TRANSPORT USER BENEFITS CALCULATION WITH THE "RULE OF A HALF" FOR TRAVEL DEMAND MODELS WITH CONSTRAINTS

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ABSTRACT

The importance of user benefits in transport projects assessments is well-known by transport planners and economists. Generally they have the greatest impact on the result of cost-benefit analysis. It is common practice to adopt the consumer surplus measure for calculating transport user benefits. Normally the well-known "Rule of a Half", as a practical approximation for the integral of the demand curve, is used to determine the change of consumer surplus. In this paper we enter into the question of whether the Rule of a Half is valid in the case of travel demand models with multiple constraints. Such models are often used for travel demand modeling of large-scale areas. The most discussed and well-known model in transport modeling field is the doubly constrained gravity model. Beside this model with inelastic constraints there are also more flexible models with elastic constraints. The theoretical analysis in this paper provides a mathematical proof for the validity of the concept of the Rule of a Half for travel demand models with multiple elastic and inelastic constraints. In this case the Rule of a Half is also a correct approximation of the change of consumer surplus.

Keywords: consumer surplus, Rule of a Half, travel demand model, multinomial logit model, constraints

1 INTRODUCTION

Investments in the transport sector are hallmarked by a long-term useful life and high investment costs. Due to restricted public financial resources not all transport projects are realizable and they have to be economically appraised. For this purpose different methodical approaches exist. The most important concept is the cost-benefit analysis, which is preferentially used worldwide.

Generally in transport project appraisals, transport user benefits have the most impact. The user benefits result from the changes of user costs like travel time and travel cost between the "Do-Minimum" situation (the most likely transport situation over the course of the appraisal period if no intervention were to occur) and the "Do-Something" situation (the Do-Minimum with transport intervention). The economic theory – as the theoretical basis of the cost-benefit analysis – provides the concept of the consumer surplus for the calculation of the transport user benefits. This measure can be derived by the mathematical integral of the demand function (See, for example, Varian, 1992).

The importance of user benefits in transport projects assessments is well-known by transport planners and economists. But the calculation of the exact integral of multiple travel demand functions is quite challenging. Hence, normally the "Rule of a Half", as a practical approximation of the integral of the demand function, is used to estimate the change of consumer surplus. This approach is proven and correct for travel demand models without multiple sets of constraints.

But for travel demand modeling of large-scale areas, models dealing with two sets of constraints are often used. The most discussed and well-known model in the transport modeling field is the doubly constrained gravity model. Beside this model with inelastic constraints, there are also more flexible models with elastic constraints. For doubly (or more) constrained models (regardless whether inelastic or elastic) the correctness of the Rule of a Half has not yet been proved. Because of the impact and importance of the correct transport user benefits measure for cost-benefit analysis, the investigation of this aspect is highly relevant.

In the following, the focus is on travel demand modeling whose results are the basis for impact estimation and the evaluation of a transport investment. In this context a travel demand model with elastic and inelastic constraints will be introduced. Additionally the multinomial logit model is considered. Because of its economic foundation, it is used as the methodical base concept for further considerations. Thus, a theory-consistent calculation of the change of the consumer surplus is in principle enabled. In the next section the derivation of a "doubly constrained multinomial logit model" follows, whose results equals the amount of travel demand calculated by the constrained travel demand model discussed before hand.

After the reflections on travel demand modeling, the concept of the Rule of a Half for calculating transport user benefits is presented. On the basis of the doubly constrained multinomial logit model for calculating travel demand, a mathematical analysis of the correctness of the Rule of a Half follows. Finally, a short discussion of the findings for practical uses closes the paper.

2 TRAVEL DEMAND MODELS

For travel demand modeling many different theories and approaches exist (for a good overview see, for example, Ortúzar and Willumsen, 2011). Beside the main goal of realistic travel demand modeling the theoretical base of the models is also highly important for the context here discussed. Thereby one can differentiate between doubly constrained models, which satisfy the given total numbers of trips originating in a travel zone **and** trips attracted to a travel zone respectively. Such constraints are powerful instruments and enable more realistic results especially for travel demand modeling of large-scale areas. The total number of trips is a result of the trip generation (first step of the 4-step-algorithm) and an input variable of the trip distribution. But the economic consistency of such models is unclear, because of their theoretical derivations from analogies of natural laws (e.g. gravitation, entropy maximization).

Beside traditional doubly constrained models, discrete choice models are highly relevant for the context considered here. The most famous representative of them is the multinomial logit model. But the model is only able to satisfy one set of constraints:

1. the total number of produced trips in each zone or
2. the total number of attracted trips in each zone or
3. the total number over all zones.

However, the multinomial logit model is derived from the economic concept of utility maximization and is completely economically consistent. Both approaches – doubly constrained models and the multinomial logit model – have advantages and disadvantages in the considered context and their strengths should be combined. In particular, the constraints should be combined with the economical interpretability of the multinomial logit model.

2.1 The Doubly Constrained Trip Distribution Model

Constraints are used to satisfy the given number of trips in all travel zones. They are a very useful and pragmatic method to achieve good results for trip distribution for large-scale areas. The most well-known model is the doubly constrained gravity model, using inelastic constraints solely (see Wilson, 1967). Another, more flexible, model is the "EVA model" (from the German terms for production (**E**rzeugung), distribution (**V**erteilung) and mode choice (**A**ufteilung)). It is a simultaneous model for production, distribution and mode choice but here we only consider the distribution part for the sake of simplicity (for a more detailed view see Vrtic et al., 2007). Differently to the "Wilson model", the EVA model deals with inelastic and/or elastic constraints.

The content of this paper is based on the EVA model. Since it is not possible to show all facets, in the following the model specification with a set of inelastic constraints for the origins and a set of elastic constraints for the destinations will be used. The EVA model is then:

$$\begin{aligned} &T_{ij} = f\left(g_{ij}\right) \cdot a_i \cdot b_j \\ &\left.\begin{aligned} &O_i^{min} \le \sum_j T_{ij} = O_i \le O_i^{max} \quad with\ O_i^{min} = O_i^{max} \\ &D_j^{min} \le \sum_i T_{ij} = D_j \le D_j^{max} \quad with\ D_j^{min} < D_j^{max} \end{aligned}\right\} constraints \end{aligned} \tag{1}$$

with

a_i, b_j	balancing factors to satisfy the constraints
f	deterrence function with regard to the generalized costs g_{ij} from zone i to zone j
O_i	total number of trips originating at zone i
D_j	total number of trips attracted to zone j
T_{ij}	number of trips from zone i to zone j

Thereby the number of trips is calculated taking into account generalized costs and the two sets of constraints. The constraints are satisfied by the balancing factors, which are solved iteratively. For solving this optimization problem different methods can be used, e.g. the Furness algorithm (Furness, 1965). The constraints are calculated in the trip generation a priori, but in contrast to the fixed origin constraints ($O_i^{min} = O_i^{max}$) only minimal and maximal values are defined for the destination constraints. In this intermediate range the resulting number of trips attracted to the different zones is calculated in the trip distribution. Thus, the number of attracted trips does not depend only on the number of attractors (e.g. number of persons) but also on the accessibility of the zones. In particular, this is important for substitutable trips like shopping.

There are many different versions of the above-mentioned deterrence function, although the exponential function has gained a very high importance and will be applied below. The integrated generalized costs g_{ij} are composed by travel time and travel cost from origin i to destination j and the value of travel time for monetizing (Goodwin, 1974). Hence, the function can be written:

$$f\left(g_{ij}\right) = e^{-g_{ij}} \quad with\ g_{ij} = c_{ij} + vot \cdot t_{ij} \tag{2}$$

with

c_{ij}	travel cost from zone i to zone j
t	travel time from zone i to zone j
vot	value of time

The value of time can be derived either from special travel behavioral surveys (e.g. stated response surveys) or from the model itself (as a result of model calibration).

2.2 The Multinomial Logit Model

Discrete choice modeling and in particular the (here considered) multinomial logit model (MNL model) is probably the most used decision concept in the travel modeling context currently. These models are grounded on the economic concept of utility maximization and the individuals are assumed choosing the most beneficial (optimal) alternative (trip from i to j). Unlike the constrained models, the MNL model calculates probabilities for the discrete alternatives and not the trips directly. These are definable by multiplying the probabilities with the total number of trips.

The comparison of alternatives is on the basis of the alternative specific utilities. Therefore utility functions including decision-relevant variables like travel costs and travel time have to be defined. Due to travel modelers' not being able to observe all decision relevant components, as well as travelers' inaccurate "perception" of the objective costs, the utility function additionally includes, beside objective components, a stochastic portion of utility standing for the unobservable part. The utility function can then be described as (Ben-Akiva and Lerman, 1985):

$$u_{ij} = \bar{u}_{ij} + \varepsilon_{ij} \tag{3}$$

with
u_{ij} utility of an alternative (trip from zone i to zone j)
$\bar{u}_{ij}$ observable part of utility (deterministic)
ε_{ij} unobservable part of utility (stochastic)

The chosen distribution of the stochastic part of utility determines different discrete choice models. The MNL model results from the use of Gumbel distribution and the calculation of the probabilities of trips from zone i to zone j can be written:

$$P_{ij} = \frac{e^{\bar{u}_{ij}}}{\sum_{i'} \sum_{j'} e^{\bar{u}_{i'j'}}} \tag{4}$$

with
P_{ij} probability of choosing alternative i-j
e exponential function

The wanted number of trips from zone i to zone j result from a globally fixed MNL model (satisfies only the total number over all zones):

$$T_{ij} = P_{ij} \cdot T \tag{5}$$

with
T total number of trips over all zones

The observable part of utility, which is here defined by the negative generalized costs, is highly relevant. Thus, it is:

$$\bar{u}_{ij} = -g_{ij} = -c_{ij} - vot \cdot t_{ij} \tag{6}$$

The simplicity of calculating travel demand on the ground on the MNL model is one important model advantage but there is another good point in addition. In several publications and especially in a ground-breaking theoretical analysis by McFadden, the embedding of discrete choice models in the economic theory could be shown. This is of great value for integrated demand modeling and benefit calculating. Hence, travel demand thus derived is economically consistent and the integral of the travel demand function provides the consumer surplus.[1] The great practical and theoretical advantages of the MNL model are accompanied by the disadvantage of the absence of two sets of constraints.

3 THE DOUBLY CONSTRAINED MULTINOMIAL LOGIT MODEL

The combination of economic consistence on the one hand and the simplicity of travel demand estimation on the other hand is a big plus of the MNL model. However the limitation of considering multiple constraints is a serious drawback for several transport planning problems. Hence, a doubly constraint MNL model has to be formulated enabling the satisfaction of production **and** attraction constraints. Thus, the objective is the formulation of the EVA model in terms of the MNL model. For the sake of simplicity, the derivation is carried out only for the case with a set of inelastic production and a set of elastic attraction constraints. Of course other combinations are possible as well.

The utility function is the central element to account for decision-relevant characteristics in the MNL model. In order to consider two sets of constraints the utility function in (6) origin and destination "shadow prices" has to be added to the generalized costs (Neuburger and Wilcox, 1976). The term "shadow" is used, since these "prices" are not definable a priori but derivable from the balancing factors a_i and b_j of the EVA model a posteriori (after the iteration process). The given number of originating and attracting trips contain additional information about the underlying decision process of the transport users. This information is implemented in the utility function by the shadow prices which serve the emendation of the "a priori utility". Then, by means of the shadow prices, the utility maximization process provides the probabilities of alternatives producing a result with satisfied constraints. The utility function can then be written:

[1] Furthermore, the antiderivative is known for the MNL model and the exact integral is defined. See for more information e.g. Williams, 1977 or Small and Rosen, 1981. In the present paper the approximation of the integral with the Rule of a Half is considered.

$$\begin{aligned}\bar{u}_{ij} &= -c_{ij} - vot \cdot t_{ij} + \theta_i + \tau_j \\ &= -g_{ij} + \theta_i + \tau_j\end{aligned} . \tag{7}$$

with

θ_i, τ_j shadow prices of origin i an destination j

The shadow prices in this function – as well as the generalized costs – are monetary factors. By implementing this function in the MNL model we get the doubly constrained MNL model for estimating trips T_{ij}:

$$\begin{aligned}&T_{ij} = P_{ij} \cdot T = \frac{e^{\left(-g_{ij}+\theta_i+\tau_j\right)}}{\sum_{i'}\sum_{j'} e^{\left(-g_{i'j'}+\theta_{i'}+\tau_{j'}\right)}} \cdot T \\ &O_i^{min} \le \sum_j T_{ij} = O_i \le O_i^{max} \quad with\, O_i^{min} = O_i^{max} . \\ &D_j^{min} \le \sum_i T_{ij} = D_j \le D_j^{max} \quad with\, D_j^{min} < D_j^{max}\end{aligned} \tag{8}$$

The doubly constrained MNL model is formally defined by this equation, though the shadow prices are unknown, the definition of which requires further considerations and mathematical redrafts. First of all the doubly constrained MNL model can be written as (with omitted constraints):

$$T_{ij} = \frac{e^{\left(-g_{ij}\right)} \cdot e^{\theta_i} \cdot e^{\tau_j}}{\sum_{i'}\sum_{j'} e^{\left(-g_{i'j'}\right)} \cdot e^{\theta_{i'}} \cdot e^{\tau_{j'}}} \cdot T . \tag{9}$$

With $f\left(g_{ij}\right) = e^{\left(-g_{ij}\right)}$, $a_i = e^{\theta_i}$ und $b_j = e^{\tau_j}$ results:

$$T_{ij} = \frac{f\left(g_{ij}\right) \cdot a_i \cdot b_j}{\sum_{i'}\sum_{j'} f\left(g_{i'j'}\right) \cdot a_{i'} \cdot b_{j'}} \cdot T \tag{10}$$

and in addition it is:

$$T = \sum_i \sum_j f\left(g_{ij}\right) \cdot a_i \cdot b_j . \tag{11}$$

The resulting of insertion of (11) in (10) equates to the EVA model in (1). Hence, the derivation of the shadow prices is straightforward by logarithmic transformations of the balancing factors:

$$a_i = e^{\theta_i} \rightarrow ln(a_i) = \theta_i$$
$$b_j = e^{\tau_j} \rightarrow ln(b_j) = \tau_j \quad . \qquad (12)$$

So we can see that no further adaptations are required and all utility characteristics are on hand. As a consequence of the shown interrelationships it is obvious that an MNL model can be formulated in mathematical terms to satisfy multiple constraints.

4 THE RULE OF A HALF

The concept of the Rule of a Half (RoH) as an approximation of calculation of the change of consumer surplus arising from specific transport intervention was used for the first time at the end of the 1960s (Williams, 1977). In all likelihood it is the most-commonly applied instrument in practice for estimating user benefits. For example, the RoH is recommended by the Economic Commission for Europe of the United Nations (United Nations, 2003). Its popularity is based on the easy calculability and the universal applicability.

For a short presentation of the RoH the starting point is the consideration of the consumer surplus, defined by Alfred Marshall as

> "the excess of the price which [the consumer] would be willing to pay rather than go without the thing, over that which he actually does pay" (Marshall, 1920).

Furthermore, the concept is transferable to market demand (all transport users) without difficulty. For economic evaluation of transport interventions, the **change** of consumer surplus is much more interesting still. The comparison of consumer surpluses of the Do-Minimum and the Do-Something provides information about the (monetary) user benefits of the project.

Assuming that only generalized costs have influence over the travel demand, one can say, that a change in consumer's surplus accompanying a fall in generalized costs from g_{ij}^0 to g_{ij}^1 may then be equated to the area to the left of the demand curve between the initial (Do-Minimum) and final (Do-Something) costs (see figure 1).[2] Due to the decreased costs, the number of trips T_{ij} rises from T_{ij}^0 to T_{ij}^1. The blue shaded area represents the consumer surplus of the Do-Minimum and the sum of the blue and red shaded areas represents the consumer surplus of the Do-Something. The desired change in consumer surplus corresponds to the red shaded area. Mathematically, the change in consumer surplus results from the integral of the travel demand function between the initial and final generalized costs.

[2] The marked points 0 and 1 stand for the equilibrium in the initial and final state. The supply curves, in fact necessary for defining the equilibriums, are omitted for the sake of clarity.

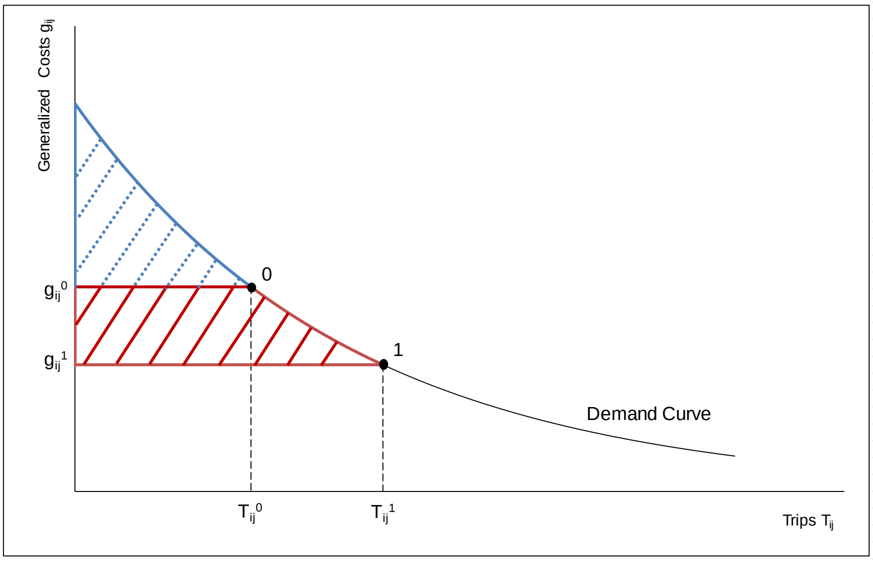

Figure 1 – Consumer Surplus (Change)

Generally, an intervention in the transport sector affects the costs of several competing alternatives, whose costs and demands are interdependent. Hence, the particular demand curves are not "stable" but rather shifting elements according to all alternative specific generalized costs. Thus, a consideration of a general equilibrium is necessary. Because of the immense computing time, generally, the desired data are only calculated for the initial and final states and the demand curves are only known for these two single (equilibrium) situations. So, an assumption of the shape of the demand curve between the initial and final state is necessary to estimate the change in consumer surplus. If one neglects the curvature of the demand function, a linear line results. That case is in accordance with the RoH and the change in consumer surplus over all transport users is:

$$\begin{aligned} \Delta CS &= \sum_i \sum_j \Delta CS_{ij} \\ &= \sum_i \sum_j \left(\frac{1}{2} \cdot \left(g_{ij}^0 - g_{ij}^1 \right) \left(T_{ij}^0 + T_{ij}^1 \right) \right) \end{aligned} \quad (13)$$

with

ΔCS change in consumer surpluses of all transport users and trip relations i-j

ΔCS_{ij} change in consumer surpluses of all transport users T_{ij}

The principle of the RoH is shown in Figure 2 and the approximate nature of the approach is distinguishable, whose accuracy depends on the shape of the demand curve. The more the generalized costs differ between the initial and final state, the more inaccurate is the result. Nevertheless the concept is used worldwide because of its simplicity, and for most transport investment evaluations the results are sufficient and close to the exact measure. But there are approaches allowing for the direct integration of the travel demand model and the finding of the exact amount of change in consumer surplus. The most well-known approach is the difference of the logsum-terms of the MNL model (see, for example, Train, 2003). For the doubly constrained MNL model the exact formula is also found (Winkler, 2011). However this issue is not the subject of that paper.

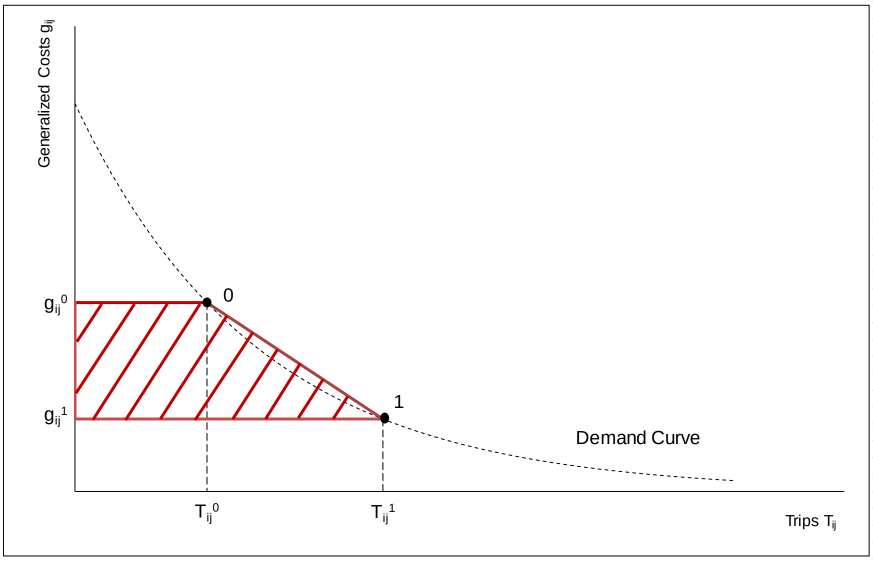

Figure 2 – The Rule of a Half

The RoH is a pragmatic and comprehensible method to estimate the change in consumer surplus. Williams (1976) contributed a mathematical analysis of the concept, which proved the correctness of the RoH as an approximation of the true benefit measure in economic sense. Hence, the methodical approach is theoretically consistent. But the main question here is whether this is true for the case of travel demand models with multiple constraints.

5 THE RULE OF A HALF FOR TRAVEL DEMAND MODELS WITH CONSTRAINS

The RoH method aims to measure the user benefits by an approximate integration of the travel demand function between the initial and final generalized costs. In the case of the doubly constrained MNL model, however, the demand is influenced by shadow prices as well. Now, the question is, which consequences do these additional (model) factors have on the change of consumer surplus quantified by the RoH? At first, of course, the RoH can be used for the doubly constrained MNL model. Then, all deterministic components of the utility function – including shadow prices – have to be considered by integrating the demand function. So we obtain:

$$\begin{aligned}\Delta CS^* &= \sum_i \sum_j \Delta CS_{ij}^* \\ &= \sum_i \sum_j \left(\frac{1}{2} \cdot \left(\bar{u}_{ij}^1 - \bar{u}_{ij}^0 \right) \cdot \left(T_{ij}^0 + T_{ij}^1 \right) \right) \\ &= \sum_i \sum_j \left(\frac{1}{2} \cdot \left(\left(g_{ij}^0 - \theta_i^0 - \tau_j^0 \right) - \left(g_{ij}^1 - \theta_i^1 - \tau_j^1 \right) \right) \cdot \left(T_{ij}^0 + T_{ij}^1 \right) \right)\end{aligned} \tag{14}$$

ΔCS^* stands for the approximate integral of the doubly constrained MNL model, in which the influence of shadow prices is included. However, the desired change in consumer surplus is only a function of the real quantities of generalized costs (Williams, 1976). So ΔCS^* has to be adjusted for the contribution of the constraints (by shadow prices), since (Winkler, 2011):

$$\Delta CS^* \neq \Delta CS. \tag{15}$$

It now needs clarifying, whether the RoH, taking only generalized costs into consideration (and neglecting the shadow prices), gives a correct (approximate) benefit measure in the case of a doubly constrained MNL model. The mathematical decomposition of the integral of the doubly constrained MNL model is needed to prove this.

The following differential is the starting point of the analysis:

$$\Delta CS^* = CS^{1*} - CS^{0*} = \int_{PA} dCS^*. \tag{16}$$

The differential dCS* corresponds to the (total) demand function (Varian, 1992). So we obtain:

$$dCS^* = \sum_i \sum_j T_{ij}\left(\bar{\boldsymbol{u}}\right) d\bar{u}_{ij}. \tag{17}$$

Here, $\bar{\boldsymbol{u}}$ represents the vector of all alternative specific deterministic utility (negative generalized costs and shadow prices), since every trip T_{ij} depends on the costs of all alternatives. The insertion in (16) provides:

$$\Delta CS^* = \int_{PA} \sum_i \sum_j T_{ij}\left(\bar{\boldsymbol{u}}\right) d\bar{u}_{ij}. \tag{18}$$

The integral is a line integral, whose outcome depends on the path of integration PA, i.e. the sequence of integration of the various utility components. Hence, a detailed analysis of the integral of the doubly constrained MNL model is necessary. To begin with, the total differential $d\bar{u}_{ij}$ can be expressed by the sum of the partial differentials (Chiang and Wainwright, 2005):

$$d\bar{u}_{ij} = -\frac{\partial \bar{u}_{ij}}{\partial g_{ij}} dg_{ij} + \frac{\partial \bar{u}_{ij}}{\partial \theta_i} d\theta_i + \frac{\partial \bar{u}_{ij}}{\partial \tau_j} d\tau_j. \tag{19}$$

The components g_{ij}, θ_i and τ_j are added unweighted in the utility function u_{ij}. Thus, the partial derivations in (19) are 1 and we obtain:

$$d\bar{u}_{ij} = -dg_{ij} + d\theta_i + d\tau_j. \tag{20}$$

With $T_{ij}\left(\bar{\boldsymbol{u}}\right)=T_{ij}$ and consideration of the bounds of integration, it follows:

$$\Delta CS^{*}=\int_{\bar{\boldsymbol{u}}^{0}}^{\bar{\boldsymbol{u}}^{1}}\sum_{i}\sum_{j}T_{ij}\left(-dg_{ij}+d\theta_{i}+d\tau_{j}\right). \tag{21}$$

Furthermore, the decomposition of the integral is necessary and possible (Merziger et al., 1999). Then we can write:

$$\Delta CS^{*}=-\int_{\boldsymbol{g}^{0}}^{\boldsymbol{g}^{1}}\sum_{i}\sum_{j}T_{ij}\,dg_{ij}+\int_{\boldsymbol{\theta}^{0}}^{\boldsymbol{\theta}^{1}}\sum_{i}\sum_{j}T_{ij}\,d\theta_{i}+\int_{\boldsymbol{\tau}^{0}}^{\boldsymbol{\tau}^{1}}\sum_{i}\sum_{j}T_{ij}\,d\tau_{j}\,. \tag{22}$$

The desired component ΔCS, which encompasses the change of consumer surplus due to the change in real costs (generalized costs), is represented only by the first term on the right side. In contrast, the effect of the shadow prices has to be extracted. Initially, for finding of ΔCS equation (22) can be expressed by:

$$\Delta CS=\Delta CS^{*}-\int_{\boldsymbol{\theta}^{0}}^{\boldsymbol{\theta}^{1}}\sum_{i}\sum_{j}T_{ij}\,d\theta_{i}-\int_{\boldsymbol{\tau}^{0}}^{\boldsymbol{\tau}^{1}}\sum_{i}\sum_{j}T_{ij}\,d\tau_{j}\,. \tag{23}$$

The term ΔCS* can be approximated by the RoH (as shown above) and is here revised to the portion of the shadow prices. Unfortunately the two shadow price integrals are not readily solvable, but further simplifications are possible. Both integrals are one-dimensional and cover the dimension of the production or attraction side only. Since a change in θ_i does not have any impact on the total number of attracted trips D_j (the same applies to τ_j and O_i), it can be written (with $\sum_{j}T_{ij}=O_{i}$ and $\sum_{i}T_{ij}=D_{j}$):

$$\Delta CS=\Delta CS^{*}-\int_{PA}\sum_{i}O_{i}\,d\theta_{i}-\int_{PA}\sum_{j}D_{j}\,d\tau_{j}\,. \tag{24}$$

After these simplifications and mathematical redrafts, the integration of functions of the total number of originated and attracted trips over the shadow prices is now necessary. But there are no (explicit) closed formulas for the calculus of the O_is and D_js and integration cannot easily be achieved.

It is of great importance for the solution of the integrals, whether the initial and final total number of trips O_i and D_j, respectively, are equal or not. Generally, in the case of inelastic constraints, the numbers are equal. On the other hand with elastic constraints changes almost always occur. As hitherto, for solving the integrals, we consider inelastic constraints on the total number of originating trips and elastic constraints on the total number of attracting trips in each zone.

Case $O_i^0 = O_i^1$

The O_is are results of the trip generation and are constant in the trip distribution. In consequence they are independent of the shadow prices. Hence, there are no formal interdependences of the numbers of trips of the zones and the O_is can be taken outside the integrals. Thus, the integration is path-independent, because of the independent integrability of all integration variables θ_i. So we obtain:

$$\int_{PA} \sum_i O_i \, d\theta_i = \sum_i O_i \cdot \int_{\theta_i^0}^{\theta_i^1} d\theta_i \, . \tag{25}$$

Then the integration of the differentials $d\theta_i$ provides the integration variables θ_i. Next we insert the bounds of integration and the solution for the inelastic case is:

$$\int_{PA} \sum_i O_i \, d\theta_i = \sum_i O_i \cdot \left(\theta_i^1 - \theta_i^0 \right). \tag{26}$$

All required factors are available – the O_is from the trip generation and the θ_is from the trip distribution. So there is no problem calculating the first portion of benefits.

Case $D_j^0 \neq D_j^1$

In the case of changing number of trips we need another approach, since the number of total trips D_j is not constant and we cannot take it outside the integral readily.[3] So, in principal, it is necessary to integrate the (unknown) functions of the resulting D_js. But we only have information about the D_js for the two equilibrium (initial and final) states. Hence, we have to make an assumption about the shape of the function. According to the RoH we assume a linear form.

With the assumption of a linear path of integration it can be written:

$$\int_{PA} \sum_j D_j \, d\tau_j = \sum_j \left(\frac{D_j^0 + D_j^1}{2} \right) \cdot \int_{\tau_j^0}^{\tau_j^1} d\tau_j \, . \tag{27}$$

Now the integral can be solved analogically to the inelastic case and we obtain:

$$\int_{PA} \sum_j D_j \, d\tau_j = \sum_j \left(\frac{D_j^0 + D_j^1}{2} \right) \cdot \left(\tau_j^1 - \tau_j^0 \right). \tag{28}$$

In this case all required factors are given by the trip distribution.

[3] This is the normal case for elastic constraints. Changing numbers of trips are also possible for inelastic constraints, but only for zone-specific changes (between the initial and final state) of the trip numbers. This is, in particular, an interesting point for long-term cases.

In order to solve equation (24) we have to simplify equation (14) firstly:

$$\begin{aligned}\Delta CS^{*} &= \sum_i \sum_j \frac{1}{2}\cdot\left(g_{ij}^{0}-g_{ij}^{1}\right)\cdot\left(T_{ij}^{0}+T_{ij}^{1}\right)\\ &+\sum_i \sum_j \frac{1}{2}\cdot\left(\theta_i^{1}-\theta_i^{0}\right)\cdot\left(T_{ij}^{0}+T_{ij}^{1}\right)+\sum_i \sum_j \frac{1}{2}\cdot\left(\tau_j^{1}-\tau_j^{0}\right)\cdot\left(T_{ij}^{0}+T_{ij}^{1}\right)\end{aligned}. \tag{29}$$

Furthermore we can write:

$$\begin{aligned}\Delta CS^{*} &= \sum_i \sum_j \frac{1}{2}\cdot\left(g_{ij}^{0}-g_{ij}^{1}\right)\cdot\left(T_{ij}^{0}+T_{ij}^{1}\right)\\ &+\sum_i \frac{1}{2}\cdot\left(\theta_i^{1}-\theta_i^{0}\right)\cdot\left(O_i^{0}+O_i^{1}\right)+\sum_j \frac{1}{2}\cdot\left(\tau_j^{1}-\tau_j^{0}\right)\cdot\left(D_j^{0}+D_j^{1}\right)\end{aligned}. \tag{30}$$

The insertion of the solved integrals (26) and (28) as well as the simplification (30) in equation (24) provide:

$$\begin{aligned}\Delta CS &= \sum_i \sum_j \frac{1}{2}\cdot\left(g_{ij}^{0}-g_{ij}^{1}\right)\cdot\left(T_{ij}^{0}+T_{ij}^{1}\right)\\ &+\sum_i \frac{1}{2}\cdot\left(\theta_i^{1}-\theta_i^{0}\right)\cdot\left(O_i^{0}+O_i^{1}\right)+\sum_j \frac{1}{2}\cdot\left(\tau_j^{1}-\tau_j^{0}\right)\cdot\left(D_j^{0}+D_j^{1}\right).\\ &-\sum_i O_i\cdot\left(\theta_i^{1}-\theta_i^{0}\right)-\sum_j\left(\frac{D_j^{0}+D_j^{1}}{2}\right)\cdot\left(\tau_j^{1}-\tau_j^{0}\right)\end{aligned} \tag{31}$$

Considering, that is $O_i^0 = O_i^1 = O_i$ for the inelastic case, we obtain:

$$\sum_i \frac{1}{2}\cdot\left(\theta_i^{1}-\theta_i^{0}\right)\cdot\left(O_i^{0}+O_i^{1}\right)=\sum_i O_i\cdot\left(\theta_i^{1}-\theta_i^{0}\right). \tag{32}$$

Furthermore, it is:

$$\sum_j \frac{1}{2}\cdot\left(\tau_j^{1}-\tau_j^{0}\right)\cdot\left(D_j^{0}+D_j^{1}\right)=\sum_j\left(\frac{D_j^{0}+D_j^{1}}{2}\right)\cdot\left(\tau_j^{1}-\tau_j^{0}\right). \tag{33}$$

Consequently, the last four terms in equation (31) cancel out. So then we can see, that the RoH is a correct approximation of the change of consumer surplus for travel demand models with multiple constraints as well. We have derived the approaches for inelastic constraints on the total number of originating trips and elastic constraints on the total number of attracting trips. Of course, the approaches are applicable in any combination (See Winkler, 2011).

6 CONCLUSION

On the one hand we have been able to show that doubly constrained travel demand models can be expressed in terms of the MNL model. On the other hand we have presented a proof for the correctness of the widely used RoH on the basis of doubly constrained travel demand models. There is no need for correction terms. The shadow prices do not have any effect on real user benefits and can be disregarded in benefits calculation.

The findings are of theoretical interest, since they show the possibility of an economic formulation of travel demand models with multiple constraints and the applicability of the RoH. For practical use the results are very important too, since there is a verification of all previous user benefits evaluations by the RoH independent of the used travel demand model – with or without multiple constraints. The same applies to all following evaluations which use this concept.

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