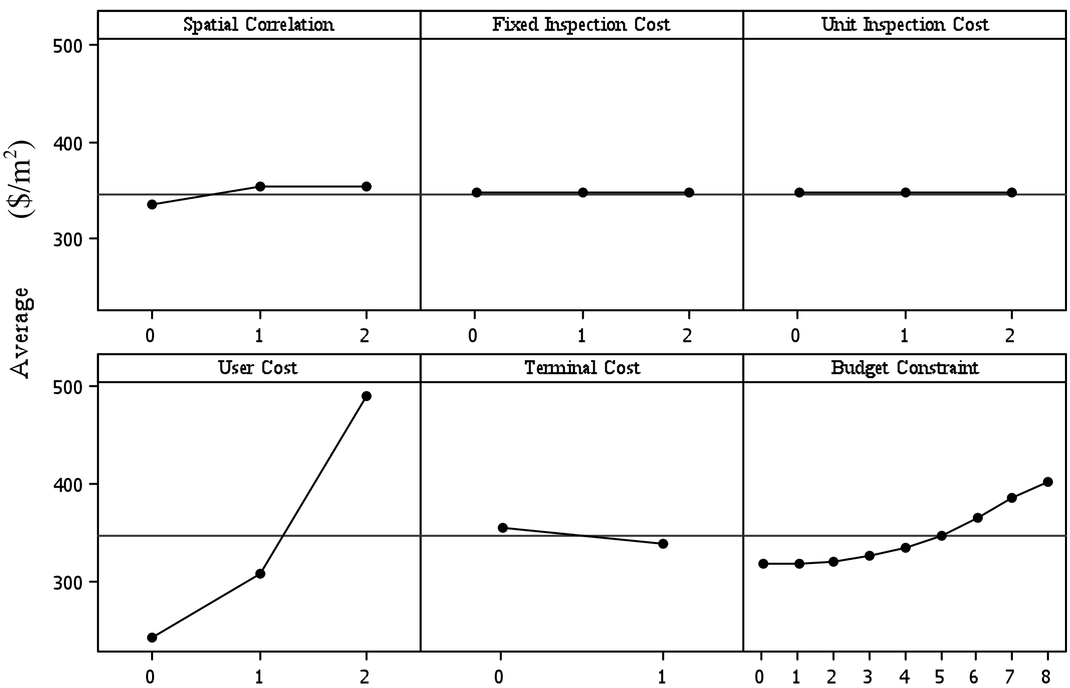

Figure 1 – Average expected total cost at optimality versus factor levels

4. SUMMARY AND FUTURE RESEARCH

This paper presents a methodology developed to address the IM&R decision-making at the network level whereby the uncertainty due to condition sampling is captured and sample sizes over time and across the facilities forming the network are included as decision variables in the optimization. In addition, a sensitivity analysis is conducted to explore the effect of various factors on the optimal solution. The sensitivity analysis revealed that the user cost, annual budget constraint, terminal cost, and the spatial correlation function have appreciable impact on the optimal solution. Among these four factors, the impacts of the user cost and annual budget constraint are the most marked.

In light of the identified single factor effects, it is important to investigate joint factor effects as well. In addition, it is important to conduct an evaluation to quantify the value of capturing sampling uncertainty and including sampling as a decision variable for the network case. As mentioned above, such an evaluation has been conducted for the facility level problem in the absence of a budget constraint (Mishalani and Gong 2008), however, it remains to be undertaken as part of future research for the network level problem in the presence of a budget constraint.

Regarding methodology related future research, it is important to capture facility interactions. Durango-Cohen and Sarutipand (2007) captured important interactions, however, condition sampling was not considered. Developing a decision-making framework that simultaneously takes into account facility interactions and includes condition sample sizes across facilities and over time as decision variables would be worthwhile.

ACKNOWLEDGEMENTS

This research was supported by the Region V University Transportation Center funded by the Research and Innovative Technologies Administration (RITA), U.S. Department of Transportation (US DOT), with additional financial support provided by The Ohio State University (OSU). The views, opinions, findings, and conclusions reflected in this paper are the responsibility of the authors only and do not represent the official policy or position of US DOT or RITA.

REFERENCES

Carnahan, J.V., E.J. Davis, M.Y. Shahin, P.L. Keane and M.I. Wu (1987). Optimal maintenance decisions for pavement management. *Journal of Transportation Engineering*, Vol. 113(5), pp. 554-572.

Durango-Cohen, P.L. and P. Sarutipand (2007). Capturing Interdependencies and Heterogeneity in the Management of Multifacility Transportation Infrastructure Systems. Journal of Infrastructure Systems, Vol. 13(2), pp. 115-123.

Harper, W., J. Lam, A. Al-Salloum, S. Al-Sayyari, S. Al-Theneyan, G. Ilves and K. Majidzadeh (1990). Stochastic Optimization Subsystem of a Network-Level Bridge Kuhn, Management System. Transportation Research Record, No. 1268, pp. 68-74.

Kuhn, K.D. (2010). Network Level Infrastructure Management Using Approximate Dynamic Programming. Journal of Infrastructure Systems, Vol. 16(2), pp. 103-111.

Gopal, S. and K. Majidzadeh (1991). Application of Markov Decision Process to Level-of-Service-based Maintenance Systems. Transportation Research Record, No. 1304, pp. 12-17.

Ellis, H., M. Jiang and R.B. Corotis (1995). Inspection, Maintenance, and Repair with Partial Observability. Journal of Infrastructure Systems, Vol. 1(2), pp. 92-99.

Golabi, K., R.B. Kulkarni and G.B. Way (1982). A statewide Pavement Management System. Interfaces, Vol. 12(1), pp. 5-21.

Golabi, K. and R. Shepard (1997). Pontis: A System for Maintenance Optimization and Improvement of US Bridge Networks. Interfaces, Vol. 27(1), pp. 71-88.

Gong, L. (2006). Optimal Spatial Sampling of Infrastructure Condition: a Life-cycle-based Approach under Uncertainty. Ph.D. Dissertation, Department of Civil and Environmental Engineering and Geodetic Science, The Ohio State University, Columbus, OH.

Madanat, S.M. (1993). Incorporating inspection decisions in pavement management. Transportation Research, Part B: Methodology, 27B, pp. 425-438.

Madanat, S.M. and M. Ben-Akiva (1994). Optimal inspection and repair policies for infrastructure facilities. Transportation Science, Vol. 28(1), pp. 55-62.

Mishalani, R.G. and L. Gong (2009). Optimal Infrastructure Condition Sampling Over Space and Time for Maintenance Decision-Making under Uncertainty. Transportation Research Part B: Methodology, Vol. 43(3), pp. 311-324.

Murakami, K. and M. Turnquist (1995). A Dynamic Model for Scheduling Maintenance of Transportation Facilities. Transportation Research Record, No. 1030, pp. 8-14.

Mishalani, R.G. and H.N. Koutsopoulos (2002). Modeling the Spatial Behavior of Infrastructure Condition. Transportation Research Part B: Methodology, Vol. 36(2), pp. 171-194.

Mishalani, R.G. and L. Gong (2008). Evaluating the Impact of Pavement Condition Sampling Advances on Life-Cycle Management. Transportation Research Record, No. 2068, pp. 1-9.

Smilowitz, K. and S.M. Madanat (2000). Optimal Inspection and Maintenance Policies for Infrastructure Networks. Computer-Aided Civil and Infrastructure Engineering, Vol. 15(1), pp. 5-13.