MULTIPLE CRITERIA STOCHASTIC RANKING OF VARIANTS OF THE DISTRIBUTION SYSTEM

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ABSTRACT

The paper presents an original methodology of solving a real-world multiple criteria stochastic decision problem (MCSDP) that consists in evaluating and ranking different variants of the distribution system. It originates in the analysis of the existing distribution system and development of its redesign scenarios (variants). The operations of these redesign scenarios have been simulated and their evaluation characteristics – criteria have been defined. Due to randomness of simulation these parameters have stochastic character. Thus, the resulting multiple criteria ranking problem has a stochastic character, too. The authors of the paper propose an original computational procedure that helps the decision maker (DM) to solve the above mentioned MCSDP and select the most desirable (compromise) concept (variant) of the distribution system. The proposed approach is a combination of a traditional, deterministic multiple criteria ranking method (e.g. Electre III) and a classification algorithm (e.g. Bayes classifier).

Keywords: multiple criteria stochastic decision problem, multiple criteria ranking method, classification procedure, distribution system

INTRODUCTION

Distribution is a business-oriented activity focused on moving materials and goods (products) from their origins (points of sourcing, manufacturing) to their destinations (points of utilization, purchasing, consumption) in a certain supply network composed of raw material extraction and processing units, semi-finished and finished products manufacturing plants, wholesalers, retailers and final consumers (Coyle, Bardi, Langley 2003; Kapoor, Kansal 2003; Ross 1996). Distribution consists in delivering goods to customers efficiently, in proper quantities, condition and in a timely manner. It encompasses two major activities, i.e.: material storing & handling and material moving - transporting. Two major categories of distribution are recognized, i.e. (Coyle, Bardi, Langley 2003; Kotler 1994):

- procurement distribution,
- physical distribution.

The former is responsible for delivering raw materials, components and semi-finished products to manufacturing plants where they are used to produce the finished goods. Thus, procurement distribution covers the initial part of the supply chain. The physical distribution focuses on delivering finished goods to final consumers. It is responsible for moving goods from manufacturing plants to the points of final consumption and thus covers the terminal part of the supply chain (Coyle, Bardi, Langley 2003; Kotler 1994). In this paper the considered research topic refers to the physical distribution of goods.

The evaluation of the distribution systems is discussed in many reports (Abrahamsson, Brege, Norrman 1998; Lumsden, Dallari, Ruggeri 1999; Sawicka, Zak 2010). Some authors prove that the assessment of distribution systems may be based on the general concept of "logistics excellence" featured by the idea of "7 rights" (Kapoor, Kansal 2003; Kotler 1994). Many authors agree that the evaluation of distribution systems should involve the analysis of their complex and dynamic character, interactions between partners in the supply chain and many aspects of their operations (technical, economic, social, environmental). P. Kotler (Kotler 1994) and several other authors discuss the question of many stakeholders that exist in the distribution systems and insist on considering their interests in the evaluation of the distribution activity. For the reasons mentioned above some authors apply multiple criteria analysis (MCA) (Sawicka, Zak 2010) in evaluating distribution systems. Several reports demonstrate that evaluation of distribution systems combined with their comprehensive analysis/ diagnosis leads to the assessment of their overall condition. If this condition is poor or at least unsatisfactory certain corrective actions, restructuring processes and/ or redesign projects are recommended to adjust and improve the operations of the distribution system and enhance the standards of customer service. In these restructuring process different logistics solutions can be proposed and various variants of distribution systems can be designed.

In general, there are two major approaches to the quantitatively oriented redesign of the distribution systems (Sawicka, Zak 2010; Zak 2010), i.e.: the restructuring based on the application of traditional optimization (mathematical programming) methods or intuitive, expert – oriented redesign supported by simulation techniques and in some cases multiple criteria decision making/aiding (MCDM/A) methods. The former consists in generating the best - optimal structure of the distribution system, while the latter focuses on designing different variants of the distribution system and ranking them from the best to the worst. The second of these two approaches is considered in this paper.

The authors face a challenge of evaluating and ranking different variants of the distribution system designed as a result of its complex evaluation. In the design phase the operations of all the variants have been simulated which resulted in the generation of a set of their characteristics and parameters. Due to randomness of the simulation process these parameters have a stochastic character. To satisfy the interests of different stakeholders and guarantee a complete assessment of the proposed solutions several criteria have been defined to carry out this analysis. As a result a multiple criteria stochastic ranking problem has been formulated whose solution leads to an order of redesign scenarios of the distribution system of goods (dsg).

The proposed solution procedure is based on the application of three principal methodologies, including: multiple criteria decision making/ aiding (MCDM/A) (Roy 1985, Vincke 1992), classification theory (Berger 1985; Demsar 2006; Mitchell 1997) and simulation (Law, Kelton 2000).

Simulation is a controlled statistical sampling technique for estimating the performance of complex stochastic systems when analytical models do not suffice (Hillier, Lieberman 1990). Simulation is applied to visualize the operations of different objects, processes and systems and observe their behavior, mutual interactions between considered components and their impact on the surrounding environment. The simulation model describes the operations of the system in terms of individual events of the individual components of the system. Thus, in the simulation process the system is divided into elements whose behavior can be predicted, at least in terms of probability distributions, for each of the various possible states of the system and its inputs (Law, Kelton 2000; Hillier Lieberman, 1990). In this research the authors apply simulation to design different variants of the dsg and visualize their operations as well as to generate a sample of random values of criteria, evaluating the variants of the dsg.

MCDM/A is a field which aims at giving the DM some tools in order to enable him/her to solve a complex decision problem in which several, often contradictory points of view must be taken into account. Instead of finding an optimal solution MCDM/A concentrates on suggesting a “compromise solution”, that is based on the analysis of trade-offs between criteria and the DM’s preferences (Vincke 1992). MCDM/A provides tools and methods that assist the DM in solving multiple criteria decision problems (Zak 2010). These problems are the situations in which having defined a set of actions (decisions, variants) *A* and a consistent family of criteria *F* the DM tends to (Roy 1985): define a subset of actions (decisions, variants) being the best on *F* (choice problematic), divide the set of actions (decisions, variants) into subsets according to certain norms (sorting problematic), rank the set of actions (decisions, variants) from the best to the worst (ranking problematic). In this paper a specific MCDM/A method is applied by the authors to generate the ranking of the variants of the dsg.

Classification theory (Smola, Vishwanatan 2008); Demsar 2006) is a quantitatively oriented methodology that helps the DM to classify and categorize objects into predefined classes. There is a wide variety of classification methods reported in the literature (Berger 1985; Demsar 2006; Mitchell 1997), including:

- classification by the induction of decision trees (Pisetta, Jouve, Zighed 2010; Quinlan, 1986),

- k-NN – nearest-neighbor classification (Bremner et al. 2005; Hall, Park, Samworth 2008),
- Bayesian classification (Berger 1985; Mitchell 1987; Smola, Vishwanatan 2008),
- rough sets theory (Pawlak 1982; Yao 2011),
- neural networks (Cetiner, Sari, Borat 2010; Wei, Schonfeld 1993).

Many authors (Berger 1985; Mitchell 1987) claim that one of the most efficient method to solve real-world classification problems is Bayesian classification. Its advantage is precision and effectiveness when solving problems associated with the manipulation on the large sets of data. In this research the Bayesian classifier is applied to produce appropriate relations between considered variants of the dsg, based on the stochastic data.

THE METHODOLOGY OF SOLVING A MULTIPLE CRITERIA STOCHASTIC DECISION PROBLEM

The theoretical background

As described in the introduction the authors' approach has been based on the application of a traditional – deterministic MCDM/A ranking method e.g. Electre III combined with a classification method e.g. Bayes classifier.

The first of the above mentioned methods i.e. Electre III (Roy 1985; Vincke 1992) represents the European school of MCDM/A based on the outranking relation. The method requires to determine the model of DM's preferences by the indifference q_j, preference p_j, and veto v_j thresholds as well as weights w_j for each criterion j. The aggregation procedure starts from the calculation of concordance and discordance indices. The first one measures the arguments in favor of the statement that variant a outranks variant b, while the second index represents the strengths of evidence against the above hypothesis. Based on these indices the outranking relation is calculated. The ranking of variants is based on two classification algorithms: descending and ascending distillations. Descending distillation procedure starts from choosing the best variant i.e. the one with the highest value of qualification index $Q(a)$ and placing it at the top of the ranking. $Q(a)$ equals the difference between the number of variants, which are outranked by the variant a and the number of variants that outrank this variant. In the consecutive steps, the best variant from the remaining set of variants is selected and placed in the second position of the ranking. The procedure stops, when the set of variants is empty. Ascending distillation procedure starts from choosing the worst variant and placing it at the bottom of the ranking. Then the worst variant from the remaining set of variants is selected and placed in the second worst position of the ranking. The final graph corresponding to the outranking matrix is the intersection of the two distillations. It constitutes a graphical representation of indifference I, preference P and incomparability R relations (between variants) that construct the outranking matrix.

The second method applied in this research is Bayes classifier which seems to be particularly appropriate to handle a complex, stochastic decision problem with large sets of data.

In the Bayes theory two major categories of probabilities can be distinguished, i.e.: a priori probabilities of certain events and their a posteriori equivalents. The former are defined beforehand (prior to the computational classification experiments) and constitute the input data for all computations carried out in the classification experiment. The latter are the results of the carried out computations.

The algorithm of Bayesian classification is composed of 3 phases, such as:

- phase 1 – the construction of training set *T* i.e. classifier,
- phase 2 – testing of the training set *T*,
- phase 3 – classification of the vector of observations to predefined classes (decision attributes).

In the first phase, the random set of samples from the data set is selected. The random set should equal 70% of the data set. Based on this sample the training set *T* i.e. classifier is constructed. Its role is the assignment of the decision attribute value to samples based on description attributes. It is assumed, while constructing the training set *T*, that the following information is known: the decision attribute and the conditional a priori probability of the vector of observations x(*a*) that it belongs to $C^{(z)}$ class i.e. $P(x(a)|C^{(1)})$, $P(x(a)|C^{(2)}),\dots,$ $P(x(a)|C^{(Z)})$.

In the second phase, the training set *T* is tested. The decision attribute is assigned to each sample of the training set based on classifier constructed in the first phase. Conditional a posteriori probability for each sample is calculated.

The conditional a posteriori probability that the vector of observations x(*a*) belongs to the class $C^{(z)}$ is formulated with respect to Bayes rule as follows (Mitchell 1997):

$$P\left(C^{(z)}|x(a)\right)=\frac{P\left(x(a)\,|C^{(z)}\right)P\left(C^{(z)}\right)}{P(a)} \tag{1}$$

where:

P(a) – a priori probability of description attribute *a*, which is constant for each class $C^{(z)}$;

$P(C^{(z)})$ – a priori probability of class $C^{(z)}$ estimated by the relative frequency of appearance of class $C^{(z)}$ in a training set *T*;

$P(x(a)|C^{(z)})$ - a conditional a priori probability of x(*a*) conditioned on $C^{(z)}$, which is formulated as follows:

$$P\left(x(a)\,|C^{(z)}\right)=\prod_{k=1}^{K}P\left(x_k|C^{(z)}\right) \tag{2}$$

where:

x_k - is a value of an attribute A_k for a sample *a.* $P(x_k|C^{(z)})$ is the number of samples of class $C^{(z)}$ in a training set *T* having the value x_k for the attribute A_k, divided by the frequency of appearance of class $C^{(z)}$.

The class of maximum value of a posteriori probability is assigned to every classified vector of observation. This class is defined by the following formula (Mitchell 1997):

$$f(a)=\arg\max_{c} P\left(C|x(a)\right) \tag{3}$$

It is assumed that the value of a decision attribute is known. This allows the evaluation of the classification error β, which is formulated as follows (Mitchell 1997):

$$\beta = \frac{B_T - B_C}{B_T} \quad (4)$$

where:
B_T – the number of testing sets,
B_C – the number of correctly classified testing sets.
The classification error can obtain the values in the range [0,1]. Thus, the lowest the value of this error is, the better the classification is.
In the third phase, the vector of observations with a priori unknown decision attribute is classified into the predefined classes. For this purpose the classifier constructed in the first phase is applied.

The proposed approach

The proposed approach to solving the stochastic multiple criteria ranking problem is composed of the following six steps:

- step I – collection of the stochastic data,
- step II – random selection of deterministic numbers using simulation technique e.g. ExtendSim,
- step III – solving a multiple criteria deterministic problem with an application of MCDM/A method e.g. Electre III,
- step IV – classification of deterministic relations between variants using classification method e.g. Bayes classifier,
- step V – construction of final ranking of variants with an application of computational tool e.g. MS Excel,
- step VI – the recommendation of the compromise solution based on a stochastic final ranking of variants.

These steps are presented in figure 1.

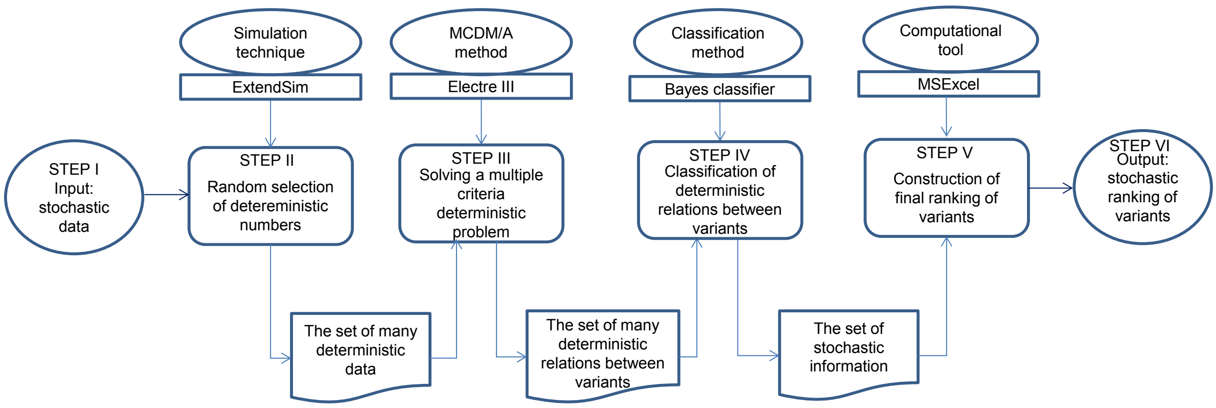

Figure 1. The paradigm of solving a multiple criteria stochastic decision problem

In the first step the stochastic evaluations of variants are collected and model of the DM's preferences is constructed.

In the second step the stochastic evaluations of variants are transformed to random deterministic parameters. The simulation method e.g. ExtendSim is applied to generate these values within at least 100 iterations (Marinoni 2005).
In the third step the outranking relations between variants for the results of at least 100 simulation iterations are constructed. The computational experiments are carried out with an application of a traditional, deterministic MCDM/A method e.g. Electre III.
In the fourth step the training set and vector of observations are selected, and deterministic relations between variants are classified to decision attributes (classes). A classification method e.g. Bayes classifier is applied.
In the fifth step the final ranking of variants constituting the vector of observations is constructed based on the computational experiments carried out with the application of a spread sheet e.g. MSExcel. The probabilities of occurrence of particular relations between variants are computed.
In the sixth step the final ranking of variants is analyzed and conclusions are drawn.
The next chapter presents a practical application of the above described computational procedure. The real-world decision situation, described and formulated as a multiple criteria stochastic ranking of different variants of dsg, is solved.

THE DESCRIPTION OF THE DECISION SITUATION

The existing distribution system of goods (dsg) operated by a Polish electrotechnical company is considered in this paper. The system is composed of 24 warehouses spread all over the country. They are divided into three categories: central level - 1 warehouse, regional level - 12 warehouses and local level - 11 warehouses. A comprehensive evaluation of the dsg leading to the diagnosis of its strengths and weaknesses has been carried out. This strategic diagnosis has included detailed evaluation of different elements and aspects in such areas as: transportation and logistics, infrastructural and technological potential, marketing and sales (customer service), financial analysis, organizational behaviour and human resources. The diagnosis revealed certain drawbacks of the distribution system, such as: under-utilization of available fleet, high level of inventories, low utilization of the warehouse space, highly depreciated warehousing equipment on a regional level, low marketing budget, over-employment, inaccurately defined employees functions, repetition of tasks in different organizational units, high share of holding costs in total distribution costs. Based on these weaknesses the management team of the company decided to redesign the system. It has been assumed that a redesign process should have an intuitive character and a limited number of variants of the distribution system should be constructed.

The project has been divided into two steps. The first step has been focused on designing different components of the distribution system. The following changes have been introduced: relocation and correction of the number of warehouses, redefinition of relations between warehouses, new arrangement of the transportation system, redefinition of the number and type of vehicles and warehousing equipment, introduction of different product portfolios (articles in the stock), reassignment of human resources to tasks. As a result 4 new variants of the real-world dsg have been developed. The existing dsg, denominated as W0, has constituted the benchmark for the remaining variants WI-WIV of the dsg. An object – oriented computer simulation package ExtendSim (Krahl 2003) has been applied to describe the dynamic character of the system behavior, to model the periodic (discrete) changes of its state and to represent some random (stochastic) input components. More information about the simulation modelling of the system and the variants is presented in the papers by H. Sawicka and J. Zak (Sawicka, Zak 2010; Sawicka, Zak 2006). For all variants the simulation software ExtendSim has generated a set of many data records.

Based on them, in the second phase, a systematic and comprehensive evaluation of the set of variants has been carried out. The set of seven evaluation criteria has been proposed. The data records, e.g. delivery time of every order made, utilization of each vehicle delivering products, rotation level of each product stored in each warehouse, corresponding to these criteria, have been collected and their statistical analysis has been carried out. As a result, expected values of these parameters and their variation have been calculated. The following characteristics have been distinguished: K1 - delivery time [days], minimized characteristic; K2 - monthly distribution costs [mln PLN], minimized characteristic; K3 – fleet utilization [-], maximized characteristic; K4 – inventory rotation level [days], maximized characteristic; K5 - utilization of human resources (deviation from the optimum value) [%], minimized characteristic; K6 - share of outsourced transportation orders [%], maximized characteristic; K7 - difference between the levels of investments and divestments [mln PLN], minimized characteristic. The expected values and ranges of variations of these characteristics are presented in table I. The graphical representation of the variants' distance to the ideal point (represented by the most desirable criteria values) and the nadir point (characterized by the most undesirable criteria values) (Vincke 1992) is shown in figure 2.

Table I. The table of performance for each variant of the dsg considered in its redesign process

Criterion		Variants				
		W0	WI	WII	WIII	WIV
K1	Expected value	4	3	2	1	1
	Range of variation*	[3,87; 4,13]	[2,76; 3,20]	[1,78; 2,30]	[0,95; 1,25]	[0,95; 1,13]
K2	Expected value	1,1	1,0	1,4	0,8	0,9
	Range of variation	[0,95; 1,17]	[0,87; 1,05]	[1,24; 1,60]	[0,65; 0,87]	[0,72; 1,04]
K3	Expected value	0,52	0,43	0,50	0,80	0,80
	Range of variation	[0,51; 0,53]	[0,41; 0,45]	[0,44; 0,57]	[0,76; 0,85]	[0,77; 0,83]
K4	Expected value	32	30	28	36	37
	Range of variation	[31,33; 32,67]	[29,84; 30,24]	[27,19; 29,49]	[34,96; 37,44]	[36,68; 37,24]
K5	Expected value	50	50	30	15	10
	Range of variation	[48,80; 52,00]	[48,80; 52,00]	[28,09; 31,51]	[12,57; 17,43]	[8,80; 12,00]
K6	Expected value	21,2	18,6	22,7	0	70
	Range of variation	[19,14; 23,34]	[17,95; 19,17]	[22,54; 22,90]	-	[68,37; 71,71]
K7	Average value	0	-1,0	2,7	7,5	5,8

* *range of variation for 1-α = 0,9*

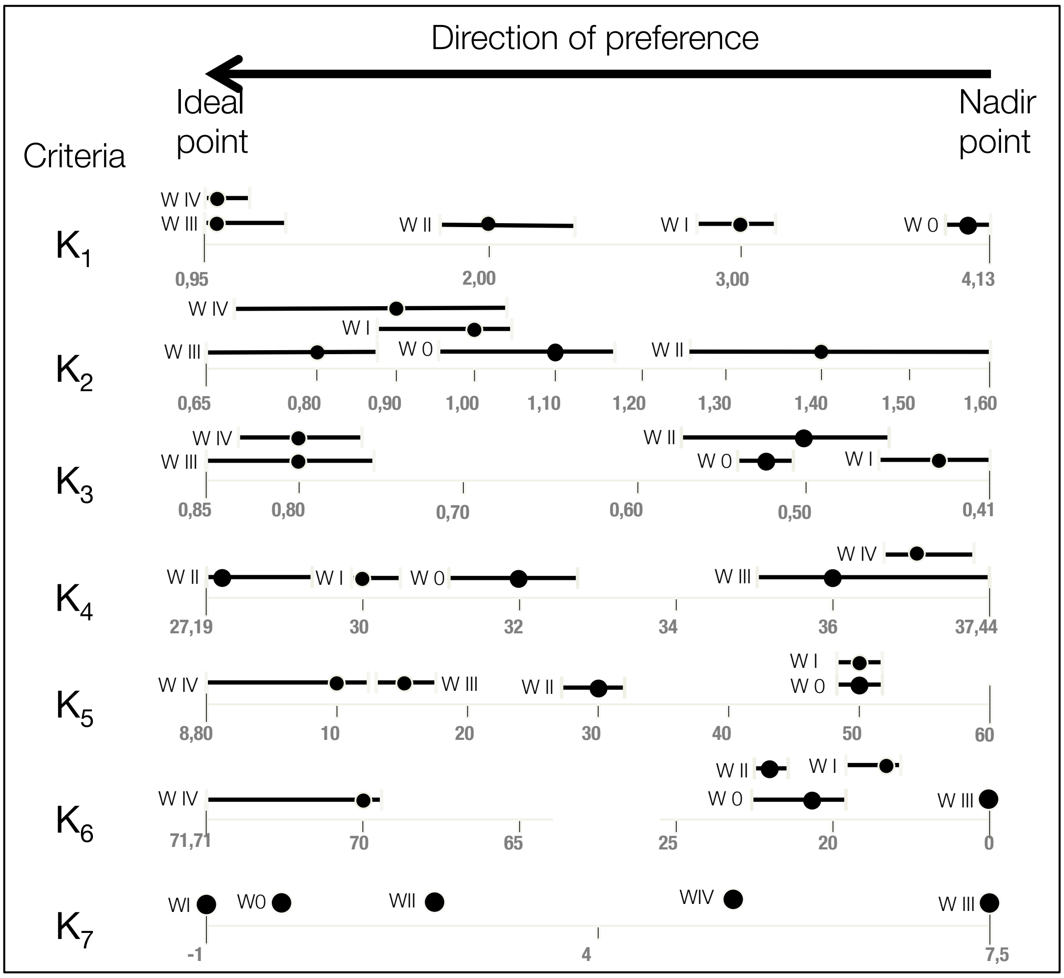

Figure 2. Variants' positions regarding evaluation criteria

Based on the information presented in table I and figure 2 it is hard to decide which variant is the compromise solution. The following aspects have the most important influence on this situation:

- ambiguity of variants' evaluation i.e. some of them have the best values on one criterion while on the other are close to the worst position, e.g. variant WIV has the best value on criterion K6 and its position with respect to criterion K4 is close to the worst;
- ambiguity of the values of variants, which have different spreads of ranges of variation, e.g. variant WII has a very wide range of variation on criterion K3, while the range of variation of variant W0 regarding this criterion is narrow and it is contained in the range of variation of variant WII; thus, it is not obvious which one of these variants is better than the other; this situation becomes more complex while considering the position of variant WI evaluated by the criterion K3 i.e. in a very narrow section of the range of variation, its position is the same as the position of variant WII;
- uncertainty of the variants' mutual position regarding each criterion i.e. in some criteria their ranges of variations are almost evenly distributed between the ideal and nadir point e.g. positions of variants regarding criterion K1, while on the other criteria part of the set of variants is cumulated around one value e.g. variants W0, WI, WII regarding criterion K3 and distant from the other part of variants e.g. WIII and WIV;

- difficulty in evaluating such a large collection of information i.e. many criteria and variants, many data records, different levels of criteria importance expressed by the DM.

Based on these ambiguities, uncertainties and difficulties with interpretation of data the stochastic multiple ranking problem has been formulated and solved with an application of an original computational procedure, being a combination of the deterministic MCDM/A method (Electre III) and Bayes classifier. Computational experiments performed with an application of this procedure are presented in the next chapter.

Computational experiments

In the first step of the computational procedure, based on the set of 5 variants (W0-WIV) and the family of 7 criteria (K1-K7), decision maker's preferences have been modeled according to the principles of Electre III method. They are presented in table II.

Table II The model of DM's preferences for the considered decision problem

Criteria	Weights	Thresholds		
	w	Indifference *q*	Preference *p*	Veto *v*
K1	10	0,60	1,10	2,20
K2	9	0,05	0,10	0,20
K3	5	0,10	0,20	0,30
K4	5	2,00	4,70	8,30
K5	6	6,00	14,00	19,00
K6	4	3,00	5,00	15,00
K7	7	0,50	1,00	3,00

The DM has assigned different weights to criteria, using the scale of 1 to 10 points, representing the least and the most important criteria, respectively. He/she has decided that criterion K1 – delivery time has the greatest importance and, thus assigned the largest value of weight - 10 to this criterion. Another very important criterion for the DM is criterion K2 - monthly costs of distribution with the weight of 9. The least important parameter is criterion K6 – the share of outsourced transportation orders. However, the DM has not totally neglected its role, assigning the weight of 4 to it.

The definition of thresholds has been a complex task. The DM has decided to assign the values of *q, p, v* using the information presented in table I. Thus, the analysis of difference between criteria values has been carried out based on the expected values and ranges of variation, too. The results of this tedious task indicate that the DM is highly sensitive to changes of some values of criteria e.g. K2 (one of the most important criterion), where the indifference threshold equals 0,05, preference threshold equals 0,10 and veto threshold equals 0,20; and his/her sensitivity to changes is rather low regarding other criteria e.g. K6 (the least important criterion), where the indifference threshold equals 3,00, preference threshold equals 5,00 and veto threshold equals 15,00.

In the second step, the random numbers of each criterion value have been generated. For this purpose, the simulation model in simulation tool called ExtendSim has been constructed.

More information about this model is presented in H. Sawicka doctoral dissertation (Sawicka 2012).

The number of iterations generating random numbers of criteria evaluating each variant equals 150. This value corresponds to the number suggested by Marinoni (Marinoni 2005) and it provides large sample of random deterministic parameters. The example of results is presented in table III.

Based on the data records presented in table III one can conclude that the generated values have been randomly dispersed in the ranges of variations and some values equal the minimum or maximum value of these ranges.

Table III Deterministic values of criteria randomly generated in ExtendSim simulation tool

Criteria	Iterations								
	1	2	3	…	50	…	100	…	150
	"WO"	***"WO"***	***"WO"***		***"WO"***		***"WO"***		***"WO"***
K1	3,87	4,07	4,08		4,06		4,06		4,11
K2	1,14	0,98	1,10		0,95		1,16		1,08
K3	0,53	0,52	0,52		0,52		0,52		0,51
K4	31,73	31,43	31,95		31,79		31,36		31,47
K5	50,26	49,08	49,82		50,43		50,91		51,74
K6	23,05	19,71	22,44		19,18		20,20		21,55
K7	0,00	0,00	0,00		0,00		0,00		0,00
	""	""	""		""		""		""
	"WI"	***"WI"***	***"WI"***		***"WI"***		***"WI"***		***"WI"***
K1	2,87	2,79	2,89		3,02		3,10		3,02
K2	0,99	1,04	0,97		0,89		0,96		0,99
K3	0,43	0,43	0,45		0,43		0,45		0,43
K4	30,03	30,14	30,02		30,16		29,95		29,97
K5	51,24	50,40	49,88		48,96		51,23		51,43
K6	19,11	18,15	18,87		18,72		18,79		18,93
K7	−1,00	−1,00	−1,00		−1,00		−1,00		−1,00
	""	""	""		""		""		""
	"WII"	***"WII"***	***"WII"***		***"WII"***		***"WII"***		***"WII"***
K1	2,19	1,94	2,12		2,05		2,28		2,07
K2	1,29	1,31	1,31		1,55		1,49		1,48
K3	0,46	0,47	0,46		0,44		0,47		0,55
K4	28,83	28,04	28,64		29,00		27,78		29,29
K5	29,80	29,80	29,49		30,39		29,43		28,24
K6	22,70	22,71	22,85		22,82		22,66		22,64
K7	2,70	2,70	2,70		2,70		2,70		2,70
	""	""	""		""		""		""
	"WIII"	***"WIII"***	***"WIII"***		***"WIII"***		***"WIII"***		***"WIII"***
K1	1,06	1,24	1,04		1,07		1,09		1,24
K2	0,74	0,82	0,70		0,87		0,79		0,66
K3	0,82	0,84	0,84		0,82		0,80		0,77
K4	36,65	35,49	37,28		36,16		35,82		37,07
K5	16,08	16,72	17,31		14,87		15,07		17,36
K6	0,00	0,00	0,00		0,00		0,00		0,00
K7	7,50	7,50	7,50		7,50		7,50		7,50
	""	""	""		""		""		""
	"WIV"	***"WIV"***	***"WIV"***		***"WIV"***		***"WIV"***		***"WIV"***
K1	0,99	0,97	1,07		1,13		1,06		1,10
K2	0,98	0,94	0,94		0,81		0,80		0,93
K3	0,78	0,81	0,80		0,80		0,80		0,81
K4	37,18	36,76	36,89		36,91		36,90		37,08
K5	8,85	10,35	11,22		11,80		9,76		10,43
K6	69,31	70,55	70,27		69,02		68,37		71,42
K7	5,80	5,80	5,80		5,80		5,80		5,80

In the third step, for the set of data of every iterations (the total number of iterations is 150), the computational experiments with an application of MCDM/A method Electre III/IV method have been carried out.

The final ranking matrices of four randomly selected computational experiments are presented in figure 3.

5	W0	WI	WII	WIII	WIV
W0	I	P-	P-	I	P-
WI	P	I	P	P	I
WII	P	P-	I	P	P-
WIII	I	P-	P-	I	P-
WIV	P	I	P	P	I

6	W0	WI	WII	WIII	WIV
W0	I	P-	P-	P-	P-
WI	P	I	P	P	P
WII	P	P-	I	I	I
WIII	P	P-	I	I	I
WIV	P	P-	I	I	I

7	W0	WI	WII	WIII	WIV
W0	I	P-	P-	I	P-
WI	P	I	P	P	I
WII	P	P-	I	P	P-
WIII	I	P-	P-	I	P-
WIV	P	I	P	P	I

8	W0	WI	WII	WIII	WIV
W0	I	P-	P-	I	P-
WI	P	I	P	P	I
WII	P	P-	I	P	P-
WIII	I	P-	P-	I	P-
WIV	P	I	P	P	I

Figure 3. Randomly selected four final ranking matrices of variants

These matrices present the indifference *I,* preference *P* and reciprocal of preference P^- relations between variants. Particular relations between redesign scenarios vary in some matrices, e.g. the relation between variants W0 and WIII is *I* (indifference) in ranking matrices 5, 7 and 8, while it is P^- (reciprocal of preference) in the ranking matrix 6. Analyzing the total set of data composed of 150 matrices, the authors of this paper have discovered many similar differences and thus, have decided to classify the relations between variants to predefined classes, which have corresponded to the definition of decision attributes. This classification constitutes **the fourth step** of the method. Bayes classifier has been applied to solve the classification problem.

Initially, 150 matrices have been split into 15 sets with 10 matrices in each of them. For each set the relations between variants represented by the description attributes (i.e. indifference *I*, preference *P*, reciprocal of preference P^- and incomparability *R* relations) have been recognized and the probabilities of occurrence of particular relationships in the whole set have been calculated (see table IV). The values of description attributes are in the range [0,1]. Based on them the decision attributes have been constructed. The following rules have been adopted:

- if the probability of the description attribute for a relation between variants has the value greater than 0,5 then the decision attribute is the same as the description attribute;
- if the probabilities of description attributes *I* and *P* for a relation between variants have values 0,5 then the decision attribute is a weak preference relation (Q);

- if the probabilities of description attributes I and P^- for a relation between variants have values 0,5 then the decision attribute is a reciprocal of weak preference relation (Q^-).

As a result 5 decision classes (attributes) $C^{(z)}$ has been distinguished (see table III), such as:

- $C^{(I)} = I$ – indifference relation between variants,
- $C^{(P)} = P$ – strong preference relation between variants,
- $C^{(P-)} = P^-$ – reciprocal of strong preference relation between variants,
- $C^{(Q)} = Q$ – weak preference relation between variants,
- $C^{(Q-)} = Q^-$ – reciprocal of weak preference relation between variants.

Table IV The results of computational experiments of the share of relations I, P, P⁻, and R between pairs of variants for a fragment of testing set and their assignment to decision attributes

	Description attribute				Decision attribute
Relation between variants	I	P	P-	R	Class $C^{(z)}$
W_0 - W_0	1,0	0	0	0	I
W_0 - W_I	0	0	1,0	0	P-
W_0 - W_{II}	0	0	1,0	0	P-
W_0 - W_{III}	0,3	0	0,7	0	P-
W_0 - W_{IV}	0	0	1,0	0	P-
W_I - W_I	1,0	0	0	0	I
W_I - W_0	0	1,0	0	0	P
W_I - W_{II}	0	1,0	0	0	P
W_I - W_{III}	0	1	0	0	P
W_I - W_{IV}	0,3	0	0,7	0	P-
W_{II} - W_{II}	1,0	0	0	0	I
W_{II} - W_0	0	1,0	0	0	P
W_{II} - W_I	0	0	1,0	0	P-
W_{II} - W_{III}	0,2	0,8	0	0	P
W_{II} - W_{IV}	0,2	0,1	0,7	0	P-
W_{III} - W_{III}	1,0	0	0	0	I
W_{III} - W_0	0,3	0,7	0	0	P
W_{III} - W_I	0	0	1,0	0	P-
W_{III} - W_{II}	0,2	0	0,8	0	P-
W_{III} - W_{IV}	0,3	0	0,7	0	P-
W_{IV} - W_{IV}	1,0	0	0	0	I
W_{IV} - W_0	0	1,0	0	0	P
W_{IV} - W_I	0,3	0	0,7	0	P-
W_{IV} - W_{II}	0,2	0,7	0,1	0	P
W_{IV} - W_{III}	0,3	0,7	0	0	P
…					
W_{II} - W_{III}	0,5	0,5	0	0	Q
W_{III} - W_{IV}	0,5	0	0,5	0	Q-
…					
W_0 - W_{IV}	0	0	1	0	P-

Next, the training set T has been randomly selected. It is composed of 10 sets of matrices i.e. around 70% of 15 sets considered in the problem. Based on the information collected in the training set T the a priori probabilities $P(C^{(z)})$ of classes $C^{(z)}$ have been calculated. They are as follows:

- $P(C^{(I)}) = 0,3649635$
- $P(C^{(P)}) = 0,2189781$
- $P(C^{(P-)}) = 0,3430657$

- $P(C^{(Q)}) = 0{,}0291971$
- $P(C^{(Q-)}) = 0{,}0437956$

The conditional a priori probabilities $P(x(a)|C^{(z)})$ have been also calculated. The results are presented in table V.

The conditional a priori probability $P(I \geq 0{,}5|I)$ = 1,00, presented in the table V (column $P(x(a)|I)$) means that the indifference relation *I* is assigned to the pairs of variants when the relation *I* between these variants is at least 0,5. If this relation between variants does not exceed 0,5 i.e. $P(I<0{,}5|I)$ – see table V, column $P(x(a)|I)$, then the conditional a priori probability of the assignment of indifference relation *I* to this pair of variants equals 0,02.

Table V The conditional a priori probability $P(x(a)|C^{(z)})$

$P(x(a)\|I)$	$P(x(a)\|P)$
$P(I \geq 0{,}5\|I) = 1{,}00$	$P(I \geq 0{,}5\|P)$
$P(I<0{,}5\|I) = 0{,}02$	$P(I<0{,}5\|P)$
$P(P \geq 0{,}5\|I)$	$P(P \geq 0{,}5\|P)$
$P(P<0{,}5\|I)$	$P(P<0{,}5\|P)$
$P(P^- \geq 0{,}5\|I)$	$P(P^- \geq 0{,}5\|P)$
$P(P^-<0{,}5\|I)$	$P(P^-<0{,}5\|P)$
$P(R \geq 0{,}5\|I)$	$P(R \geq 0{,}5\|P)$
$P(R<0{,}5\|I)$	$P(R<0{,}5\|P)$

In the next phase of the fourth step, the testing of a training set *T* has been carried out. The remaining 5 sets of matrices i.e. around 30% of all sets considered in the problem, have been utilized in the computational experiments. The conditional a posteriori probability has been calculated for each pair of variants. The example of the results of computational experiments carried out for relation between one pair of variants (W0-WIII) are presented in table VI.

Table VI The results of computational experiments of conditional a posteriori probability of the testing sample between the relation of variants W0 and WIII

	Decision attribute	Description attribute / Conditional a priori probability $P(x(a)\|C^{(z)})$				A priori probability of class $C^{(z)}$	Conditional a posteriori probability
Relation between variants	Class $C^{(z)}$	I	P	P^-	R	$P(C^{(z)})$	$P(C^{(z)}\|x(a))$
W0 - WIII	*P-*	*0,3*	*0,1*	*0,6*	*0*		
	I	0,02	0,02	0,02	0,02	0,364963504	0,000000058
	P	1,00	0,03	0,03	1,00	0,218978102	0,000243309
	P-	1,00	1,00	1,00	1,00	0,343065693	0,343065693
	Q	0,25	0,25	0,25	0,25	0,029197080	0,000114051
	Q-	0,17	0,17	1,00	0,17	0,043795620	0,000202758

The column *Decision attribute* presents the known a priori decision attribute P^- for the considered pair of variants in one testing set. The columns *Description attribute* show the values of description attributes *I, P,* P^- and *R*. Values presented below i.e. for the classes *I, P,* P^-, *Q,* Q^- correspond to the conditional a priori probabilities presented in table V e.g. if the description attribute *I* equals 0,3 (column *Description attribute I* in table 6) then conditional a priori probability that the relation between variants W0-WIII belongs to the class *I* equals 0,02 (column *P(x(a)|I)* as *P(I<0,5|I)*=0,02 in table V).

Based on the values of the conditional a priori probabilities and the a priori probability of class $C^{(z)}$, the conditional a posteriori probability $P(C^{(z)}|x(a))$ has been calculated – see equation 1. The results have been presented in column *Conditional a posteriori probability* of table VI. The highest value of conditional a posteriori probability i.e. 0,343065693 indicates that the decision attribute P^- should be assigned to the considered pair of variants. This result also shows the correct a priori assignment of relation P^- between variants of the testing set.

For each pair of variants from the testing set the classification error β has been calculated – see equation 4. It equals 0,07, which means that the training model has been constructed correctly and Bayes classifier is a good tool for the analyzed problem.

Finally, the vector of observations with a priori unknown decision attributes has been selected. This vector has been composed of 20 relations between pairs of variants (20 relations correspond to the number of relations between variants in one ranking matrix) and has been characterized by the description attributes (see table VII). Based on the description attributes 20 decision attributes concerning the relations between variants have been defined. According to the decision attributes 10 pairs of variants have been classified to class *P*, while the remaining 10 pairs to class P^-. No pairs of variants have been classified to the decision attributes *I, Q* and Q^-.

Table VII. The results of computational experiments of conditional a posteriori probability of the vector of observations with unknown decision attribute

	Description attribute				Decision attribute
Relation between variants	*I*	*P*	P^-	*R*	Class
W_0 - W_I	0,007	0,000	0,993	0,000	P^-
W_0 - W_{II}	0,007	0,000	0,987	0,007	P^-
W_0 - W_{III}	0,467	0,007	0,520	0,007	P^-
W_0 - W_{IV}	0,000	0,000	1,000	0,000	P^-
W_I - W_0	0,007	0,993	0,000	0,000	*P*
W_I - W_{II}	0,007	0,993	0,000	0,000	*P*
W_I - W_{III}	0,000	1,000	0,000	0,000	*P*
W_I - W_{IV}	0,467	0,527	0,007	0,000	*P*
W_{II} - W_0	0,007	0,987	0,000	0,007	*P*
W_{II} - W_I	0,007	0,000	0,993	0,000	P^-
W_{II} - W_{III}	0,267	0,707	0,027	0,000	*P*
W_{II} - W_{IV}	0,267	0,013	0,720	0,000	P^-
W_{III}- W_0	0,467	0,520	0,007	0,007	*P*
W_{III}- W_I	0,000	0,000	1,000	0,000	P^-
W_{III}- W_{II}	0,267	0,027	0,707	0,000	P^-
W_{III}- W_{IV}	0,313	0,000	0,687	0,000	P^-
W_{IV}- W_0	0,000	1,000	0,000	0,000	*P*
W_{IV}- W_I	0,467	0,070	0,527	0,000	P^-
W_{IV}- W_{II}	0,267	0,720	0,130	0,000	*P*
W_{IV}- W_{III}	0,313	0,687	0,000	0,000	*P*

In the fifth step the final matrix and ranking of variants have been generated. The results have been presented in figure 4a – final matrix of variants and 4b – the final graph.

The final relations between variants have been presented in the matrix as bold text. The information of probabilities of occurrence of particular relations between variants have been also presented, e.g. the probability of occurrence of relation P^- between variants W0 and WI is 0,993.

This stochastic ranking matrix based on probability relations between variants has been transformed into the final ranking of variants (see figure 4b). The compromise solution has been presented at the top of the ranking, while the worst redesign scenario at the bottom. Dotted rectangular boxes have been drawn for the variants for which the probabilities of occurrence of preference *P* and indifference *I* relations are close to 0,5. These pairs of variants have been recognized by the analysts as almost indifferent.

In the final step the ranking has been analyzed. The leader of the ranking is variant WI. It is featured by slight changes in the existing distribution system of goods. The second position goes to the variant WIV, which represents the most radical changes in the dsg. The value of preference relation *P* (0,527) of variant WI against variant WIV is similar to the value of indifference relation *I* (0,467) between these variants. In the next position in the final ranking is variant WII. The preference relation between variants WIV and WII is strong and the probability of occurrence of this relation equals 0,720. Variant WII is strongly preferred to variants WIII and W0. The probability supporting strong preference relation between variants WIII and W0 equals 0,520. The indifference relation between these redesign scenarios equals 0,467. Variant W0 has the worst position in the ranking. It represents existing distribution system of goods.

a)

Stochastic ranking matrix

	W0	WI	WII	WIII	WIV
W0	***I*** *I* (1,000)	***P^-*** P^- (0,993) *I* (0,007)	***P^-*** P^- (0,987) *I* (0,007) *R* (0,007)	***P^-*** P^- (0,520) *I* (0,467) *P* (0,007) *R* (0,007)	***P^-*** P^- (1,000)
WI	***P*** *P* (0,993) *I* (0,007)	***I*** *I* (1,000)	***P*** *P* (0,993) *I* (0,007)	***P*** *P* (1,000)	***P*** *P* (0,527) *I* (0,467) P^- (0,007)
WII	***P*** *P* (0,987) *I* (0,007) *R* (0,007)	***P^-*** P^- (0,993) *I* (0,007)	***I*** *I* (1,000)	***P*** *P* (0,707) *I* (0,267) P^- (0,027)	***P^-*** P^- (0,720) *I* (0,267) *P* (0,013)
WIII	***P*** *P* (0,520) *I* (0,467) P^- (0,007) *R* (0,007)	***P^-*** P^- (1,000)	***P^-*** P^- (0,707) *I* (0,267) *P* (0,027)	***I*** *I* (1,000)	***P^-*** P^- (0,687) *I* (0,313)
WIV	***P*** *P* (1,000)	***P^-*** P^- (0,527) *I* (0,467) *P* (0,007)	***P*** *P* (0,720) *I* (0,267) P^- (0,013)	***P*** *P* (0,687) *I* (0,313)	***I*** *I* (1,000)

b)

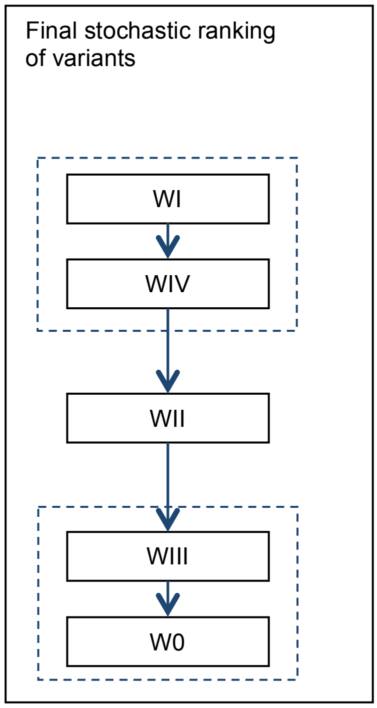

Figure 4. (a)The stochastic ranking matrix representing the mutual relations between variants (b) The final stochastic ranking of variants demonstrating the positions of variants in the graphical form

Based on the analysis of the final stochastic ranking of variants the authors of this paper have proposed the following stepwise path of changes:

- introduction of the evolutionary changes represented by variant WI in the first phase,
- more radical transformation from variant WI to variant WIV in the second phase.

The proposed path should have provide smooth and multistage changes of the distribution system. It could also be redesigned in a long time horizon.

CONCLUSIONS

In this paper the authors proposed a computational procedure that allows solving a real-world multiple criteria stochastic ranking problem and construct the hierarchy of different variants of the distribution system. The algorithm assists the DM in selecting the most desirable concept (variant) of the distribution system, which is considered as a compromise solution.

The authors demonstrated a practical example of the application of this procedure. The decision problem has been formulated as a multiple criteria stochastic ranking problem and it has been based on evaluating and ranking of alternative redesign scenarios of a dsg and final selection of the best candidate.

Based on the proposed procedure the following aspects could have been included while solving the multiple criteria problem:

- incorporating into the computational experiments stochastic data obtained from the simulation of variants and represented by the expected value and range of variation;
- modeling DM's preferences, including the definition of the importance of criteria represented by weights and his/her sensitivity to the changes of the criteria values, represented by thresholds;
- ranking variants from the best to the worst and provide additional information of probabilities of occurrence of particular relations between variants i.e. indifference *I*, preference *P*, reciprocal of preference P^- and incomparability *R*.

Further research should be directed towards the stochastic multiple criteria analysis and evaluation of other systems, distributing such products as: fuel, pharmaceuticals, food. The proposed stochastic MCDM/A method for evaluation and ranking of distribution system redesign scenarios should be verified on a larger set of distribution systems. The application of different MCDM/A methods and different classification methods should be tested. The research should also be guided towards the aggregation of steps and making it less time-consuming and more user friendly.

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