THE GRAPHICAL HIERARCHY PROCESS FOR DECISION MAKING

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ABSTRACT

Decision making and appraisal often involve multicriteria evaluation since several variables are not easy to translate into money and in some cases they are not even quantitative. Any simple multicriteria procedure is based in a utility function that maps the n-dimensional space of the decision variables or criteria over the real segment [0, 1] to produce an ordered ranking. These methods are simple but are too "normative" in the sense that they clearly numerically rank the alternatives in the mentioned [0, 1] segment and decision makers do not like the "dictatorship of the numbers". Any sophisticated multicriteria method that avoids this mapping is usually too difficult to be understood and used by decision makers.

Decision makers may prefer a simple picture or drawing to see which alternatives are best according to different weights for the criteria as opposed to a numerical ranking. We present the Graphical Hierarchy Process, a multi-criteria decision making method based on graphical representations of preference maps: the contribution lays on the hierarchy process rather than in the mathematical algorithm for the aggregation process. By simulating all possible criteria weights with Monte Carlo simulation, the decision maker visualizes at a glance all the possibilities and the relevant issues in the decision making process. By working out a complex hierarchy tree with up to three branches, it is possible to take complex decisions…. with graphics (dots, segments or triangles).

The Graphical Hierarchy Process may be of great interest for decision makers since the appraisal process (evaluation of each alternative according to each criteria) and the hierarchy process are objective and transparent. They still have some maneuvers to make the final decision since they have represented all the possible outcomes of all possible technical weights. This decision making procedure provides a new philosophy to understand and to implement rational decision making in complex environments and it can be of interest for planning in developing countries and to address complex issues like urban mobility.

Keywords: Decision making, multicriteria analysis, graphical hierarchy process, Monte Carlo simulation

MULTICRITERIA DECISION MAKING

Decision making forms part of our everyday life: we often have to make choices that will have more or less influence on our life, and these decision makings can be very important for firms and institutions. In view of a personal decision between several alternatives, a mixed decision process can apply two successive steps (with the corresponding feedbacks), the first guided by the brains and the second by the heart (1). One should select a set of well-defined criteria (quantitative or qualitative), generate the table of partial evaluation and define a benchmarking of minimum values for these criteria; if these values are not reached, the alternative is discarded. After a first pre-selection of a maximum of 3 to 5 alternatives (because of the 7±2 rule of ergonomics does not enable to distinguish more than 5 to 9 alternatives), it is the time for Psychology: the choice with the heart between the alternatives pre-selected by the reason.

But, this process for personal decisions is not extrapolable for institutional decisions. We defend the viewpoint that in complex multi-criteria evaluation processes, it is better to have a simple and transparent process than including obscure aggregation methods dealing with sophisticated mathematic model. We want simple decision methods because the decision making process is complex. Multi-criteria analysis gives a logical well-structured process for decision making.

Any multi-criteria analysis method which is based on a utility function by an explicit or implicit way, is an application between a *m*-dimensional space of the criteria and the segment [0, 1] of the line of real numbers: $U : E^m \rightarrow \Re_{[0,1]}$. This application cannot be bijective for $m > 3$.

Hierarchies are often used to divide a complex reality into groups and subgroups. By this way we can acquire detailed knowledge of the complex reality we are working with. If we are dealing with a complex decision problem, we can use a hierarchy to integrate large amounts of information, and thereby we will improve our knowledge of the situation. The Graphical Hierarchy Process (GHP), as the Analytic Hierarchy Process (AHP) developed by Saaty (2), uses hierarchy to make the process transparent.

The AHP arranges the important components of a problem (the decision goal, the alternatives for reaching it, and the criteria for evaluating the alternatives) into a hierarchical structure. In fact, it reduces the problem to pairwise comparisons between the elements of the decision hierarchy.

The pairwise comparison method measures both ordinal importance (i.e. the order of importance of the list of elements involved) and cardinal importance (i.e. the difference in magnitude between the importance of two elements). In general we compare pairs of alternatives with regard to a criterion which can be quantitative or not, so we need a scale of comparison. The scale defined by Saaty uses the following values for making such comparative judgments:

- 1- equal importance,

- 3- moderately more important,
- 5- strongly important,
- 7- very strongly important,
- 9- extremely more important.

The values 2, 4, 6 and 8 can be used as intermediate levels (we even can use any values in the segment [0, 9] of the line of real numbers, if we can make such distinctions).

The AHP uses comparison matrix to know the weight of each criterion for reaching the goal, and the weight of each alternative under each criterion. In order to have the relative weight of each element, we make pairwise comparisons, and then construct a comparison matrix A, where the element a_{ij} is the (unknown) quotient w_i/w_j. If there are *n* elements to compare, the comparison matrix is a square matrix with *n* lines and *n* columns. This matrix is positive-definite and it is reciprocal: $a_{ji} = a_{ij}^{-1}$, for each *i* and *j*. However, the consistency property: $a_{ij} = a_{ik}a_{kj}$, for all *i*,*j*,*k*, is only approximate. In this case, the absolute weights are given by the eigenvector of the maximum eigenvalue of the comparison matrix A: $Aw = \mu_{max}w$.

The consistency index (CI) is defined by CI = (μ_{max}-1) / (*n*-1). The weight vector *w* calculated as the eigenvector of the comparison matrix is accepted if CI ≈ 0.

With the pairwise comparison method, it is possible to pinpoint the decisions that contribute to the inconsistency of the judgments, and thereby, to review these judgments. In fact, we know that the approximation of the vector's components is better when more consistent is the matrix A. So the AHP allows the participants to re-evaluate their first set of responses and to modify some values in order to be more consistent.

Moreover, a high number of criteria (greater than 7) dilute the polarization of the weights towards the most important criteria such as the economics' criteria. This bias can be reduced by putting the criteria into a hierarchy structure (3).

The idea of the Graphical Hierarchy Process (GHP) is to helping the decision making with simple graphics and transparent hierarchy process. The following section of this paper introduces the "preferences map" for three criteria, whereas the third section details the general GHP method. The fourth section gives examples of graphical representations and discusses about the influence of the criteria's structure (hierarchy) on the results.

GRAPHICAL REPRESENTATION: PREFERENCE TRIANGLE

If there are only three criteria, it is possible to make a graphical representation of the best alternative according to the weight of each criterion (1). Indeed, we can consider the trihedral formed by the positive values of the weights w_1, w_2 and w_3. In the weights space $\{w_i\}$ ($i=1,2,3$), the plane $w_1 + w_2 + w_3 = 1$ intersects this positive trihedral into an equilateral triangle. Putting down this equilateral triangle, it contains the geometric place of the points that represent, by a bijective way, any combination of unit weights (w_1, w_2, w_3) with $0 \le w_i \le 1$ and $w_1 + w_2 + w_3 = 1$. For each combination of weights, one can determine which is the best alternative by estimating the "utility" of each alternative for these weights. It is obvious that the centre of gravity of this triangle corresponds to the case where $w_1=w_2=w_3$.

A "preferences map" is constructed by performing a Monte-Carlo simulation for all the weights (1, 0, 0), (1-δ, δ, 0), (1-2δ, 2δ, 0), ... (1-δ, 0, δ), ... (1-2δ, δ, δ), ... etc. (δ is the weight increment in the simulation, typically $0{,}01 \le \delta \le 0{,}1$).

Figure 1 shows an example of a preferences triangle for three alternatives. The graphical representation enables decision makers to visualize the best alternative according to a combination of weights, and then to choose the alternative that they consider as the best. For example, if decision makers give more value to criterion 1, then they should choose the second alternative. The area of an alternative in the triangle gives the probability to choose this alternative a priori supposing that all the weight combinations are equally likely. In this example, the third alternative is the one with the greatest area.

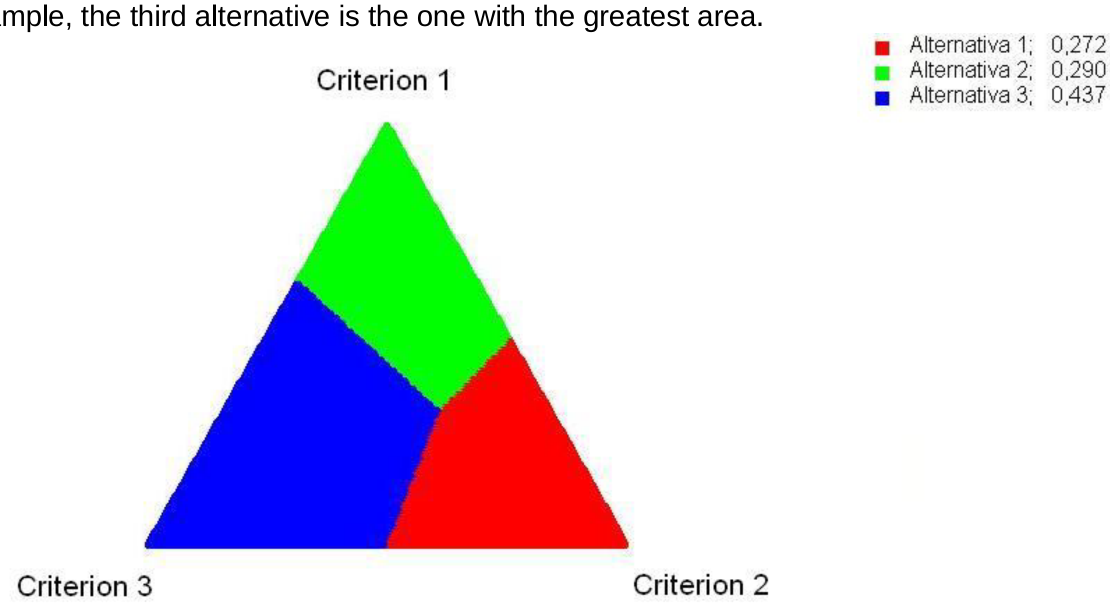

Figure 1 – Example of a preferences triangle for three alternatives

In fact, the aim of the Graphical Hierarchy Process is to provide graphical representations to decision makers whatsoever the number of criteria. However, for more than three criteria it is not possible to find bijective identification between a combination of weights and a dot on the preferences map. That is why the GHP uses a specific hierarchy of the criteria in order to make such representations: the decision tree.

THE GRAPHICAL HIERARCHY PROCESS (GHP)

Let $\{A_1, A_2, ..., A_n\}$ be *n* alternatives evaluated with *m* criteria $\{C_1, C_2, ..., C_m\}$. The GHP supposes that decision makers can organize these criteria into a tree-like hierarchy of a maximum of three branches. In fact, decision makers have to organize the criteria into a ternary tree structure where each node will correspond to a criterion. The trunk of the tree represents the decision, its branches the main criteria for the decision, and all the others nodes are subcriteria.

The GHP considers that decision makers are able to evaluate, by any existing method, each alternative for each criterion that corresponds to a leaf of the tree. These criteria will be call *leaf-criteria*. To make a decision, it is important to know the global weight of each criterion. In fact, the GHP considers that, at a level of the tree, the criteria are 'homogeneous', so that the branches have equal weight. Thereby, if a node has three branches, then each branch has a weight of $1/3$; and if the node has two branches, the weight of each branch is $1/2$.

Figure 2 is an example of ternary tree's structure. It has thirteen nodes (i.e. thirteen criteria) and the node *0* represents the global decision or goal. The nodes *'Eco-mobility'*, *'Private vehicle'* and *'Public transport'* are the main criteria of the decision and their weight is $1/3$. Moreover, the subcriterion *'Tramway'* (node *11*) has a global weight of $\frac{1}{12}$ (indeed its weight is: $\frac{1}{3}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{12}$). Let $a_{j\,11}$ be the value of the alternative *i* for the criterion *11*, and $a_{j\,12}$ be the value of the alternative *i* for the criterion *12*. Then, the value of the alternative *i* for the criterion *10* will be:

$$a_{i\,10}=\tfrac{1}{2}a_{i\,11}+\tfrac{1}{2}a_{i\,12}\,.$$

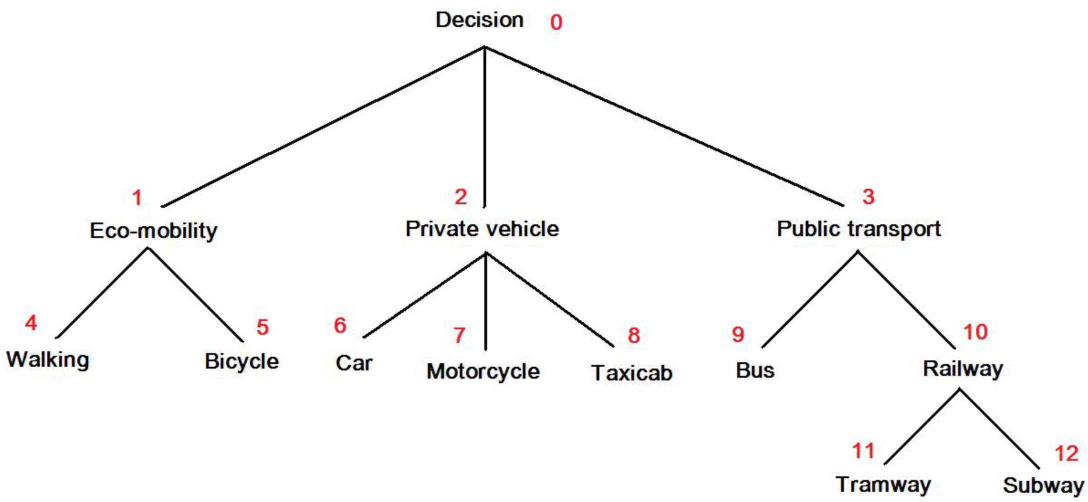

Figure 2 – Example of structure of a ternary tree

In a general way, let a_{ij} be the value of the alternative *i* for the criterion *j*. The value of an alternative i for a criterion *j*, which does not correspond to a leaf of the tree, can be obtained by the following formula:

$$a_{i,\,j}=\frac{1}{m_j}\sum_k a_{i,\,k}$$

where m_j is the number of branches of the node-criterion *j*, and *k* corresponding to the criteria of the branches of *j*.

If *j* corresponds to the trunk of the tree, i.e. *j* represents the decision, then the value a_{ij} is the ranking of the alternative *i*. However, the ranking of an alternative is only a number, and if it is trivial that the best alternative is the one of greater ranking, decision makers do not have a great liberty of choice. Furthermore, they cannot visualise their decision.
In the section 2, we have seen the 'general' case of the GHP: when there are three main criteria. In fact, there are two other cases explained below: two criteria branches and one criterion.

A trivial case is when the trunk has a single branch. Obviously, a single branch can be represented by a point, which will correspond to the best alternative among all. In fact, if there is a single branch, then its weight is 1, so that the value of each alternative for the node or for the branch is the same. However, this representation has no utility because it doesn't give additional information compared with the ranking. In fact, we can easily transform this non-interesting or superfluous case (a weight of 1) into a 'normal' case by putting this node as the trunk of the tree, as it is shown in figure 3.

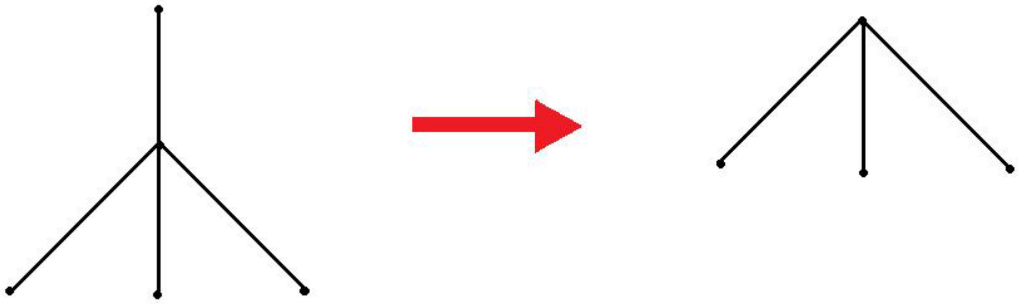

Figure 3 – How to get rid of a (superfluous) single branch

If the trunk of the tree has two branches, then the graphical representation is a segment. To each point of the segment is associated a couple of weight *(w_1, w_2)* where $w_2 = 1 - w_1$ with $0 \le w_i \le 1$ *(i = 1, 2)*. Then, each point of the segment will correspond to the best alternative for this couple of weight. For example, in order to have the best alternative for branches of equal weight, we just have to take the alternative in the middle of the segment.

The following example (figure 4) with two criteria and three alternatives gives an illustration of a possible choice that decision makers may have to make. The alternative 2 is the best for criteria of equal weight: the middle of the segment is in the area of alternative 2. But, one can also understand the choice of alternative 3 because it is the alternative with the greatest area; this choice corresponds to a "blind decision". Finally, if criterion 1 is more important for the decision than criterion 2, the choice of alternative 1 is possible too. Typically, if the criterion 1 represents the economic criterion, most of people would choose alternative 1.

This example shows that the graphical representation offers a liberty of choice in the process of decision making. Indeed, in this case, if decision makers only have the ranking of each alternative, they would certainly choose the alternative 3, while others choices are possible.

Obviously, when we change the tree's structure, we change the hierarchy of the criteria and the weight of each criterion, and so we change the results. Only the number of main criteria determines the shape of the graphical representation (a segment or a triangle). However, all the branches of the tree have an influence on the results and on the graphical representation by determining the value of each alternative on each node of the tree, and thereby their values on the main criteria.

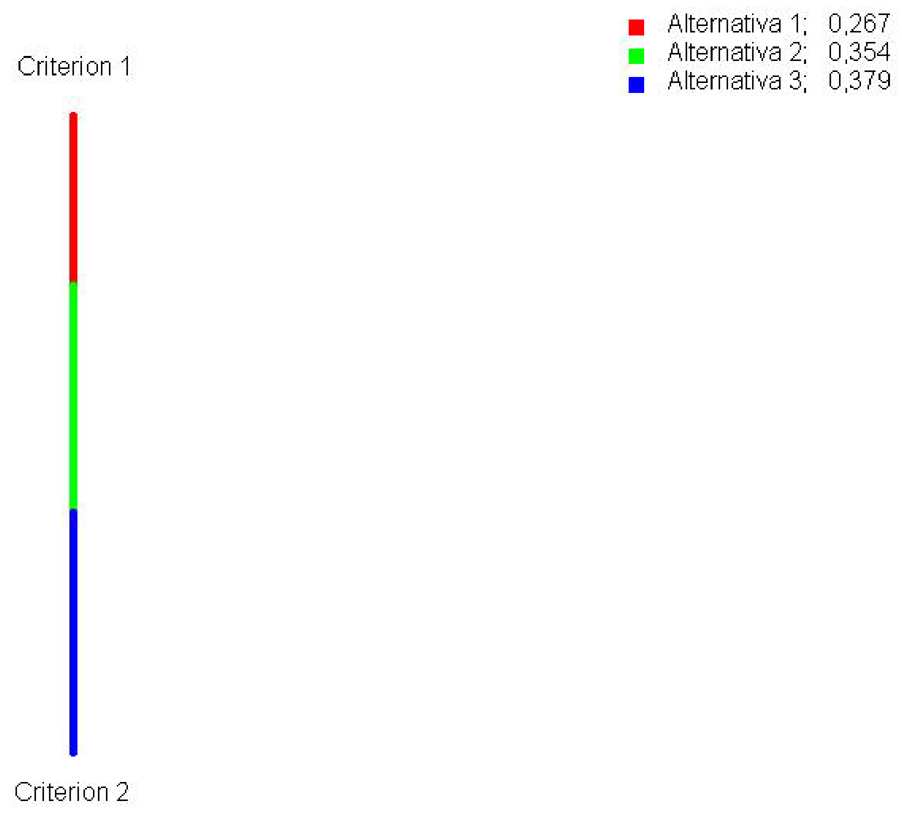

Figure 4 – Example of a decision between three alternatives and for two main criteria

In Figure 5, in both cases, the graphical representation will be a triangle. But, the results can be different according to the values of each alternative on the nodes. The only case where the two triangles will be exactly the same is if the values we get (after calculation) for each alternative, in nodes 1, 2 and 3 of tree Nº 2 are equal to the values (given) of each alternative on nodes 1, 2 and 3 of tree Nº 1.

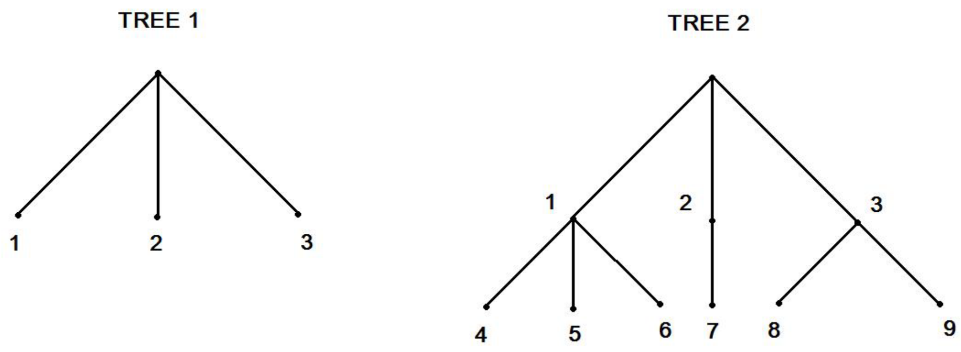

Figure 5 – Two different trees, two different results

In fact, the tree gives the global weight of each criterion. For example, in the figure 6, the weight of criterion X is important in tree Nº 1, whereas it is quite small in tree Nº 2. So the tree's construction process is very important. The greater weight possible is ½ and then it goes down according to the tree's structure.

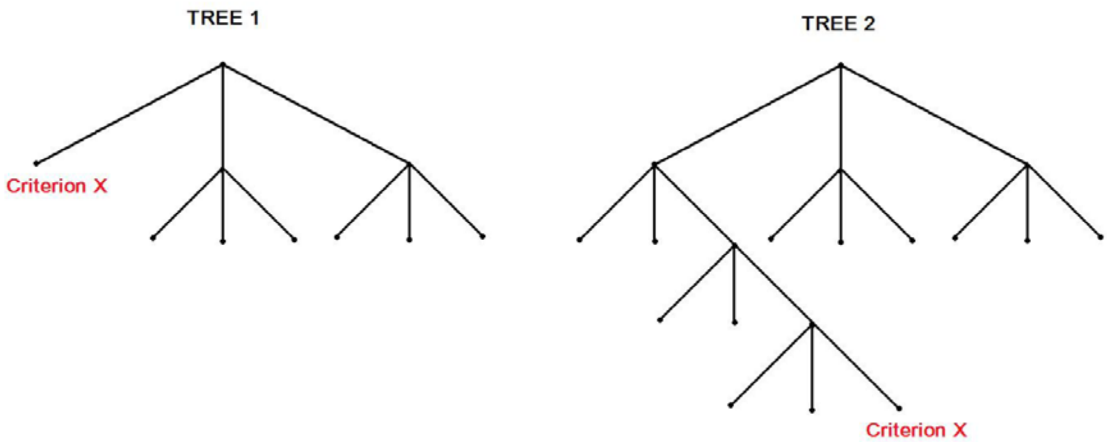

Figure 6 – Link between the tree's structure and the global weight of the criteria

If decision makers really want a criterion with a weight greater than ½, then the graphical representation gives this possibility. Then, making the tree's structure is the more complex step for decision makers. However, it is completely feasible.

The tree's structure has an influence on the results and on the graphical representation, so the hierarchy of the criteria is very important and must be thought seriously. If the tree well represents the relative importance of each criterion on the decision, then the graphical representation will give a 'good solution', that is to say a solution that will well represent the best alternatives according to the 'main criteria' for the decision makers.

The graphical representation gives the possibility to decision makers to see the best alternative according to the weight of the main criteria that they have chosen and to select their 'favourite' alternative (i.e. the alternative they consider as the best one).

Influence of the tree's structure and some examples

The hierarchy of the criteria has an influence on the results because it changes the global weight of the criteria. However, this influence is small when the global weights involved are small too. For example, the decision makers that had made the hierarchy of criteria of the figure 2 may have asked themselves if the structure of the figure 7 would not have been better for their problem. Using the same values for the leaf-criteria, the graphical representations obtained are similar, as seen in figure 8a and 8b. The weights of three criteria (*Bus*, *Tramway* and *Subway*) have changed, and so have done the results: the alternatives 1 and 2 have a greater area in figure 8b than in figure 8a, and the alternatives 3 and 4 a smaller area; but there are not important changes: alternative 3 is still the one with the greatest area.

If the decision makers consider that the criterion *Eco-mobility* is more important than the criteria *Public transport* and *Private vehicle*, they can choose the alternative 2 by looking at the graphical representations of figure 8a or 8b, or they may choose to do another hierarchy for the criteria as shown in figure 9. The graphical representation is a segment, and, keeping the values used for the figures 8, the preferences segment is shown on figure 10. The results are very different from the figures 8: the alternative 3 becomes the one with the smaller area. Obviously, the alternative 2 is still the best alternative for criterion *Eco-mobility*.

So these examples show that if the differences between the criteria's hierarchy of figure 2 and 7 probably would not have a great influence on the decision making, the choice of the criteria's hierarchy of figure 9 instead of figure 2 probably would have.

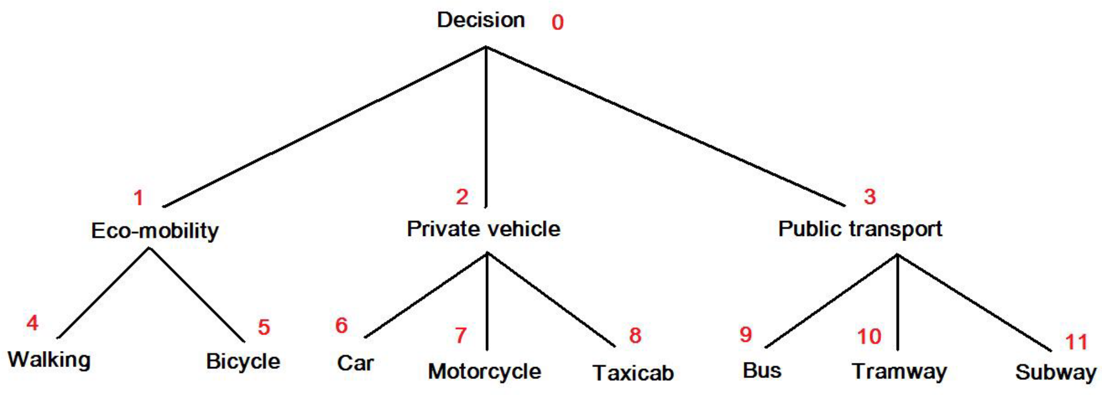

Figure 7 – Another possibility of hierarchy for the criteria of figure 2

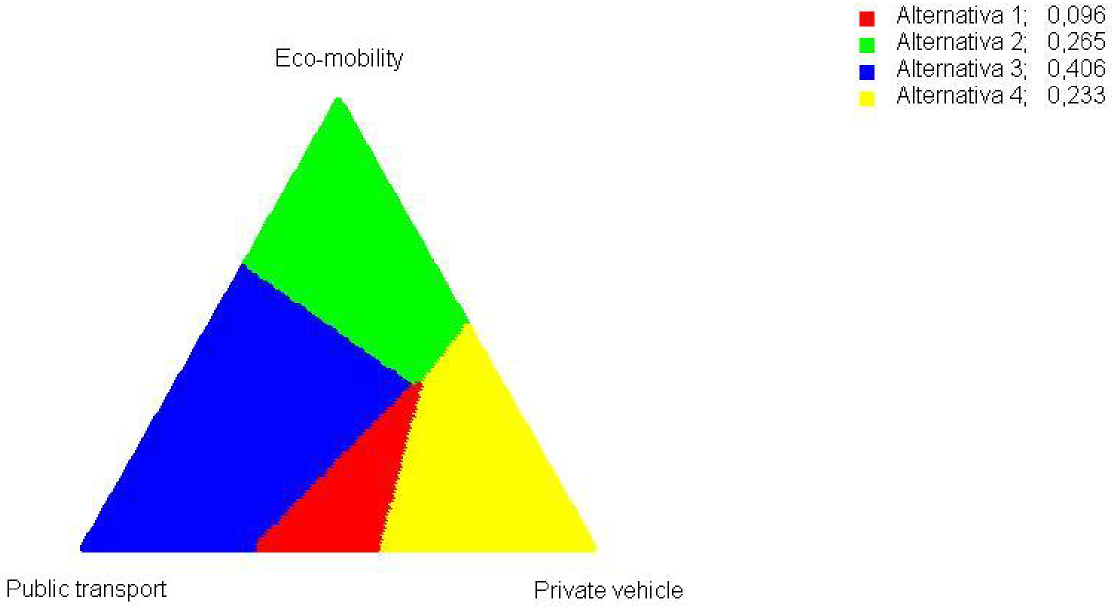

Figure 8a – Preferences triangle for the criteria's structure of Fig. 2

Furthermore, in some specific cases, changes in the criteria's hierarchy have involved important changes on the graphical representations. This occurs when an alternative was the best for one criterion for the first structure and became lightly worse for this criterion than another for the new structure. Then, if this alternative was never better than the others for the others criteria, it can disappear from the graphical representation. However, this occurs when the weight of some criteria – where the alternative had 'good' values – diminished, and thereby they became less important for the decision making; whereas others weights criteria – where the alternative had lower values – increased, and thereby they became more important for the decision making.

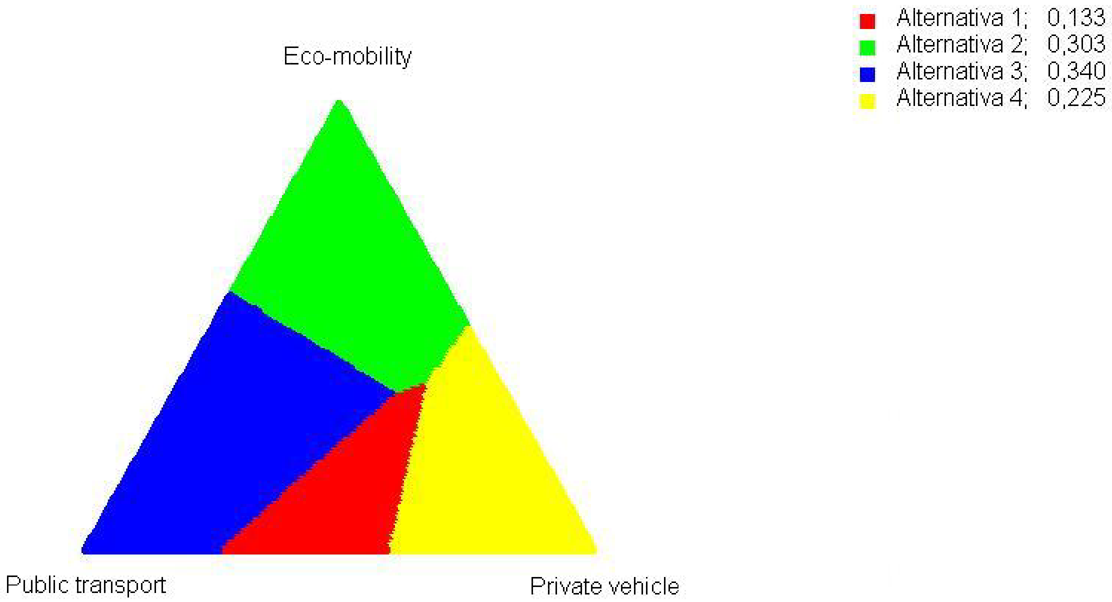

Figure 8b – Preferences triangle for the criteria's structure of Fig. 7

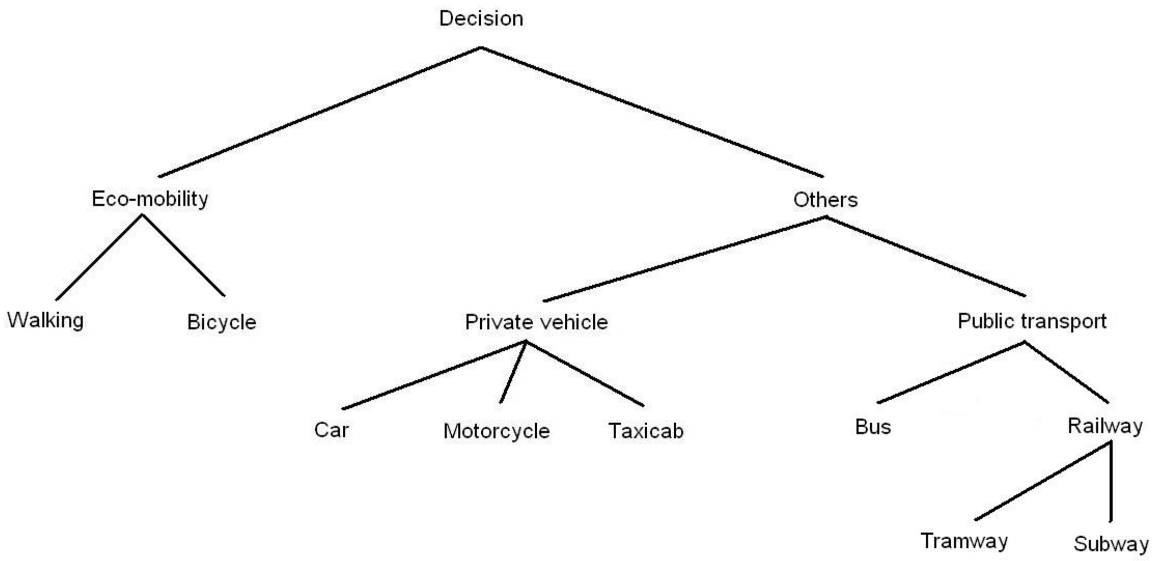

Figure 9 – Another possibility of hierarchy for the criteria of Fig. 2, with two main criteria

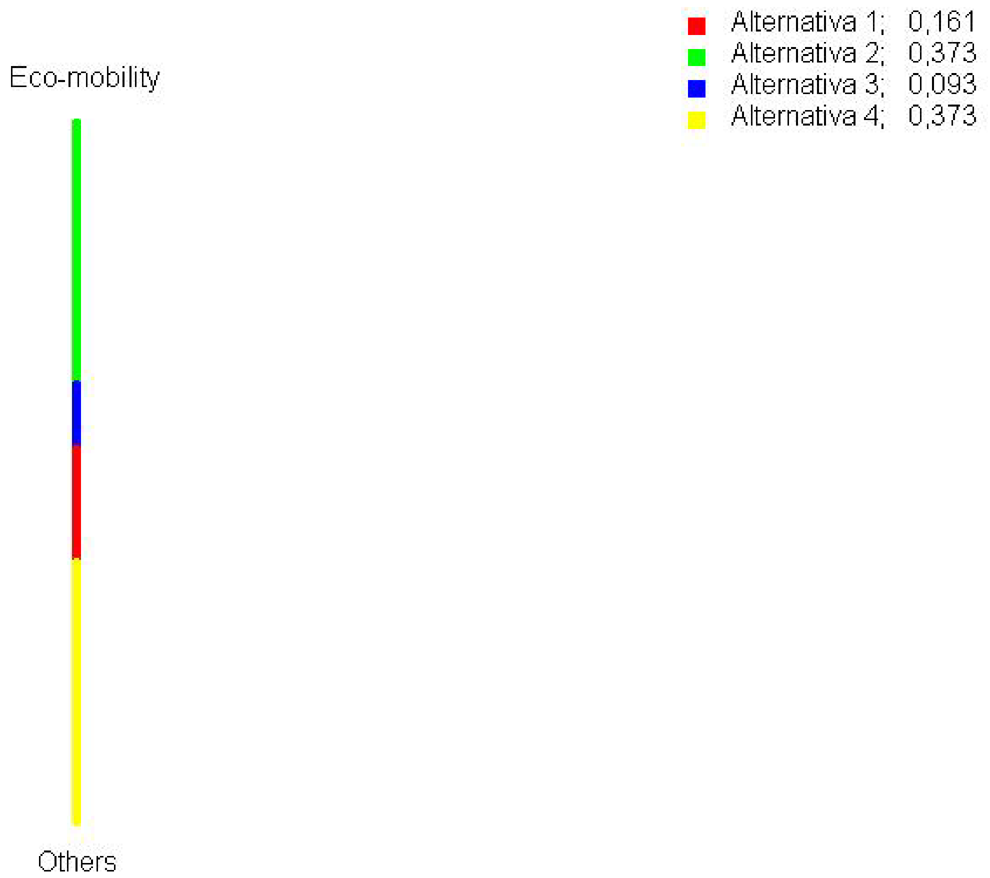

Figure 10 – Preferences segment for the criteria's structure of Fig. 9

CONCLUSIONS

The Graphical Hierarchy Process (GHP) is a new and simple tool for decision making that uses a hierarchy of criteria to make simple graphical representations of the best alternatives according to all the combination of weights of the main criteria. Decision makers have to define the tree's structure (maximum of three branches coming out of a trunk) into a transparent way, and still have a margin of manoeuvre to make their decisions (this does not happen with utility-based decision making methods such as AHP. They also have to know the values of the alternatives for the leaf-criteria. The GHP deduces from this hierarchy the weight of each criterion, and then determines the values of the alternatives for the two or three main criteria of the decision. Finally, a graphical representation is drawn by performing a Monte-Carlo simulation for the main criteria.

Computers can be easily used to calculate the values of the alternatives for the main criteria from the data of the criteria's structure and the values for the leaf-criteria. Software can be used to do the Monte-Carlo simulation and to draw the graphical representation.

With GHP, decision makers can "see" the best alternative according to the weight of the main criteria they have chosen. They have a visual aid to discuss the choice of the alternative with the stakeholders and it is a simple and visual way to communicate or interact with citizens (better than values on a table or rankings of opaque quantitative methods). The criteria's hierarchy has an influence on the results, but this influence depends on the changes made by the decision makers and on the values for the leaf-criteria.

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