A HEURISTIC FOR DESIGNING INVENTORY-ROUTING NETWORKS CONSIDERING LOCATION PROBLEM AND HETEROGENEOUS FLEET

Esther Segura, Universidad Nacional Autónoma de México (UNAM), Institute of Engineering C.U. Coyoacán 04510, D.F., México, ESeguraP@iingen.unam.mx

Rafael Bernardo Carmona Benítez, Universidad Anáhuac México Norte, School of Business and Economics, Av. Universidad Anáhuac No.46. Col. Lomas Anáhuac, Huixquilican, 52786, State of Mexico, Mexico, rafael.carmona@anahuac.mx

Angélica Lozano, Universidad Nacional Autónoma de México (UNAM), Institute of Engineering C.U. Coyoacán 04510, D.F., México, ALozanoC@iingen.unam.mx

ABSTRACT

This paper presents a model to design inventory-routing distribution networks considering the location of distribution centres (DC) and heterogeneous fleet of vehicles. Specifically, the distribution network under study is characterized by one DC, a set of facilities into regions, and homogeneous fleet of vehicles. Since finding the optimal solutions for an inventory routing problem (IRP) is a NP-hard problem, this paper proposes a methodology based on two phases. The first phase is to locate a number of DC equal to P, and to assign end facilities to such DC. The second phase is to route a heterogeneous fleet of vehicles. In this paper, a new algorithm to route a heterogeneous fleet of vehicles has been developed. The algorithm determines the number of trips between facility pairs. It minimizes the total transportation capacity needed to transport the demanded product in a certain period of time T. The aim is to design the best network based on the minimization of transportation costs, inventory costs, and opportunity costs of having certain amount of product on stock at each facility, while satisfying customer demand according with the best re-order time per facility. In this paper, the case study is based on shipping one type of product from the main facility to the set of DC, and then from the DC to the rest of facilities. The study has been motivated by a real life application related to a company operating in Mexico.

Keywords: *Inventory-routing problem (IRP), location-allocation routing problem (LRP), p-median problem, heterogeneous fleet*

INTRODUCTION

The design of a distribution network faces a number of issues to be optimized. Some of them are: location, inventory management, fleet assignment, and routing. Each of these problems involves decisions with their own characteristics. These models have been solved independently and in combined ways. For example, the Vehicle Routing Problem (VRP) combines routing decision, and fleet assignment with either homogeneous or heterogeneous vehicles (Laporte, 2009; Caric and Gold, 2008; Golden et al., 2010). The Location-allocation Routing Problem (LRP) combines location decision and customer allocation (Wesolowsky and Trustcott, 1975; Halper, 2011; Köksalan and Süral, 1999). The design of distribution networks are intimately linked to problems of location, fleet assignment and routing.

Routing and location decisions are handled simultaneously by the LRP. A classification of LRP can be found in Min et al., (1998). The type of LRP depends on the number of depots to be located, depots' capacity, route length, time constraints, and the form of the objective function. In these problems, the main facility or main distribution centre is generally linked to other distribution centres, and those distribution centres to end facilities. The solution problem locates all the distribution centres and stores. Some location-routing models for realistic scenarios are reported by Melkote and Daskin (2001), Bruns and Klose (1995), Caramia et al., (2007), Lenstra and Rinnooy-Kan (1981), Lin et al., (2006), Gribkovskaia et al., (2007), Laporte et al., (2000), and Ambrosino et al., (2009).

Inventory Routing Problem (IRP) arises when inventory and routing decisions must be taken simultaneously, which yields a difficult combinatorial optimization problem (Halper, 2011; Wesolowsky and Trustcott, 1975; Köksalan and Süral, 1999). The IRP is originated into the distribution and inventory management contexts, where a supplier coordinates the replenishment process of a number of customers. This is the case of vendor-manager inventory systems. Under this strategy, the supplier decides when to visit its customers, how much to deliver to each of them and how to combine them into vehicle routes. Applications include the distribution of liquefied natural gas and ship routing problems (Andersson et al. 2010; Christiansen, 1999). The IRP has received considerable attention in the last decade. A recent literature review was carried out by Andersson et al., (2010).

The IRP can consider the location of a single or multiple facilities; homogeneous or heterogeneous fleet, and different inventory policies. Hence, there is not a unique formulation of this problem, which is highly combinatorial. Then, heuristics techniques have been developed for solving the IRP in polynomial time (Coelho and Laporte, 2012) (Ambrosino, 1999).

There are special cases such as those presented by Coelho and Laporte, (2012), which studies the IRP with multiple vehicles (i.e. MIRP) and solve a set of instances with its own characteristics using the exact branch and bound technique. For this problem, several heuristics have been proposed (Archetti et al., 2011; Bertazzi et al., 2002; Christiansen, 1999).

The main goal of designing distribution networks is determining the best way to transport any product or commodity from supply sources to demand points. The design network structures must satisfy customer demand and minimize the total distribution cost. It is usually expressed in terms of transportation, set up, and inventory costs (Ambrosino et al., 2009).

This paper presents and studies a problem composed of the following sub-problems: DC location, fleet assignment, inventory management at each facility, and routing. Its objective is to minimize transport costs, inventory costs, and opportunity costs of having certain amount of product on stock at each facility. Additionally, the presented problem considers heterogeneous fleet, special location and a particular inventory policy. Then, the presented problem has not been treated in literature in a comprehensive and integrated way. Then, the proposed solution procedure cannot be compared with another one in literature, but it is compared with the current situation.

Statement of the problem

Currently, the company distributes a product from the main distribution centre to other facilities. It uses a homogeneous fleet of vehicles with the same capacity. Figure 1 describes the company's current transportation system.

The product is transported from the main facility (φ) to each facility. Transporting product from φ to each facility implies high transportation costs. It does not consider the opportunity of shipping high amount of product to certain facilities, to try to minimize transport costs per unit and get optimum inventory levels. A homogenous fleet could not optimize the quantity of product to be shipped between facility pairs, as it could be done by a heterogeneous fleet. Transportation costs could be reduced by using the cheapest transport truck. Also, the use of full trucks reduces the difference between truck's capacity and the demand of product in a time T. Company quotations on trip price indicate that trucks with higher capacities have cheaper transport cost per unit than the truck currently used by the company. The company needs to consider the possibility of using trucks that have different capacities, in order to reduce transportation costs.

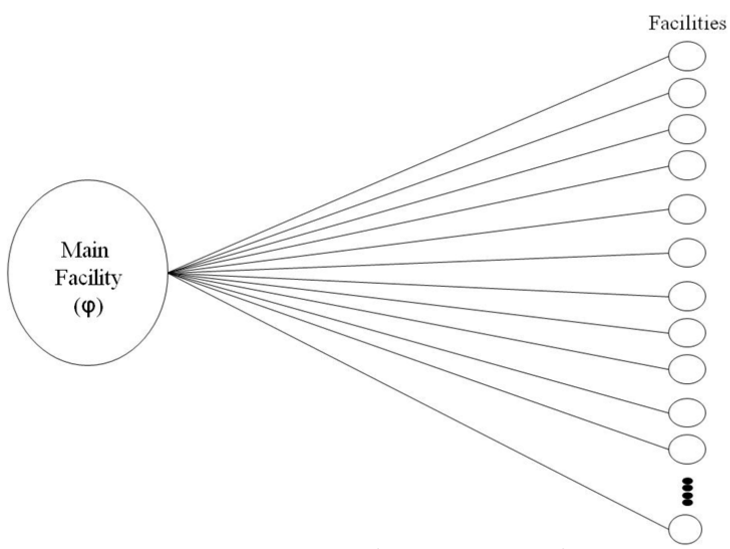

Figure 1. From the main facility (φ) to each facility

Inventory levels are important because they determine when a facility needs to be supplied, and they have an impact on inventory costs and opportunity costs. Inventory management establish a relationship between demand and supply. Time (T) is an important variable because, the determination of the optimum shipment frequencies and number of shipments to each facility depend on T

The nature of the product does not allow shipping to several facilities with the same truck. Each truck can deliver the complete amount of product to just one destination facility. The transportation company charges full fees depending on the truck capacity. It does not matter if the truck is full. Currently, trucks travel on toll highways.

This paper proposes a model to design an inventory-routing network for heterogeneous fleet of vehicles, linked to a DC location problem. The aim is to minimize transport costs, inventory costs, and opportunity costs of having certain amount of product on stock at each facility while satisfying the demand according with the best re-order time at each facility. Figure 2 presents the general structure of this problem solution.

The solution of the previously described problem is a challenge because it is a NP-complete problem. The proposed solution is based on the application of the p-medium problem solved by applying construction and local search heuristics, and a new algorithm to route the heterogeneous fleet. This algorithm has been developed to solve the IRP problem considering the DC location problem and a heterogeneous fleet of vehicles.

The current operation (shipments directly from φ to end facilities) will be compared with several scenarios which consider a set of distribution centres (which send product to end facilities) and heterogeneous fleet.

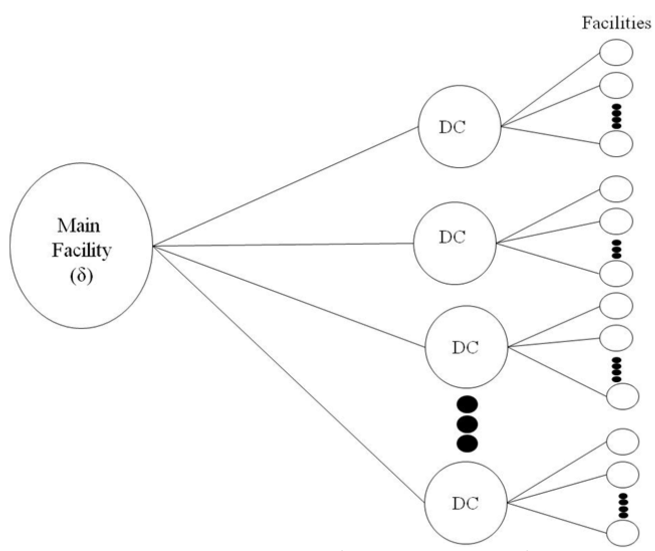

Figure 2. From the main facility (φ) to each facility

The solution of our problem must include the following: the number and location of DC, the set of facilities served by each DC, the shipment frequencies to each facility, the truck type for each shipment to each facility, and the re-order period for each facility.

Main Research Questions

The aims of this paper are the following:

- Proposing a mathematical model for designing inventory-routing distribution networks considering the DC location and heterogeneous fleet of vehicles. The objective function is to minimize transportation costs, inventory costs, and opportunity costs of having certain amount of product on stock at each facility, while satisfying customer demand according with the best re-order time at each facility.

- Designing a methodology to solve the NP-complete problem. This methodology has two phases. The first phase includes the location of P distribution centres and the assignment of end facilities to each DC, by solving a LRP problem. The second phase is to route a heterogeneous fleet of vehicles by solving the IRP problem.

- Designing a network which is better than the current one for the company, based on the minimization of total cost.

In Section 2, the model for the designing of inventory routing networks considering the DC location and heterogeneous fleet is developed. In Section 3, the methodology to solve this NP-complete problem is presented. In Section 4, different case studies are described and solved. Finally, conclusion and references are included.

MODEL TO DESIGN INVENTORY-ROUTING NETWORKS, CONSIDERING DC LOCATION AND HETEROGENEOUS FLEET

Our problem objective is to determine the routes frequencies for a heterogeneous fleet (trucks with different capacity). Routes frequencies are dependent on the demand at each facility. The supply at each facility is limited by its storage capacity, and depends on inventory levels at each facility.

Inventory costs and opportunity costs of having certain amount of product on storage are determined by considering a security inventory and an operative inventory. Inventory level depends on transportation costs, inventory costs, and demand in a time period (T). So, for example, a higher number of shipments by using different capacity vehicles could be a better solution than few shipments by using high capacity vehicles, whereas transportation costs are cheaper than inventory and opportunity costs.

In this case study, the inventory model that calculates the optimum level of inventory for each facility is a Continuous Review Inventory System, with type 1 service level (Equation 1). The company knows the level of on-hand inventory in real time.

The average demand per day (λ) for each facility is also known. The probability (α) of not stocking out in the lead time must be 1 because of company policy. It means that any demand always has to be satisfied. The parameter α is important in the mathematical model because it let to determine the level of on-hand inventory at each facility, always ensuring that 100 percent of demands will be satisfied.

$$s = \left[\tau \quad \sigma_D + D^2\,\sigma_\tau\right]^{1/2} Z \quad = \left[\tau \quad \sigma_D + D^2\,\sigma_\tau\right]^{1/2} F(\alpha \quad)^{-1} \tag{1}$$

Table 1 shows the input data and decision variables for the mathematical model.

Even though demand per day λ is known, the distribution of the product depends on trucks and facilities capacities. The distributions depends on optimum number of trips by trucks with different capacities, facilities capacities, transportation costs, inventory costs, opportunity costs, and demands determined by certain length of time. Then, Time (T) is the most important parameter because the amount of product to supply must fill the demand of an optimum number of days (T) at each facility. T is the parameter that can fix the quantity of product to supply by the mathematical model. T allows calculating the optimum demand to satisfy in T (Equation 2).

$$D = \lambda\, T \tag{2}$$

T is the parameter that decides the optimum time to place an order to satisfy demand at each facility during T. T determines the optimum re-ordering point (R) (Equation 3).

$$R = s + \lambda\, T \tag{3}$$

The mathematical optimization model designs networks by taking into account different vehicles capacity and, facilities capacity. It determines what routes to operate, what fleet of vehicles to use, optimum number of trips per route, optimum demand to supply in time T, and optimum R.

Table 1 – Mathematical model input data and decision variables.

	Name	Variable	Parameter
S	safety stock		X
$D_{0,r}$	average demand at the main facility		X
$D_{i,r}$	average demand at CD's		X
$D_{j,r}$	average demand at CD's		X
$\lambda_{0,r}$	average demand per day at the main facility		X
$\lambda_{i,r}$	average demand per day at CD's		X
$\lambda_{j,r}$	average demand per day at end facilities		X
$\tau_{0,r}$	lead time at the main facility		X
$\tau_{i,r}$	lead time at CD's		X
$\tau_{j,r}$	lead time at end facilities		X
$\sigma_{Dd\ j,r}$	standard deviation of demand		X
σ_{τ}	standard deviation of lead time		X
Z	standard Normal Distribution value for α		X
A	level of service	X	
$C_{i,j,r,w}$	Cost of transporting the product from origin node to destination node		X
$INV_{0,r}$	Investment at the main facility		X
$INV_{i,r}$	Investment at DC's		X
$INV_{j,r}$	Investment at end facilities		X
$Iop_{0,r}$	Operative inventory at the main facility		X
$Iop_{i,r}$	Operative inventory at the DC's		X
$Iop_{j,r}$	Operative inventory at end facilities		X
h_0	Holding Cost or opportunity cost at the main facility		X
h_i	Holding Cost or opportunity cost at DC's		X
h_j	Holding Cost or opportunity cost at end facilities		X
$T_{j,r}$	Time variable	X	
$X_{i,r}$	Decision variable to locate DC's	X	
$Y_{0,i}$	Decision variable to open routes between the main facility and DC's	X	
$Y_{i,j,r}$	Decision variable to open routes between DC's and end facilities	X	
$\delta_{0,r}$	Decision variable to determine if an investment is required at the main facility	X	
$\delta_{i,r}$	Decision variable to determine if an investment is required at DC's	X	
$\delta_{j,r}$	Decision variable to determine if an investment is required at end facilities	X	
$n_{0,j,r,w}$	number of trips between the main facility and DC's	X	
$n_{i,j,r,w}$	number of trips between DC's and end facilities	X	
$n_{e,0}$	number of trips entering to the network through the main facility	X	
W	truck type w 1, …, W		X
W	total number of trucks with different product transportation capacity		X
I	Facilities that work as DC		X
I	set of facilities that belong to region r equal P	X	
J	End facilities j 1, ., K		X
K	set of facilities that belong to region r, K must be equal to I	X	
r	number of regions, r 1, …., R		X
R	set of regions	X	
0	Main facility		X
$Q_{0,i,r,w}$	truck capacity from main facility to CD's		X
$Q_{i,j,r,w}$	truck capacity from CD's to end facilities		X
$Q_{i,j,r,w}$	truck capacity from CD's to end facilities		X
$Q_{e,0}$	truck capacity entering to main facility		X

Table 1. – Mathematical model input data and decision variables.

	Name	Variable	Parameter
$Cap_{0,r}$	Maximum storage capacity at the main facilities		X
$Cap^{N}_{0,r}$	Nominal or actual storage capacity at the main facilities		X
$Cap^{I}_{0,r}$	Extra storage capacity at the main facility if investment is made		X
$Cap^{I}_{0,r}$	Extra storage capacity at the main facility if investment is made		X
$Cap_{i,r}$	Maximum storage capacity at DC's		X
$Cap^{N}_{i,r}$	Nominal or actual storage capacity at DC's		X
$Cap^{I}_{i,r}$	Extra storage capacity at DC's if investment is made		X
$Cap_{j,r}$	Maximum storage capacity at end facilities		X
$Cap^{N}_{j,r}$	Nominal or actual storage capacity at end facilities		X
$Cap^{I}_{j,r}$	Extra storage capacity at end facilities if investment is made		X

The objective function minimizes transport costs, inventory costs, investment costs, and opportunity costs (Equation 4).

$$\sum_{i=1}^{I}\sum_{j=1}^{K}\sum_{r=1}^{R}\sum_{w=1}^{W}\{[C_{ijrw}n_{ijrw}Y_{ijr}+h_j(IOp_{jr}+s_{jr})+h_i(IOp_{ir}+s_{ir})]X_{ir}\ +h_0(IOp_0+s_0)+INV_{ir}\delta_{ir}+INV_{jr}\delta_{jr}+INV_0\delta_0\} \tag{4}$$

The constraints of the problem are described as follows:

The amount of product supplied from a DC to end facilities has to be lower than the storage capacity of end facilities (Equation 5).

The supply to end facilities "j", has to be higher than or equal to its demand in time T_j (Equation 6).

The supply from the main facility (0) to all the DC has to be lower than or equal to the DC's capacities (Equation 7).

The supply from all the DC to end facilities has to be higher than the demand at end facilities in time T_i (Equation 8).

The supply from the main facility (0) to all the DC has to be lower than or equal to the storage capacity of the main facility (0) (Equation 9).

The supply from the main facility (0) to all DC has to be higher than or equal to the demand of all the DC plus the demand of end facilities in time T_0 (Equation 10).

The amount of product entering to the main facility (0) from outside the network has to be equal to the total network demand in time T_0 (Equation 11).

The amount of product entering to the main facility (0) from outside the network has to be lower than or equal to the storage capacity of the main facility T_0 (Equation 12).

The amount of product entering to the main facility (0) from outside the network has to be equal to the demand of product at the main facility plus the amount of product shipped to each DC (Equation 13).

The amount of product entering to the main facility (0) must be equal to the amount of product demanded at the main facility plus the amount of product shipped to each DC (Equation 14).

Let consider "*m*" to be the number of potential DC, and "*p*" the total number of CD. In this study case, $p = m$. Then, the total number of alternative solutions that must be exanimated is: $\binom{m}{p} = \frac{m!}{p!(m-p)!}$. If p increases, the number of possibilities increases exponential making difficult to find an optimal solution in a polynomial time.

Given that the geographical area can be divided into regions, it is possible to find a good solution in a polynomial time. In this study case, p indicates the total number of DC to be located. In each region, p is equal to 1 (Equation 15).

The connection or route is only possible between DC pairs (i) and end facilities in the region where the DC is located (Equation 16).

Every DC from each region must be connected to the main facility (0) (Equation 17).

The number of trips between facilities cannot be negative (Equation 18).

The re-order time at each facility cannot be negative (Equations 19, 20 and 21).

The inventory level of service using a Continuous Review Inventory System, with type 1 service level (Equation 1) must be a number between 0.5 and 1 (Equation 22).

$$\left(\sum_{w=1}^{W} Q_{ijwr}\, n_{ijwr} Y_{ijr}\right) X_{ir} - \left(\left(Cap_{jr}^{N} + Cap_{jr}^{I}\delta_{jr}\right) T_{jr} - IOp_{jr} - \left[\tau_{jr}\sigma_{D_{jr}} + D_{jr}^{2}\sigma_{tjr}\right]^{1/2} F(\alpha_{jr})_{jr}^{-1}\right) \leq 0 \tag{5}$$

$$\left(\sum_{w=1}^{W} Q_{ijwr}\, n_{ijwr} Y_{ijr}\right) X_{ir} - \left(T_{jr}\lambda_{jr}\right) \geq 0 \tag{6}$$

$$\left(\sum_{w=1}^{W} Q_{0iwr} n_{0iwr} Y_{0ir}\right) X_{ir} - \left(Cap_{ir}^{N} + Cap_{ir}^{I}\delta_{ir} - IOp_{ir} - \left[\tau_{ir}\sigma_{D_{ir}} + D_{ir}^{2}\sigma_{tir}\right]^{1/2} F(\alpha_{ir})_{ir}^{-1}\right) \leq 0 \tag{7}$$

$$\left(\sum_{w=1}^{W} Q_{0iwr} n_{0iwr} Y_{0ir}\right) X_{ir} - T_{ir}\left(\left(\lambda_{ir}\right) + \sum_{j=1}^{k}\left(\lambda_{jr}\right)\right) \geq 0 \tag{8}$$

$$\sum_{i=1}^{I}\sum_{w=1}^{W}\left(Q_{0iwr} n_{0iwr} Y_{0ir}\right) X_{ir} + \left(T_0\lambda_0\right) - \left(Cap_0^{N} + Cap_0^{I}\delta_0 - IOp_0 - \left[\tau_0\sigma_{D_0} + D_0^{2}\sigma_{t0}\right]^{1/2} F(\alpha_0)_0^{-1}\right) \leq 0 \tag{9}$$

$$\sum_{i=1}^{I}\sum_{w=1}^{W}\left(Q_{0iwr} n_{0iwr} Y_{0ir}\right) X_{ir} - T_0\left(\lambda_0 + \left(\sum_{i=1}^{I}\left(\lambda_{ir} + \sum_{j=1}^{k}\left(\lambda_{jr}\right)\right)\right)\right) \geq 0 \tag{10}$$

$$\mathrm{Q_{e0}n_{e0}\text{-}T_0}\left(\lambda_0 + \left(\sum_{i=1}^{I}\left(\lambda_{ir} + \sum_{j=1}^{k}\left(\lambda_{jr}\right)\right)\right)\right) = 0 \quad (11)$$

$$\mathrm{Q_{e0}n_{e0}\text{-}}\left(\mathrm{Cap_0^N} + \mathrm{Cap_0^I}\delta_0\text{-}\mathrm{IOp_0}\text{-}\left[\tau_0\sigma_{D_0} + D_0^2\sigma_{t0}\right]^{1/2}\mathrm{F}(\alpha_0)_0^{-1}\right) = 0 \quad (12)$$

$$\mathrm{Q_{e0}n_{e0}\text{-}}\sum_{i=1}^{I}\left(\sum_{w=1}^{W}\mathrm{Q_{0iwr}n_{0iwr}Y_{0ir}}\right)\mathrm{X_{ir}} + \mathrm{T_0}\lambda_0 = 0 \quad (13)$$

$$\left(\sum_{w=1}^{W} Q_{0iwr}n_{0iwr}Y_{0ir}\right)X_{ir} - \sum_{j=1}^{K}\left(\sum_{w=1}^{W} Q_{ijwr}\, n_{ijwr}Y_{ijr}\right)X_{ir} - T_{ir}m_{ir} = 0 \quad (14)$$

$$\sum_{i=1}^{I} X_{ir} = p \quad (15)$$

$$Y_{ijr} - X_{ir} \leq 0 \quad (16)$$

$$Y_{0i} = 1 \quad (17)$$

$$n_{ijwr} \geq 0 \in Z^{+} \quad (18)$$

$$T_j \geq 0 \quad (19)$$

$$T_i \geq 0 \quad (20)$$

$$T_0 \geq 0 \quad (21)$$

$$0.5 \leq \alpha_{jr} \leq 0.999999 \quad (22)$$

X_{ir} are decision variables. They locate all the DC per region (Equation 23). Y_{0ir} are decision variables. They connect the main facility with all the DC (Equation 24). Y_{ijr} are decision variables. They connect all the DC with end facilities (Equation 25).

δ_j are decision variables. They decide if a facility requires an investment (Equation 26). δ_i are decision variables. They decide if a DC requires an investment (Equation 27). δ_j are decision variables. They decide if the main facility requires an investment (Equation 28).

$$X_{ir} \in 1{,}0 \quad (23)$$

$$y_{0ir} \in 1{,}0 \quad (24)$$

$$y_{ijr} \in 1{,}0 \quad (25)$$

$$\delta_{jr} \in 1{,}0 \quad (26)$$

$$\delta_{ir} \in 1{,}0 \quad (27)$$

$$\delta_0 \in 1{,}0 \quad (28)$$

PROPOSED METHODOLOGY

Computationally, the IRP problem is a NP-complete problem. In this paper, a methodology based on heuristics is proposed to solve it, which allows finding a good solution in a polynomial time. This is because a good solution has to be found in a polynomial time. Then, a regionalization of facilities into R regions is proposed, where a DC is located for each region.

The methodology is divided into two phases (Figure 3). In the first one, a Location-Allocation problem (LAP) is solved, i.e. the DC are located and the set of end facilities are assigned to each regions. In the second phase, a routing problem is solved, i.e. the network is designed by means an algorithm that routes heterogeneous fleets.

Location-Allocation Phase

In Phase 1, the methodology solves the location of DC by solving the following questions:

1. How many DC must be located to satisfy their demand plus the demand of end facilities?
2. What facilities must work as DC?
3. What facilities are supplied by each DC?

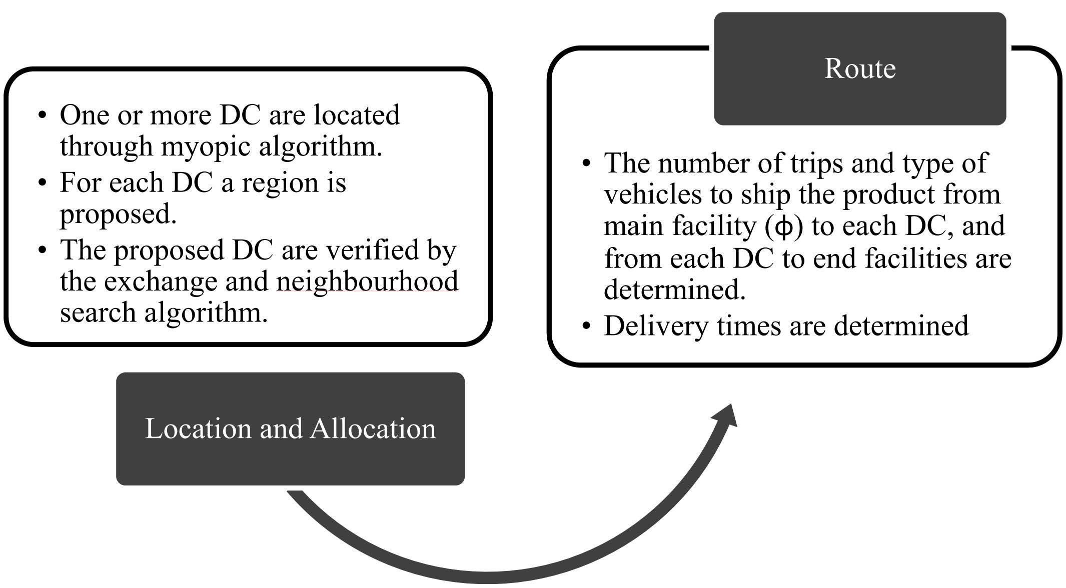

Figure 3. Proposed methodology

The p-medium problem has to be resolved to solve the first phase (LAP problem). The p-median problem can be solved in polynomial time on a tree network (Hakimi, 1965) and on a general graph it is a NP-Complete problem (Kariv and Haimi, 1979).

In this paper, the location problem is solved by applying three classes of heuristics: a myopic algorithm, an exchange heuristic, and a neighbourhood search algorithm. These heuristics can be classified into two different classes (Golden et. al., 1980): construction algorithms and improvement algorithms. The myopic algorithm is a construction algorithm. The myopic algorithm is used to build a first solution. The exchange and neighbourhood search algorithms are used to improve the first solution.

First step: The myopia algorithm identifies DC's by iteration according with the weighted demand (Figure 4). It is the initial basic or first solution. The algorithm stops when it has identified P number of DC's. This solution allows assigning end facilities to DC according with the minimum distance.

Second step: The neighbourhood search algorithm (Figure 5) and the exchange or substitution algorithm improve the initial basic solution (Figure 6). The neighbourhood algorithm checks if other facilities in the region should work as DC rather than the DC selected by the myopia algorithm. It tries to locate the best DC per region. Finally, the exchange or substitution algorithm re-checks if other facilities in the region should work as DC, better than those selected by the neighbourhood search algorithm. This algorithm considers every facility as a potential DC, and determines if a change of DC improves the previous solution.

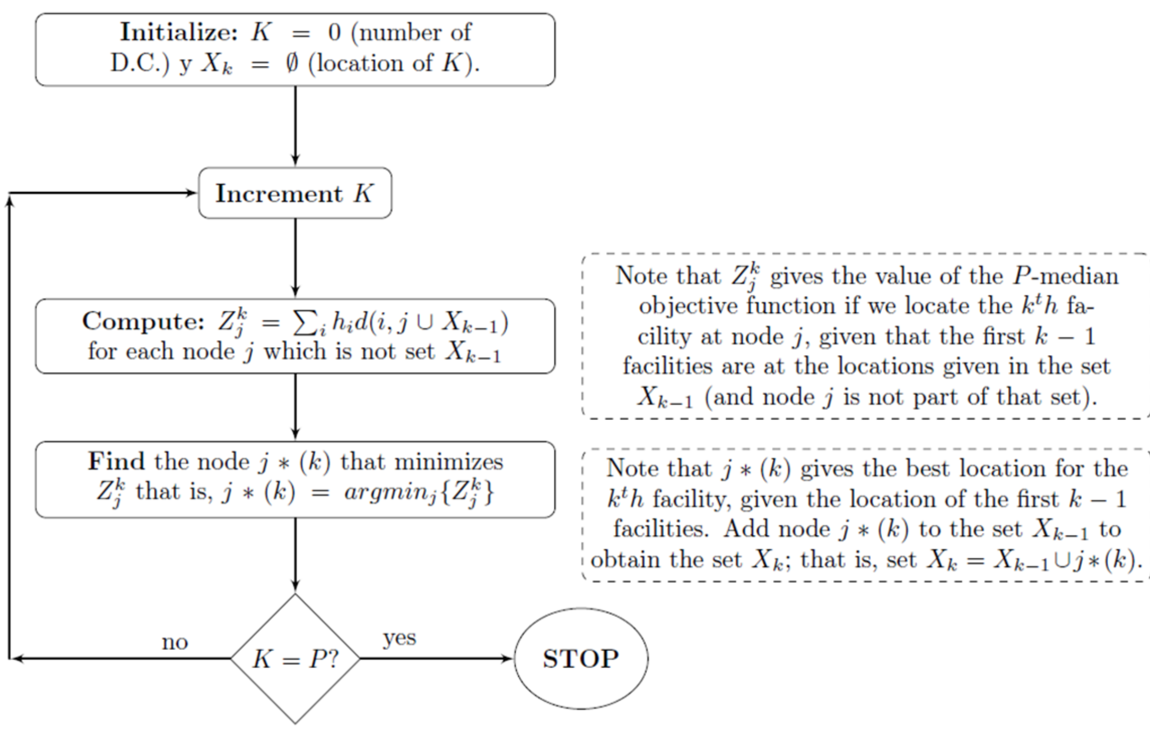

Figure 4. Myopia algorithm (Kariv and Haimi, 1979).

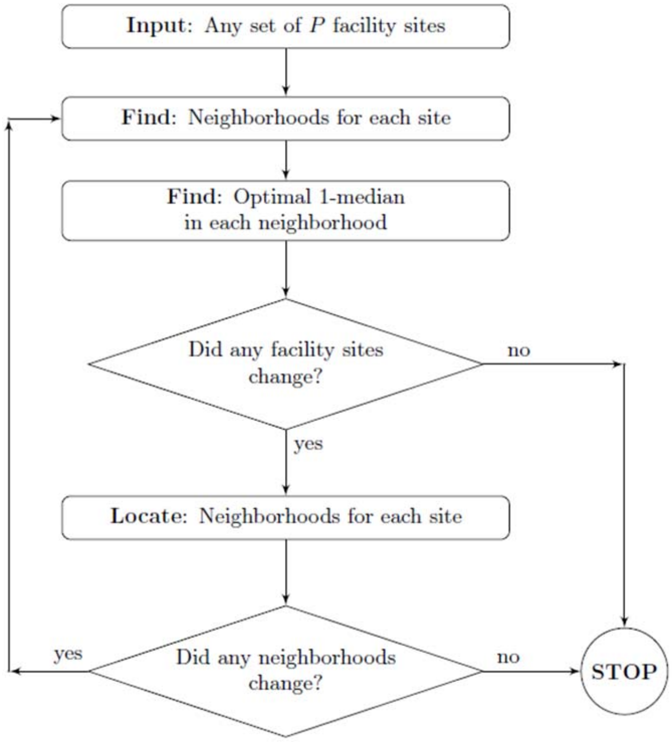

Figure 5. Neighbourhood search algorithm (Daskin, 1995)

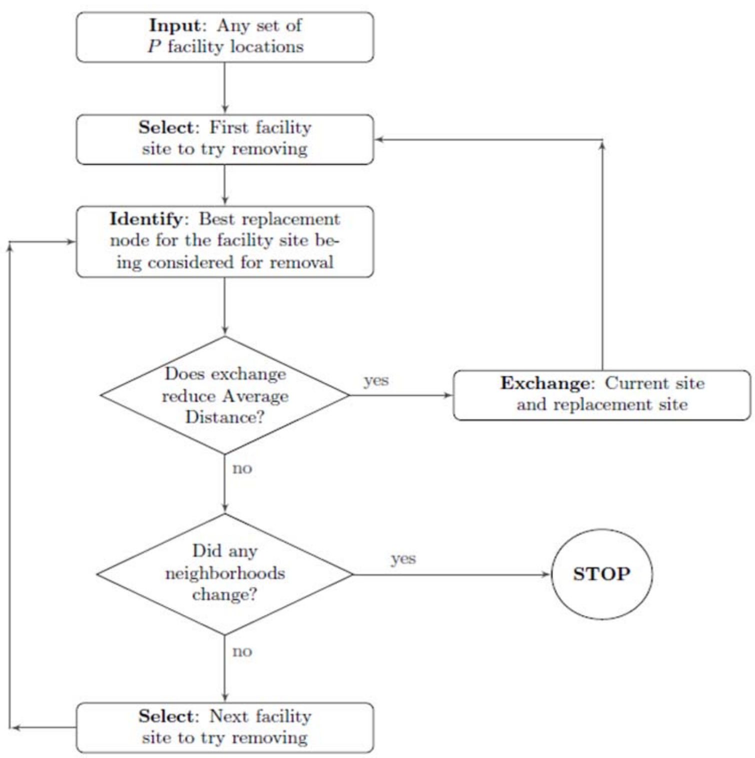

Figure 6. The exchange or substitution algorithm improves the initial basic solution

Routing phase

In Phase 2, the Algorithm routes a heterogeneous fleet.

The nature of the product does not allow a truck to visit several facilities. Each truck can visit just one facility. The transportation company charges full fees. It does not matter if the truck is loaded with few amount of product. Transportation fee depends on the truck capacity. High capacities trucks have cheaper unit cost than lower capacity trucks.

Phase 1 allows freezing facility pairs (φ, i) and (i, j). The algorithm has been designed to calculate the number of trips between each facility pair (φ, i) or (i, j). It also determines the capacity and number of trucks needed to transport the product between each facility pair (φ, i) or (i, j). The algorithm recognizes that full loaded trucks are more economic than shipping partial capacity utilization trucks, as shown in Figure 7 step 2.

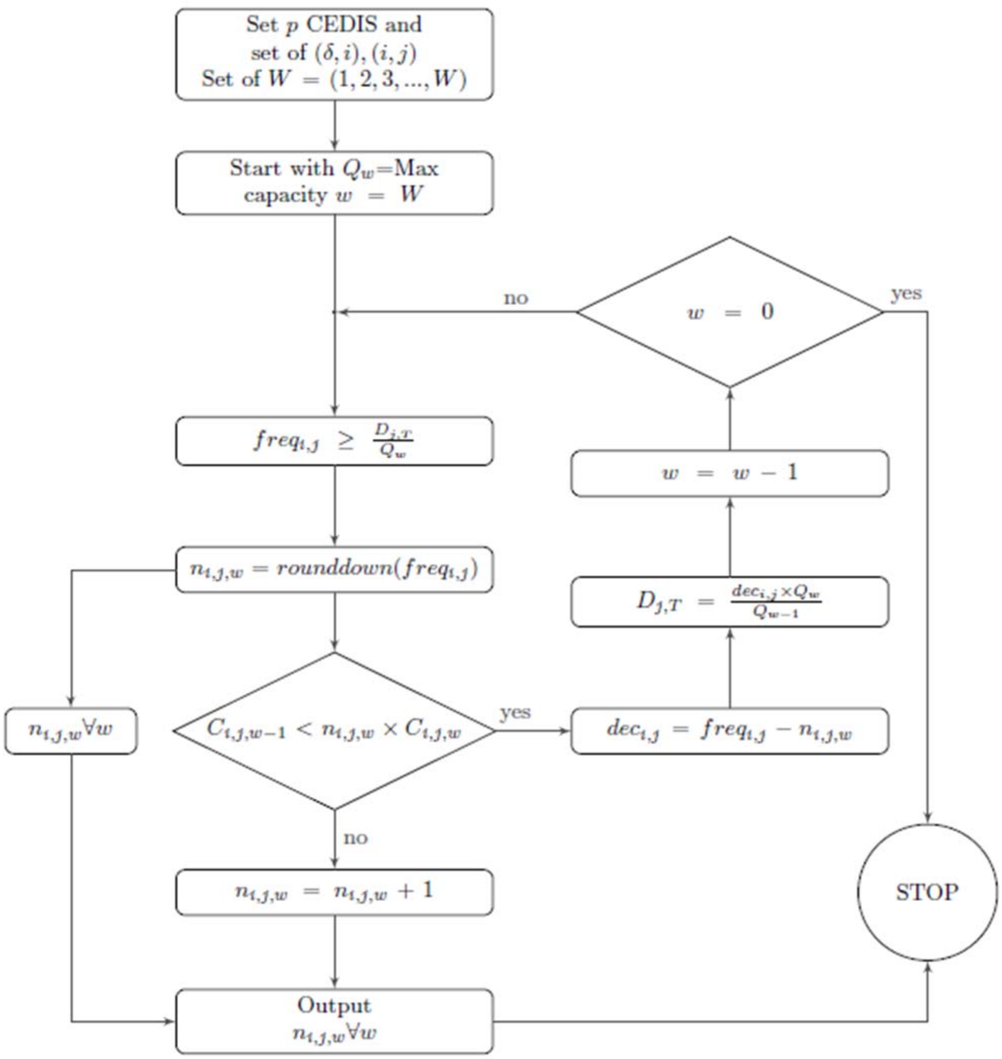

Figure 7. Algorithm that routes heterogeneous fleets

RESULTS ANALYSIS

Nine scenarios have been tested to validate the methodology with homogeneous and heterogeneous fleets. The current situation or current scenario is compared to other scenarios. Current scenario connects the main facility φ with each end facility without using DC. All scenarios consider the real road network hence only real connections between facility pairs are considered.

The results of the sensitivity analysis are shown in Table 2. The best solution is to operate seven distribution centres (p=7), dividing consequently the country in seven regions.

When a heterogeneous fleet of vehicles is allowed, the company achieves a better result (the cost is reduced) than using a homogeneous fleet (see Table 2).

The comparison of using homogeneous and heterogeneous fleets is shown in Table 3. The results prove that the algorithm finds a sub-optimal solution, satisfying demand (total demand is equal to 195.524 million of kilograms).

A total of 6,968 trips (Σn_{ijw}) are needed to transport 195.524 millions of kg using a homogeneous fleet. Capacity per vehicle is equal to 43,000 kg and the total transportation capacity is equal to 299.624 million of kg.

The heterogeneous fleet includes four capacities: 10,000, 20,000, 43,000, and 63,000 kgs. The number of trips per type of vehicle, for heterogeneous fleet, is shown in Table 3. The total transportation capacity is equal to 242.811 million of kg.

Using heterogeneous fleet reduces the total transportation capacity of the network by 19%. Therefore, total demand can be satisfied with a reduced total transportation capacity using heterogeneous fleet.

Table 2 – Mathematical optimization model and methodology results for p = 3, 4, 5, 6, 7, 8, and 9.

Total Cost	Homogeneous (MXP)	Heterogeneous (MXP)
Only φ	6,649,309.93	5,180,192.18
3 DC	4,740,268.73	4,147,797.03
4 DC	4,051,187.06	3,532,254.35
5 DC	3,742,828.72	3,128,572.58
6 DC	3,544,867.86	3,008,869.31
7 DC	**3,465,305.11**	**2,949,488.81**
8 DC	3,476,280.31	3,037,967.14
9 DC	3,474,457.28	3,021,470.56

Table 3 – Comparison of using homogeneous and heterogeneous fleets, p = 7 (resulting from the route algorithm).

Vehicle capacity (kg)	Number of trips homogeneous (n_{ij})	Total transportation capacity (millions of kg)	Vehicle capacity (kg)	Number of trips heterogeneous (n_{ijw})	Total transportation capacity (millions of kg)
43,000	6,968	299.62	10,000	-	1.2
			20,000	60	51.643
			43,000	1,201	189.968
			62,000	3,064	242.811

CONCLUSION

This paper presents the development of a mathematical model to design inventory-routing networks, considering the location of distribution centres and heterogeneous fleet of vehicles. It is very difficult to solve this particular model because it is NP-complete.

A methodology to solve the mathematical model is proposed. The methodology is based on the application of a myopia algorithm, an exchange algorithm, and the development of the algorithm that routes heterogeneous fleet. The myopia algorithm builds a first solution to the LAP problem. The exchange algorithm improves the first solution to the LAP problem solution. The proposed algorithm that routes heterogeneous fleet of vehicles proved to be a useful tool to assign the best fleet depending on transportation costs.

The proposed methodologies solves the proposed mathematical model, and then answer the issues of DC location, fleet assignment, inventory control at each facility, and routing, trying to minimize transportation costs, inventory costs, and opportunity costs of having certain amount of product on stock at each facility.

The final result is a tool developed for a distribution company, useful to design a network to deliver its product, by means of a heterogeneous fleet, reducing costs as possible. Such tool can help companies to establish and ensure a successful business.

Given the real road network characteristics (main road network in Mexico), the country had to be divided into regions, and on CD was located in each region serving all the end facilities in such region.

For the case study, using CD and heterogeneous fleet resulted better than the current situation (without CD and with homogeneous flee).

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