THEORETICAL MICROECONOMIC MODEL OF RESIDENTIAL RELOCATION INCORPORATING AGENTS EVOLUTION, INDIVIDUAL LEARNING AND LIFE CYCLE EXPECTATIONS

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Abstract

This paper is focused on the dynamics of the residential location choices based on the microeconomic theory of urban land use in which it is assumed that each property is assigned to the agent (household, firm) that has the highest bid or willingness to pay. A model of residential location choice is developed, where process experience matters in a dynamic learning process and each agent evaluates these locations decisions according to the utility obtained. Additionally, household expectations in relocation decisions is incorporated in a microeconomic formulation by mean of transition probabilities among household clusters in the life cycle and the imitation's hypothesis of such agents under the consideration that they behave rationally. A imitation multi-objective bid function is obtained which includes an expected income per unit of time and a utility consistent with the behavior of the agents that are potentially imitables.

In the learning stochastic model a behavior LOGIT is assumed where willingness to pay are determined by a deterministic part based on the dwelling characteristics, the externalities associated with the zonal distribution in supply, household income, and the urban configuration in the previous period with the inclusion of learning phenomena plus a random part with Gumbel probability distribution. Similarly, we develop a stochastic model with the imitation effect.

Numerical examples and simulations will be presented in the conference using linear bid functions

Keywords: Residential location, learning, imitation, household life cycle, Residential Segregation, multinomial logit, Bid functions.

1 INTRODUCTION

The study of land use dynamics in an urban system, particularly in the residential relocation process, involves the description and modeling of the interaction of a variety of agents that change their characteristics and preferences over time and make different decisions in time and space. Moreover, this dynamic is, among others, the result of issues such as the joint location decision of various household which in turn affects the urban system configuration, causing variations in real estate market behavior. On the other hand, there are some processes associated with making location decisions of households that are not explicitly considered in the classical

models of short and long term urban land use equilibrium such as memory, learning, habit formation, the generation of expectations, uncertainty about resources availability and about the fluctuation and disruption in social and economic changes that establish a complex system with a structure similar to other social and natural systems (Gunderson y Holling, 2002).
Even though households have an internal dynamics such as changes in life cycles, changes in the structure (new children, divorce, job changes, level of education, etc.) which affects the consumption patterns and residential location, and on the other hand, there are variations in the urban land use due to generation of new real estate projects of private and public investment in different areas of a city, it is known that previous experiences are an important element in decision-making (Flórez, 1998) noticing that households tend to remain in the neighborhood where they have lived or in similar housing types (apartments, houses, etc.). In turn it is possible that households tend to remain in a status quo in residential location decision-making over time, because of the knowledge generated by learning the area and the apprehension to change despite of changes in family structure and the environment. In turn, this type of household's life-cycle expected dynamics or transition among possible states or household types are part of a set of possibilities that can be anticipated by like themselves and used as a basis for decision making in urban areas in the short and long term, such as residential relocation and intra-urban mobility (Li and Tu, 2011). Such relocation forces are what Huff and Clark (1977) have called cumulative inertia (resistance to movement) and residential stress given by the possible dissatisfaction with certain attributes of the current household and their surroundings, this dissatisfaction can be generated by changes in the household life cycle.
As this work, there are other analytical and empirical studies that explain the residential relocation dynamics through the previous experiences, or the expectations of change associated with possible future states. For example, in Chen et. al. (2009) the effect of past location decisions is studied by means of the spatial correlation between new and previous decisions showing that there are preferences formed over time and interact with the life cycle; this is modeled through different extreme value distributions. Habib and Miller (2010) propose a model of residential choice founded on the discrete choice theory where individuals decide to relocate based on a reference point, this selection is given by the former location. For this, they postulate a utility function, conditional on the amount of gain or loss for each attribute or feature of the property, with respect to the former location, using concepts of the risk decision theory (Tversky and Kahneman, 1991; Sugden, 2003). Páez, et. al (2008) present a discrete choice model in which agents are modeled through the random utility theory, where each good's utility depends on the decisions made in the past for themselves and a socially related group of agents. For this they use a formulation of the social distance grounded in the social network theory. The problem with this formulation is that the decision made by the agents is the only one analyzed, but not the valuation of each one of the location options or the utility perceived by them. Chen and Lin (2011) develop a life cycle econometric model for residential location and show that the marginal valuation of each property attributes is generated by a learning process that depends on past experiences, showing that this assessment is formed throughout time, using the bounded rationality concepts described in Camerer, (1998). In addition, they analyze the influence of life cycle changes (having children, age, marriage decisions, etc.) in location decisions. In other type of studies, the relocation decision is analyzed as a two-stage process, first as a relocation probability and then as a search for a new option. For example, Nijkamp, et. al (1993) obtain each household's probability of moving (logit binomial) from its current location associated to age changes, changes in the characteristics of the goods and if the agent is owner or not, and given

the information of this probability, a new property demand function is obtained. Altogether, the probability to move into a new zone is obtained and conditional to it the probability of switching to a type of housing within the area. Other several econometric works that based on logistic regression or logit binomial find probabilities to move from the current location by means of the utility thresholds obtained using longitudinal data are Clark et. al., (2003) and Sommers and Rowell, (1992). Lee and Waddell (2010) extend this type of works including sampling in the second stage of the model that corresponds to the selection of the new housing, creating a set of bounded decision alternatives given the relocation probability.

Eluru, et al, (2009) present an econometric formulation using multinomial type choices and a system of equations that defines the reason for moving and the new choice duration. They used a retrospective survey to estimate the model parameters.

Additionally, there are several empirical studies that explain the residential relocation dynamics through effects like the expected salary in the future or the importance given by the agents to the utility drawn by others with the consumption of various goods in urban areas. One way to make these making decision changes is by assessing the anticipation of them, knowing the cluster or household type expectations or future change probabilities, for example, future expected revenues can be used as an estimate of the payment capacity for a household (Kennan and Walker, 2011). In addition, other influential causes in the relocation decision making are the possible changes in the long-term activities in any of the household members in terms of work or education (Hooimeijer, 1996; Clark, and Withers, 1999; Li and Tu, 2011), changes in the household structure by departure or arrival of new members, etc.

An important concept found in some of the relocation models are transaction costs, that contain not only monetary costs but also costs associated with distances; social or psychological losses which are explained with the disutilities of change by Miller and Haroun (2000) that according to Russ (1994) and Kennan and Walker (2011) are costs that increase with the agents' age.

Moreover, some studies in psychology and sociology show that habit formation and learning, as well as socioeconomic changes in the life cycle are important factors when a household decides to make decisions about intra-urban mobility (Rossi, 1955; Ritchey, 1976; Anderson and Milson, 1989; Aarts et. al, 1998).

In addition, there are dynamic economic models that explain and analyze the consumption of continuous and discrete goods of the agents through learning, habit formation, or memory models, analyzing long-term effects and their consequences in the demand thereof (see Milani, 2004). On other hand, in the context of game theory there are theoretical studies of imitation and learning dynamics associated with strategic decision making (Alós-Ferrer and Schlag, 2009). An interesting example in this area is the work developed by Berg (2008) where the firms make the location decisions using information of the profit obtained by other firms previously located. Finally, there are some micro simulation models that attempt to model these dynamics and their influence on the household choice. For instance, Miller and Haroun (2000) have developed a model called ILUTE which simulates the behavior of individual agents in time and space. The overall purpose of ILUTE is to simulate the evolution of an entire urban region for an extended period of time analyzing the effect of changes in the transport system, in the real estate market, and other urban policies.

The objective of this work is to formulate analytically an urban equilibrium problem of residential location that captures the dynamic behavior described above and their effects in the short and long term urban configurations.

In the second section, the theoretical background and initial considerations necessary for the model development with learning or history appreciation are described. In the third section, a discrete choice deterministic microeconomic model associated with residential location is developed incorporating the utility gained by past experiences and the transition between periods; likewise the households' willingness to pay is found when this type of phenomena is considered. In the fourth section, the use of household change's expectations are incorporated in a residential location discrete choice microeconomic formulation the by mean of transition probabilities among household clusters in the life cycle. Finally, there is a section on conclusions and final discussion.

2 THEORETICAL BACKGROUND

Real estate units are characterized by attributes or characteristics called "location externalities" given by the relationship between the distinctive variables that establishes the location of the agents (households, firms, schools, etc.) in a city. As one of these agents is located, changes occur in the physical and / or socio-economic setting in the chosen place; resulting in changes in the perception or the utility of the rest of the agents (Martínez and Araya, 2000). Urban economics is based on two main approaches to explain the urban agents' location choice both founded on the assumption that the housing is a quasi-unique good. In the first, called Bid, an auction-type market is assumed where agents bid for the different locations, which are awarded to the highest bidder (Alonso, 1964). The bid depends on variables such as location or housing attributes, accessibility variables, as well as the income level, consumption and utility level of the agents. The second approach, called Choice, assumes that the agents choose those locations that maximize their utility level (McFadden, 1978; Anas, 1982). Immediately after, the two approaches are briefly explained, making some necessary assumptions for the development of the learning model. Based on Martínez and Araya (2000) and Martínez (1992) the following static and deterministic problem of residential location is assumed using the discrete choice theory (1), where the household type $h \in H$ in a period t selects a real estate $i \in D$ that maximizes its utility,

$$\max_i \max_x \; U_h^t(x, Z_i^t) \text{ subject to } p^t x + r_i^t \leq I_h^t \tag{1}$$

where Z_i^t is the set of attributes of the property indexed by (i). Attributes can be divided into the proper housing attributes and neighborhood quality attributes, and accessibility and attractiveness features (Louviere and Timmermamns, 1990). r_i^t is the rent for the property (i), I_h^t is the exogenous income of the household h, and p^t is the price vector associated with a set of market goods x. Given the optimal solution of the problem (1) the indirect utility function is obtained (2) associated to the real estate (i)

$$V_{hi}^t \equiv V_h^t(I_h^t - r_i^t, Z_i^t, p^t) \tag{2}$$

Given a utility level $\overline{U}_h$, if the inverse function of V_h^t exist with respect to the rent variable then

$$r_i^t = I_h^t - V_h^{-1}(\overline{U}_h, Z_i^t, p^t) \tag{3}$$

Under the consideration of an auction market and the assumption that each property (i) is quasi-unique consumption good, the rent variable can be seen as the willingness to pay (Ellickson, 1981)

$$B_{hi}^t = I_h^t - V_h^{-1}(\overline{U}_h, Z_i^t, p^t) \tag{4}$$

It is easy to demonstrate that if the utility function is quasi-linear, i.e. $U_h(x, Z_i) = ax_0 + f(x_{-0}, Z_i)$ then the functional form of indirect utility function and the willingness to pay are, respectively:

$$V_{hi}^t(Z_i^t, I_h^t - r_i^t) = \lambda_h^t(I_h^t - r_i^t) + \lambda_h^t b_{hi}^t(Z_i^t)$$

$$B_{hi}^t = I_h^t + b_{hi}^t(Z_i^t) - \frac{\overline{U}_h^t}{\lambda_h^t}$$

where λ_h^t is the income marginal utility and $b_{h,i}^t(Z_i^t)$ is a function that measures the property attributes valuation by household side h. The previous expression of B_{hi}^t is denoted in (Martínez and Henríquez, 2007) as:

$$B_{hi}^t = a_h^t + b_{hi}^t(Z_i^t) \tag{5}$$

where the utility level reached, embedded in $a_h^t = I_h^t - \frac{\overline{U}_h^t}{\lambda_h^t}$, is obtained by a market equilibrium condition that every agent should be located. Additionally, if it is assumed that the willingness to pay can be modeled as $\widetilde{B}_{hi}^t = B_{hi}^t + \varepsilon_{hi}^t$, where ε_{hi}^t is identically and independently distributed Gumbel with dispersion parameter μ, and B_{hi}^t is the deterministic part, then the probability that household h be the highest bidder for the property i in the period t is:

$$Q_{h|i}^t = \frac{\exp(\mu B_{hi}^t)}{\sum_{g \in H} \exp(\mu B_{gi}^t)} \tag{6}$$

Using the size correction for each alternative or households cluster in (6) proposed by Mc Fadden (1978), the probability $Q_{h|i}^t$ is:

$$Q_{h|i}^t = \frac{H_h^t \exp(B_{hi}^t)}{\sum_{g \in H} H_g^t \exp(B_{gi}^t)} \tag{7}$$

where H_h^t is the cluster size of household type h in the period t.

This result was proposed by Ellickson (1981) and extended by Martínez (1992) demonstrating that this bid maximization approach is equivalent to the utility maximization approach of McFadden, (1978) and Anas, (1982)The latter in its stochastic formulation represents the probability that a household h chooses an option $(i \in I)$ given by:

$$P_{i|h}^t = \frac{S_i^t \exp(\mu V_{hi}^t)}{\sum_{i' \in I} S_{i'}^t \exp(\mu V_{hi'}^t)}$$

where S_i^t is a deterministic and exogenous supply in each period t of real estate i. In this way, the following household distribution is obtained in a period t given the probability (7) as $H_{hi}^t = S_i^t Q_{h|i}^t$. In the *RB&SM* static model (Martínez and Henrquez, 2007), included externalities associated with the urban distribution in each zone that affects the households' willingness to pay, therefore $B_{h,i}^t(H_{gj}^t, \forall g, j)$, so that $Q_{h|i}^t$ depends on $Q_{g|j}^t$, generating a fixed point system of equations. On the other hand, in the inter-temporal or dynamic extension of such a model of Martínez and Hurtubia, (2006), considered $B_{h,i}^t(H_{gj}^{t-1}, \forall g, j)$, under the hypothesis that each agent's bid in a period t is a function of the urban distribution in $t-1$, preventing the fixed point calculation, and the urban dynamics are discussed given this decision making process, where the valuation is assumed with time-lag, based on an information knowing process. In any case the rent at each location is obtained endogenously by the expected value of the maximum bid, given by the log-sum function:

$$r_i^t = \frac{1}{\mu}\left\{\ln(\sum_h H_h^t \exp(\mu B_{hi}^t) + \gamma)\right\} \tag{8}$$

γ is Euler's constant. Finally, the equilibrium condition ensures that every home locates itself, given by $\sum_i Q_{h|i}^t S_i^t = H_h^t$, obtaining

$$a_h^t = -\frac{1}{\mu}\ln\left(\sum_i S_i^t \exp(\mu(b_{hi}^t - r_i^t))\right), \ \forall h. \tag{9}$$

Since the rents r_i^t depend on each a_g^t then (9) constitutes a fixed-point system of equations described in the form $a_h^t = f(a_g^t, \forall g)$ defining the maximum utility levels possible to get at equilibrium. Note that if the direct utility is quasi-linear, then $V_{hi}^t(Z_i^t, I_g^t - r_i^t) = \lambda_h^t(I_h^t - r_i^t) + \lambda_h^t b_{hi}^t(Z_i^t)$. Seeing that this utility indicates the satisfaction degree of an individual for a real estate (i) it may no affected by the agent's income, then we obtain truncated utility function:

$$V_{hi}^t = \lambda_h^t(b_{hi}(Z_i^t) - r_i^t) \tag{10}$$

3 MEMORY EFFECT AND LEARNING

3.1 DETERMINISTIC PROCESS

In this section, a simple dynamics of endogenous learning for each household will be assumed in the discrete residential location decision-making in period t, given the convex linear combination of the current and past valuation, using the following discrete choice problem:

$$\max_i \max_x \alpha_h U_h^t(x, Z_i^t) + (1-\alpha_h)m_{hi}^{t-1}, \ \text{subject to } \ p^t x + r_i^t \leq I_h^t \tag{11}$$

where m_{hi}^{t-1} is the memory or learning accumulation factor from past experiences (before the time t) in the real estate (i), α_h is the valuation of the property features in the decision period t or the risk aversion or the uncertainty associated with location changes. This modeling assumes that the utility on a real estate is given by a continuous learning process formed from the accumulation of information over a long period of time or where the agents make decisions using information from the past as an extension of the present and future information. For example, the set of activities performed in previous periods generates a learning process on attractiveness and accessibility measures (see Martínez, 1995; Jara-Díaz and Martínez, 1998). The proposed formulation entails a continuous assessment on the part of households about their residential location where factors such as experience may generate slow processes of relocation. In addition, for each period t an endogenous household dynamics related to the housing valuation is defined, but it is worth noting that this problem remains static. It is important to note that it is not intended to seek long-term equilibriums ("perfect foresight") as developed in Anas and Arnott (1991), who do consider a dynamic interaction (from a temporal point of view) between urban agents, forcing to suppose a complete knowledge of future real estate market equilibria by the households and firms (mainly from real estate developers). In this way, this work' contribution is based on the microeconomic formulation, using as a numeric solution base other models' algorithms such as $RB\&SM$ (Martínez and Henríquez, 2007) and its dynamic extension (Martínez and Hurtubia, 2006).

The following expressions or memory measures are nominated to analyze the microeconomic formulation (11):

- **Formulation 1: Myopic memory or learning**

  In the myopic memory assumption, the agent only bases the residential location decision on the real estate characteristics in the current and immediately prior periods, in case of relocating in the same area or in the same housing type of previous period. Mathematically, the memory expression depends on the utility gained in the period $t-1$, given by

  $$mm_{hi}^{t-1} = \mathbf{1}_{h,i}^{t-1} V_{hi}^{t-1}(Z_i^{t-1}, r_i^{t-1}) = \mathbf{1}_{hi}^{t-1} \lambda_h^{t-1}(b_{hi}(Z_i^{t-1}) - r_i^{t-1}) \tag{12}$$

  where $\mathbf{1}_{hi}^{t-1} = \begin{cases} 1 & \text{if in } t-1 \text{ the agent } h \text{ was located in } i \\ 0 & \text{otherwise} \end{cases}$

  So that in period t, previous experience increases utility if the experiences are positive or decreases if they are negative in such a period. Although in the formulation (12) seems that the agent makes decisions based only on the present and immediate past, there is a clearly interrelation or recursion between periods in, since in the previous period his decision was based on two preceding periods and so on.

- **Formulation 2: memory with discount factor or average**

  In this case it is assumed that the household or agent who makes decisions values the previous periods experiences much more, so the following formulation is proposed:

  $$m_{hi}^{t-1} = \sum_{k \le t-1} (1-\alpha_h)^{t-k-1} mm_{hi}^k = \sum_{k \le t-1} (1-\alpha_h)^{t-k-1} \mathbf{1}_{h,i}^k V_{hi}^k(Z_i^k, r_i^k) \tag{13}$$

  where $\sum_{k \le t-1}(1-\alpha_h)^{t-k-1} mm_{hi}^k$ is a weighted average to discount rate $(1-\alpha_h\ )$ over past experiences. Another formulation is the use of the utilities' average associated with previous experiences:

  $$m_{hi}^{t-1} = \frac{1}{t-1} \sum_{k \le t-1} mm_{hi}^k = \frac{1}{t-1} \sum_{k \le t-1} \mathbf{1}_{hi}^k V_{hi}^k(Z_i^k, r_i^k) \tag{14}$$

  Expression (14) loses its meaning or relevance when t is too large due to changes in urban land use such as generating real estate supply, construction of public and private property, changes in transportation systems and infrastructure, as well as the aging of goods, generate considerable variation in the previously known attributes. In this way, each period utility may not be assessed with the same weighting. On the other hand, expressions (13) and (14) allow, in the face of several options for location and a frequent relocations, to grow the utility level t in a for the set of choice options with good previous experiences ($mm_{hi}^k > 0$) and it would decrease in case of negative experiences ($mm_{hi}^k < 0$).

NOTE: The expression $\alpha_h U_h^t(x, Z_i^t) + (1-\alpha_h) m_{hi}^{t-1}$ in each memory formulation case can be viewed as an average utility where $1-\alpha_h$ represents the probability of not seeking a new location for a household h and α_h would represent the probability of seeking relocation, including the current location. Using probabilistic notation and considering an expected equilibrium utility U_h^*, this is

$$\alpha_h = P(m_{hi}^{t-1} \leq U_h^*)$$

In case of obtaining a declining history valuation because of agents or system changes, such as $m_{hi}^k < m_{hi}^{k+1}$, there will be a greater probability to seek new options due to the low utility gained in previous period.■

Note that as m_{hi}^{t-1} is a constant in the consumer problem at t then the agent's bid h with associated memory to the good (i) described by m_{hi}^{t-1} is

$$\overline{B}_{hi}^t = B_{hi}^t + \frac{1-\alpha_h}{\alpha_h}\frac{m_{hi}^{t-1}}{\lambda_h^t} \tag{15}$$

where $\frac{m_{hi}^{t-1}}{\lambda_h^t}$ is the property valuation (i) in $t-1$ divided by the income level obtained in t. That is, each household updates prior experiences assessments to the current incomes increasing or decreasing the willingness to pay for each property. Also $B_{h,i}^t = a_h^t + b_{hi}^t(Z_i^t)$, with $a_h^t = I_h^t - U_h/(\alpha_h\lambda_h^t)$, at equilibrium U_h it will be conditional to the memory level with regard to the chosen property, that is to say $U_h(m_{hi}^{t-1})$ and hence each agent h will reach a different utility depending on past experiences, creating a differentiating factor between them in addition to the proper characteristics of the household in t. Moreover, formulation (15) of the willingness to pay increases the amount of household differences because it includes within attributes of the cluster all the locations background, generating a great variety of attributes. Then, each household will be described by their socioeconomic characteristics in the period t and the characteristics in previous periods that define the perceived utilities in their previous locations. Thus, the proposed formulation provides a useful tool in micro-simulation models, where each agent is analyzed in detail, but in large-scale problems it would be difficult to obtain general behavior rules and even less to analyze asymptotic or long-term equilibirum. To avoid such a large dimensionality of classification variables, in the subsequent analyzes we will only assume a myopic learning for the equilibrium study in each period, allowing obtaining further change or relocation probabilities.

On the other hand, the microeconomic modeling of residential location with memory is an interesting contribution in the social assessment of private or public projects, because two agents of an urban system with identical conditions (residential location and household type) in a period t can perceive the utility differently at to their current residential location because of the past experiences' reference. In this sense, a social planner should include within the analysis not only the perceived utility in the period t, but also how the property characteristics, where every home is located, improve or worsen with respect to previous periods. The formulation of the discrete choice problem with memory proposes a spatial hierarchy associated with past decisions or experience analysis, which is also directly related to the agent's valuation by the perceived utility in previous periods.

3.2 DYNAMIC TRANSITION OF AGENTS

In formulations (12), (13) and (14) of memory or learning it is assumed that the agents do not change cluster between periods, only residential relocation attributes, but an important factor associated with the intra-urban mobility are changes in household characteristics, such as changing jobs or income, change in family structure, for this we assume the following dynamics

of decisions.
Consider a household that changes cluster between periods $t-1$ and t from g in $t-1$ to h in t noted (h, g) in the period t, then under a myopic memory formulation, the willingness to pay for a good (i) can be written as:

$$B^t_{hg(i)} = a^t_{hg} + b^t_{h,i}(Z^t_i) + \frac{1-\alpha_h}{\alpha_h}\frac{m^{t-1}_{gi}}{\lambda^t_h}, \tag{16}$$

where $a^t_{hg} = I^t_h - U_{hg}/(\alpha_h \lambda^t_h)$. That is, as mentioned above, two agents having the same socioeconomic characteristics and located in equivalent real estates in a period t, can have different levels of utility by the generation of the experience or by the evolution of socio-economic characteristics in previous periods. More widely, the households' classification is defined by the following vector of features in the period $t:\ (h, g, j)$. The information that defines the triplet above describes a cluster to which each agent belongs to in t which is h and in the period $t-1$ corresponds to cluster g and the residential location j. For such households the bid function with myopic memory for a real estate (i) in a period t is:

$$B^t_{(h,g,j)i} = a^t_{h,g,j} + b^t_{h,i}(Z^t_i) + \mathbf{1}_{(i=j)}\frac{1-\alpha_h}{\alpha_h}\frac{\lambda^{t-1}_g}{\lambda^t_h}(b^{t-1}_{gi} - r^{t-1}_i), \tag{17}$$

where

$$\mathbf{1}_{(i=j)} = \begin{cases} 1, & if\ i = j \\ 0, & if\ i \neq j \end{cases} \tag{18}$$

Where $\frac{\lambda^{t-1}_g}{\lambda^t_h}$ is an update of the willingness to pay in periods prior to the income in the current period. Note that if $\lambda^{t-1}_g \leq \lambda^t_h$ then $I^{t-1}_g > I^t_h$ so the assessment of the past will be reduced. In turn, if $\lambda^{t-1}_g > \lambda^t_h$ then $I^{t-1}_g < I^t_h$, implying that the valuation of the past will be greater. Furthermore, $\lambda^{t-1}_g(b^{t-1}_{gi} - r^{t-1}_i)$ is the utility obtained in i in period $t-1$. The utility a^t_{hgj} in equilibrium may be different for each agent type(hgj).
In the case of having new agents in the system type h (immigrants) at time t, which will be noted as $\tilde{H}^t_h$ they will have as bid function:

$$B^t_{hi} = a^t_h + b^t_{h,i}(Z^t_i) \tag{19}$$

Namely, it is assumed that the new agents in the urban system have zero memory. Clearly, formulations (17) to (19) pose differentiating elements associated with the household evolution and the learning of the system given by the equilibrium level of utility and the former location. Furthermore, a agent h will relocate in a period t in a real estate i being located in j in $t-1$ so $j \neq i$ if $\alpha_h V^t_{hj} + (1-\alpha_h)m^{t-1}_{hj} < \alpha_h V^t_{hi}$

$$V^t_{hi} - V^t_{hj} > \frac{1-\alpha_h}{\alpha_h} m^{t-1}_{hj} \equiv ac_{hj} \tag{20}$$

Thus, value ac_{hj} can be interpreted as a adjustment cost or pull factor, given as a lower bound associated with the utility valuation in the past experiences. It also defines a resistance factor to change or transaction cost that may includes monetary, psychological and social losses (see, Miller and Haroun, 2000). Conversely, $\frac{1-\alpha_h}{\alpha_h}$ shows the valuation of the past. If $\frac{1-\alpha_h}{\alpha_h} \geq 1$ then

$\alpha_h \leq 0.5$, obtaining that the past experiences assessment is greater in regard than present utility, generated by the uncertainty of knowing the system or the agent's risk aversion h. In addition, it is likely to have $m_{hi}^{t-1} < 0$ due to changes in land used that decrease the utility of the property, for example, location of new unwanted agents (prisons, landfills, etc.) and the dynamic effects that these locations are generating over time.

3.3 STOCHASTIC MODELING

To define the stochastic model it will be assumed that there is an exogenous transition matrix of all agents in the system $P(h^t|g^{t-1})$ that defines the aggregate percentage of change among type of households and it is given by life cycle changes like income, education, children, job change, macro decision-making, exogenous economic shocks to the household, etc. This transition matrix is exogenous in this model. Thus, the number of agents belonging to the cluster defined by (h, g, j) in the period t is

$$H_{hgj}^t = P(h^t|g^{t-1})H_{gj}^{t-1}, \tag{21}$$

where H_{gj}^{t-1} is the urban distribution of equilibrium in the period $t-1$. This distribution is obtained equivalently to what is going to be described below for the period t: assuming that the willingness to pay $B_{(h,g,j)i}^t$ is a random variable, where the modeling error is assumed with Gumbel distribution with scale parameter μ. Additionally, at each modeling period the current income must be greater than the bid:

$$\bar{B}_{(h,g,j)(i)}^t \leq I_h^t, \quad \forall (h, g, j) \tag{22}$$

If the constraint (22) is not met, the agent (h, g, j) won't enter to the set of potential bidders for the household (i) during the period t.

For new agents in the system we have:

$$\bar{B}_{(h)(i)}^t \leq I_h^t, \quad \forall h \in \tilde{H}_h^t \tag{23}$$

Constraints (22) and (23) are important insomuch as in cases where the memory assessment is very high in some goods, the willingness to pay cannot exceed the current income, thus, if an income decrease between periods is faced, there is no bid for goods that had past experiences where the income was higher, that could generate relocation processes associated with economic shocks such as job loss of one or more members of the household or increased spending on consumer durables (car, education, etc). These restrictions are may be included in the modeling through the approximation obtained by the *Constrained Logit* (Martínez, et al 2009) with "*cut-off*" functions defined as the probability that a household of a cluster h accomplishes in each i, denoted as $\phi_{(h,g,j)(i)}^t$, given by

$$\phi_{(h,g,j)(i)}^t = \frac{1}{(1 + exp(w(\bar{B}_{(h,g,j)(i)}^t - I_h^t + \zeta)))} = \begin{cases} 1, & if \ \ (\bar{B}_{(h,g,j)(i)}^t - I_h^t \rightarrow -\infty) \\ \eta, & if \ \ (\bar{B}_{(h,g,j)(i)}^t - I_h^t \rightarrow 0) \end{cases} \tag{24}$$

The value of w determines how fast the probability value approach extreme one or zero values. The parameter ζ represents the tolerance of violating the income restriction, if the bid is approaching to the income level then the factor $\phi_{(h,g,j)(i)}^t$ tends to η. To achieve this we use the parameter ζ, defined by:

$$\zeta = \frac{1}{w} \ln((1 - \eta)/(\eta)) \tag{25}$$

Analogously the probability of reaching the new agents' constraint in the system is defined as:

$$\phi^t_{h(i)} = \frac{1}{(1 + exp(w(\bar{B}^t_{h(i)} - I^t_h + \zeta)))} = \begin{cases} 1, & if \ \ (\bar{B}^t_{h(i)} - I^t_h \rightarrow -\infty) \\ \eta, & if \ \ (\bar{B}^t_{h(i)} - I^t_h \rightarrow 0) \end{cases} \tag{26}$$

Given the heterogeneity in each cluster size of consumers and the variety of them generated by the inclusion of the memory effect as part of the homes' description, the set of consumers looking for location and meeting the feasibility condition is $P(h^t|g^{t-1})H^{t-1}_{gj}\phi^t_{(h,g,j)(i)}$ for agents type (h, g, j) and there will be $\tilde{H}^t_h \phi^t_{h(i)}$ new household type h that might be located in (i) in the period t.

McFadden's argument is used to introduce these heterogeneous household sizes (McFadden, 1978), obtaining a corrected probability that the household (hgj) be the highest bidder in (i) is

$$Q_{(h,g,j)|(i)} = \frac{P(h|g)H^{t-1}_{gj}\phi^t_{(h,g,j)(i)} \exp(\mu \bar{B}^t_{(h,g,j)(i)})}{A^t_i}, \tag{27}$$

where

$$A^t_i = \sum_{h',g',j'} P(h'|g')H^{t-1}_{g',j'}\phi^t_{(h',g',j')(i)} \exp(\mu \bar{B}^t_{(h',g',j')(i)}) + \sum_{h'} \tilde{H}^t_{h'}\phi^t_{h'(i)} \exp(\mu \bar{B}_{h'(i)})$$

And additionally the probability that a new agent h^t be the highest bidder is

$$Q_{(h,0)|(i)} = \frac{\tilde{H}^t_h \phi^t_{h(i)} \exp(\mu \bar{B}^t_{h(i)})}{A^t_i} \tag{28}$$

Thus, the aggregate probability that a household type h be the highest bidder and locates on the property (i) is

$$Q^t_{h|i} = Q^t_{h,0|(i)} + \sum_{g,j} Q^t_{h,g,j|(i)} \tag{29}$$

The formulation of $Q^t_{h|i}$ differs mathematically from the one stated at the $RB\&SM$, model due to memory effect. The probability of relocation for (h, g, j) in the period t is:

$$P(move|hgj) = 1 - Q_{(h,g,j)|j}$$

Additionally, the aggregate population distribution in the period t is:

$$H^t_{h,i} = Q^t_{h|i} * S^t_i \tag{30}$$

In this modeling proposal, probabilities (27) or (28) can only be evaluated in time period equilibrium, with atributes of bids obtained by means of fixed-point algorithms. In the dynamic model it is assumed that each b^t_{hi} depends on $H^{t-1}_{h',i'}$ under the lag assumption, then calculation of the fixed-point associated with the demand or urban distribution in each time period will be avoided. That is, an agent will make decisions with a myopic memory in a period t according to the urban distribution in $t-1$ and $t-2$ (memory effect).

Equivalent to (8), the rent in (i) is obtained as follows:

$$r^t_i = \frac{1}{\mu} \ln(A^t_i + \gamma) \tag{31}$$

where γ is Euler' constant.

Moreover, given the following equilibrium equation for households with memory

$$\sum_i Q_{h,g,j|i} * S_i = P(h|g)H_{gj}^{t-1}, \ \forall \ (h,g,j) \tag{32}$$

and the equilibrium equation for new agents in the urban system:

$$\sum_i Q_{h,0|i} * S_i = \tilde{H}_h^t, \ \forall \ h \tag{33}$$

Then the utility level reached in the period t by an agent (hgj) is obtained with from following equation:

$$a_{hgj}^t = -\frac{1}{\mu} \ln \left(\sum_i S_i^t * \phi_{hgj(i)} \exp \left(\mu \left\{ b_{hi}^t + \frac{1-\alpha_h}{\alpha_h} \frac{m_{g,j=i}^{t-1}}{\lambda_h^t} - r_i^t \right\} \right) \right), \ \forall (hgj) \in H_{hgj}^t \tag{34}$$

and equivalently for new households we have

$$a_h^t = -\frac{1}{\mu} \ln \left(\sum_i S_i^t * \phi_{h(i)} \exp \left(\mu \left\{ b_{hi}^t - r_i^t \right\} \right) \right), \ \forall (h) \in \tilde{H}_h^t \tag{35}$$

As $\phi_{hgj(i)}^t$ depend of a_{hgj}^t ($\phi_{h(i)}^t$ depend of a_h^t) and the rent r_i^t depends on all levels of $a_{h',g',j'}^t$ of the agents with memory and the utility level of all the new agents $a_{h'}^t$ then (34) and (35) define a nonlinear fixed point system of equations in each period t.

Some numerical simulations will be presented in the conference based on classical models of residential segregation that seek to explain the presented modeling and its effects in making location decisions

4 EXPECTATION EFFECT IN RESIDENTIAL LOCATION

In this section the expectation or imitation effect on residential location will be analyzed, it is important to note that there are several factors that explain the reason the decisions and behavior of some economic agents (firms, schools, households, people) are similar to the decisions made by other different agents or with other characteristics. For example, the households that belong to a socio-economic cluster type incorporate, within their valuations or preferences for location, preferences that other socially related households (Páez et. al. 2008) may have or other agents make decisions based on future income, employment, education, and others expectations (Kennan and Walker, 2011). In the context of the housing choice or residential relocation, all these behaviors will be called imitation, similar to the process studied in game theory and strategic behavior (Alós-Ferrer and Sclag, 2009). To understand this effect on residential location choices and the urban equilibrium in the short and long term, we propose to incorporate the expectations associated with the life cycle dynamics to this type of choices, which can be obtained through an endogenous decision (changing jobs, having children, etc.) or an exogenous schoks (layoffs, economic shocks, etc.) of the households.

Based on the experience of the econometric works as Kennan and Walker, (2011), it is assumed that an agent makes a decision on a period t of location and goods consumption, according to the goods valuation, given their current state, that is the socioeconomic cluster h in the period t. And additionally, it incorporates the possible changes in life cycle that means belonging to another cluster in the next period. For this model, we modify the utility function in problem (1) of the consumer's residential location choices as follows:

$$\begin{aligned} &\max_i \max_x \theta_h^t U_h^t(x, Z_i^t) + \theta_h^{t+1} \sum_{f \in H} P(f^{t+1}|h^t) V_f^{t+1}(E(Z_i^{t+1}), E(I_f^{t+1} - r_i^{t+1})) \\ &\text{subject to} \qquad p^t x + r_i^t \leq I_h^t, \end{aligned} \tag{36}$$

where $\theta_h^t + \theta_h^{t+1} = 1$.

In problem (36) it is assumed that an individual belonging to the cluster h in the period t has a set of change expectations in the life cycle for the period $t+1$ with probabilities $P(f^{t+1}|h^t)$ that represent changes in the household structure (job changes, income, education, having or not car, etc.) will make them transit to cluster f. Note that for simplicity, it is assumed that the individual anticipates transition possibilities only between consecutive periods, but this assumption can be easily generalized to a long-term problem. Additionally, a household f expects a utility in $t+1$ associated with real estate (i) given by $V_{fi}(E(Z_i^{t+1}), E(I_f^{t+1} - r_i^{t+1}))$, where V_f is the obtained valuation by the conditional indirect utility that households type f have, assuming as parameters the expected values associated with the residential location (i) in the period $t+1$ noted as $E(Z_i^{t+1})$ and from the expected income and rents $E(I_f^{t+1} - r_i^{t+1})$. To estimate the expectations valuation in $t+1$ a rationality argument in decision making will be used, given by the following imitation process:

$$E(Z_i^{t+1}) \approx Z_i^t \tag{37}$$

$$E(I_f^{t+1} - r_i^{t+1}) \approx I_f^t - r_i^t \tag{38}$$

that is, the agent h estimates the expectation parameters of period $t+1$ using known information of the parameters in t of the agent f, under the assumption that is rational agent. On the other hand, I_f^t can be seen as an approximation to the present value of the future income. Moreover, if we assume that $P(f^{t+1}|h^t)$ is a homogeneous transition matrix, that is $P(f^{t+1}|h^t) = P(f^t|h^{t-1})$, then, the consumer's problem would be formulated as follows:

$$\begin{aligned} &\max_i \max_x \theta_h^t U_h^t(x, Z_i^t) + \theta_h^{t+1} \sum_{f \in H} P(f^t|h^{t-1}) V_f^t(Z_i^t, I_f^t - r_i^t) \\ &\text{subject to} \qquad p^t x + r_i^t \leq I_h^t, \end{aligned} \tag{39}$$

The problem of the consumer (39) is static in t and it supposes that the agents belonging to the cluster h knows the utility parameters (*tastes*) of the other potentially imitable agents with an associated valuation parameter θ_h^{t+1} . Besides for the household in cluster h, the value of $\theta_h^{t+1} \sum_{f \in H} P(f^t|h^{t-1}) V_f^t(Z_i^t, I_f^t - r_i^t)$ is a constant in the continuous optimization problem in x, then the optimal allocation of goods consumption is conditional only to the parameters of h and r_i^t.

$$x_h^t(p^t, I_h^t - r_i^t, Z_i^t, \theta_h^t)$$

Therefore, for a household type h, the indirect utility conditional on the real estate i is:

$$V_{hi} \equiv \theta_h^t V_h^t(I_h^t - r_i^t, Z_i^t) + \theta_h^{t+1} \sum_{f \in H} P(f^t|h^{t-1}) V_f^t(Z_i^t, I_f^t - r_i^t) \tag{40}$$

To obtain the willingness to pay for the imitation problem we will build on an illustrative example. It will be assumed, without loss of generality, that there are two households (h, f) such that the agent h has the following indirect utility function associated with the real estate i,

$$V_{hi} \equiv \theta_h^t V_h^t(I_h^t - r_i^t, Z_i^t) + \theta_h^{t+1} V_f^t(Z_i^t, I_f^t - r_i^t), \tag{41}$$

and for the agent f, the conditional utility on the real estate i is given by:

$$V_{fi} \equiv V_f^t(Z_i^t, I_f^t - r_i^t), \tag{42}$$

That is, the cluster change expectations or imitation level are zero for f.

On the other hand, given a fixed utility level for household h, this can be represented as $U_{ohf} \equiv \theta_h^t U_{oh} + \theta_h^{t+1} U_{of}$, since it must be consistent with the behavior of f considering that the utility of f is U_{of}. Then for h we have

$$\theta_h^t V_h(I_h^t - r_i^t, Z_i^t) + \theta_h^t V_f^t(Z_i^t, I_f^t - r_i^t) = \theta_h^t U_{oh} + \theta_h^{t+1} U_{of} \tag{43}$$

Under the assumption of quasi-linear direct utility for each of the agents we have

$$\theta_h^t \left(\lambda_h^t(I_h^t - r_i^t) + \lambda_h^t b_{hi}(Z_i^t)\right) + \theta_h^{t+1} \left(\lambda_f^t(I_f^t - r_i^t) + \lambda_f^t b_{fi}(Z_i^t)\right) = \theta_h^t U_{oh} + \theta_h^{t+1} U_{of} \tag{44}$$

Solving the rent variable, then the willingness to pay is:

$$B_{hf(i)}^t = \theta_h^t \left(\frac{\lambda_h^t I_h^t + \lambda_h^t b_{hi}^t(Z_i^t) - U_{oh}}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} \right) + \theta_h^{t+1} \left(\frac{\lambda_f^t I_f^t + \lambda_f^t b_{fi}^t(Z_i^t) - U_{of}}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} \right) \tag{45}$$

Or equivalently

$$\begin{aligned}
B_{hf(i)}^t &= \frac{\theta_h^t \lambda_h^t}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} \left(I_h^t + b_{hi}^t(Z_i^t) - \frac{U_{oh}}{\lambda_h^t} \right) + \frac{\theta_h^{t+1} \lambda_f^t}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} \left(I_f^t + b_{fi}^t(Z_i^t) - \frac{U_{of}}{\lambda_h^f} \right) \\
&= \frac{\theta_h^t \lambda_h^t}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} \left(a_h^t + b_{hi}^t(Z_i^t) \right) + \frac{\theta_h^{t+1} \lambda_f^t}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} \left(a_f^t + b_{fi}^t(Z_i^t) \right) \\
&= \frac{\theta_h^t \lambda_h^t}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} B_{hi}^t + \frac{\theta_h^{t+1} \lambda_f^t}{\theta_h^t \lambda_h^t + \theta_h^{t+1} \lambda_f^t} B_{fi}^t
\end{aligned} \tag{46}$$

The result (46) can be extended to any agent h described by the consumer's problem (36), concluding that the willingness to pay with expectations Be_{hi}^t is of the form:

$$Be_{hi}^t = \theta_h^t \frac{\lambda_h^t}{\bar{\lambda}_h^t} B_{hi}^t + \theta_h^{t+1} \sum_{f \in H} P(f^{t+1}|h^t) \frac{\lambda_f^t}{\bar{\lambda}_h^t} B_{fi}^t, \tag{47}$$

where $\bar{\lambda}_h^t = \theta_h^t \lambda_h^t + \theta_h^{t+1} \sum_{f \in H} P(f^{t+1}|h^t) \lambda_f^t$ is interpreted as an average of the income marginal utility between periods t and $t+1$. In the case that $P(f^{t+1}|h^t)$ represents long-term probabilities, $\bar{\lambda}_h^t$ would represent an expected marginal utility per period in the long term.

Making the following change in notation:

$$\psi_{hh} = \theta_h^t \lambda_h^t + \theta_h^{t+1} P(h^{t+1}|h^t) \lambda_h^t; \qquad \psi_{hf} = \theta_h^{t+1} P(f^{t+1}|h^t) \lambda_f^t, \quad \forall f \neq h \tag{48}$$

with $\bar{\lambda}_h = \sum_{f\in H} \psi_{hf}$, we obtain that the bid function can be written as:

$$Be^t_{hi} = \sum_{f\in H} \frac{\psi_{hf}}{\bar{\lambda}_h} B^t_{fi} = \sum_{f\in H} \frac{\psi_{hf}}{\bar{\lambda}_h} (a^t_f + b^t_{fi}(Z^t_i)) \tag{49}$$

Whereby $\frac{\psi_{hf}}{\bar{\lambda}_h}$ is the percentage or valuation level on the willingness to pay of h that has the expectation of being an agent f in the future. Furthermore, the bid with imitation (49) has a new important component given by the inclusion of an expected utility level and the expected income to be noted as $\bar{a}^t_h$. This allows the analysis within the formulation of possible effects of future income change, before making decisions associated with residential location.

$$\bar{a}^t_h = \frac{1}{\sum_f \psi_{hf}} \sum_f \psi_{hf} a^t_f = \frac{1}{\sum_f \psi_{hf}} \sum_f \psi_{hf} \left(I^t_f - \frac{U^t_f}{\lambda^t_f} \right) \tag{50}$$

The formulation (39) to (47) can be extended to other contexts independent of the life cycle dynamics, for example if we change $\theta^{t+1}_h P(f^{t+1}|h^t)$ by a weight w_{hf} that measures a social relationship among this type of households (Páez et. al, 2008), then $\frac{\psi_{hf}}{\bar{\lambda}_h}$ would indicate the valuation in the willingness to pay of the agent h of the social influence that has the agent f. Thus, the formulation of the utility (39) and the willingness to pay (49) would be an extension of the classic social network studies and their impact on decision making; insomuch as the individual incorporates within his/her preferences valuations of other socially related agents, not just the decision made by themselves that in the urban economy context is defined as a location externality. It is necessary to use the imitable or related agents utility as a differentiator in the property valuation, since equivalent households can exist (belonging to the same socio-economic cluster) with different choices of residential location. Furthermore, the previously proposed modeling strategy can be used when it is unclear to which specific cluster a household belongs (fuzzy clustering, Valente de Oliveira and Pedrycz, 2007), and thus to generate more representative willingness to pay of such agents. In that sense, in microsimulation problems, where it is assumed that each agent has a value belonging to each distinctive cluster, a more representative formulation of the utility as well as the bid can be obtained of each household for each good.

In general, these imitation phenomena are analyzed and understood as a collective learning form (Alós-Ferrer and Schad, 2009) which uses information from other agents for decision-making, for example, in firms's location the profit of other previously located ones can be used (e.g. imitation location).

Analogous to what was developed in the learning model, in the stochastic version of expectation model, it will be assumed that the willingness to pay $Be^t_{h(i)}$ is a random variable, where the modeling error is assumed with Gumbel distribution with scale parameter μ. It is important to note that the bid with expectations includes an expected income within the formulation, as noted in (50); however, the bid at time t should not be higher than the revenue in this period, independent of the expectation levels, therefore if holds that:

$$Be^t_{h(i)} \leq I^t_h \tag{51}$$

If the restriction (51) is not met the agent h will not enter to the set of potential bidders for the property (i) in the period t.

Constraint (51) is important because in cases where the valuation of expectations on agents who are being imitating each other is very high in some property goods, their willingness to pay cannot exceed the current income (this income includes wage rates, loans, savings, spending on durable goods, etc). Analogous to (22) and (23), these constraints are modeled for each agent and are included in the modeling through the Constrained Logit (Martínez et al 2009) with "*cut-off*" functions defined as the probability that a household in a cluster h meets it in every i, denoted as ϕe_{hi}^t, given by

$$\phi e_{h(i)}^t = \frac{1}{(1 + exp(w(Be_{h(i)}^t - I_h^t + \zeta)))} = \begin{cases} 1, & if \ \ (Be_{h(i)}^t - I_h^t \to -\infty) \\ \eta, & if \ \ (Be_{h(i)}^t - I_h^t \to 0) \end{cases} \tag{52}$$

Thus, the population distribution H_{hi}^t at a period t is expressed as:

$$H_{hi}^t = S_i * Q_{h|i} = S_i * \frac{H_h^t * \phi e_{h(i)}^t * \exp(\mu Be_{hi}^t)}{\sum_{g \in H} H_g^t * \phi e_{g(i)}^t * \exp(\mu Be_{gi}^t)} \tag{53}$$

Equivalent to what was shown in the static model $RB\&SM$ (Martínez and Henríquez, 2007) it is considered to exist externalities associated with the population distribution in each zone that affects the willingness to pay of households; therefore $Be_{hi}^t(H_{gj}^t, \forall g, j)$; thus $Q_{h|i}^t$ depends on $Q_{g|j}^t, \forall g, j$ obtaining a fixed-point system of equations.

Furthermore, the rent at each location is obtained endogenously by the expectation of the maximum bid, given by the logsum function:

$$r_i^t = \frac{1}{\mu} \ln \left(\sum_{g \in H} H_g^t * \phi e_{g(i)}^t * \exp(\mu Be_{gi}^t) + \gamma \right) \tag{54}$$

γ is Euler's constant.

Finally, the equilibrium condition ensures that every household is allocated somewhere, given by $\sum_i Q_{h|i}^t S_i^t = H_h^t$, obtaining

$$a_h^t = -\frac{\sum_g \psi_{hg}}{\psi_{hh}\mu} \ln \left(\sum_i S_i^t * \phi e_{h(i)}^t * \exp \left(\mu \left\{ \frac{\psi_{hh}}{\sum_g \psi_{hg}} b_{hi}^t - r_i^t + \sum_{f \neq h} \frac{\psi_{hf}}{\sum_g \psi_{hg}} * B_{fi}^t \right\} \right) \right), \ \ \forall h \tag{55}$$

Analogous to (34) and (35), as rents r_i^t depend on each a_g^t , $\phi e_{h(i)}^t$ depend of B_{fi}^t and B_{fi}^t depends on a_f^t and on the utility levels of the imitable individuals by f, and then (9) constitutes a non-linear fixed-point system of equations described as $a_h^t = f(a_g^t, \forall g)$ defining the maximum levels of utility which are possible to obtain in equilibrium, with $\bar{a}_h^t = \frac{1}{\sum_f \psi_{hf}} \sum_f \psi_{hf} a_f^t$. This value is conditioned to the other agents' utility levels (both the equilibrium and the imitation effect) as well as the expected incomes $\frac{1}{\sum_f \psi_{hf}} \sum_f \psi_{hf} I_f^t$ (see equation 50).

To analyze the imitation effect on urban distribution suppose that there are two clusters f, h with an expectation or imitation normalized factor denoted as $1 > \psi_{hf} > 0$. In addition, if given a location (i) such that $B_{fi} \geq B_{hi}$ then:

$$B_{hi} \leq (1 - \psi_{hf}) B_{hi} + \psi_{hf} B_{fi} = Be_{hi} \tag{56}$$

and under the assumption that $\phi e^t_{hi} = \phi e^t_{fi} = 1$ it is obtained (income constraint is fulfilled), then

$$\frac{H^t_h * \exp(\mu B^t_{hi})}{H^t_h * \exp(\mu B^t_{hi}) + \sum_{g \neq h} H^t_g * \phi e^t_{g(i)} * \exp(\mu Be^t_{gi})} \leq \frac{H^t_h * \exp(\mu Be^t_{hi})}{H^t_h * \exp(\mu Be^t_{hi}) + \sum_{g \neq h} H^t_g * \phi e^t_{g(i)} * \exp(\mu Be^t_{gi})} \tag{57}$$

Thus:

$$H_{hi} \leq H^{imi}_{hi} \tag{58}$$

Where H_{hi} is the urban distribution with no imitation ($\psi_{hg} = 0$) and H^{imi}_{hi} is the urban distribution with imitation or expectation in i ($\psi_{hg} > 0$). Similarly, if in a real estate good (i) such that $B_{fi} \leq B_{hi}$ then $B_{hi} \geq (1 - \phi_{hf})B_{hi} + \psi_h f B_{fi} = Be_{hi}$ and $H_{hi} \geq H^{imi}_{hi}$. Note that if $\phi e^t_{hi} < 1$ and $\phi^t_{hi} = 1$, it means, if the inclusion of expectation effect makes the income constraint not met strictly, the result (57) and (58) are not necessarily true, because the current income level does not fully allow to meet the change expectations in the period t.

On the other hand, the dynamic modeling of residential relocation allows to include bid generation strategies (or utilities) that integrate interaction between different temporal and spatial information. For example, Habib and Miller (2010) and Martínez and Hurtubia (2006) use former periods information for the construction of valuations for each real estate type in the current period. One way to formulate bids with dynamic interaction is assuming that every bid function B^t_{hi} depends on urban distribution in the period $t - 1$. That is, $B^t_{h,i}(H^{t-1}_{gj}, \forall g, j)$. It means, the fixed-point calculation associated to urban demand is avoided. But not in terms of the utility $\bar{a}^t_h$ at equilibrium, which is obtained in each period using equation (55). It is important to see that in an intertemporal context the change on expectations are being renovated and updated, hence it is necessary to do the analysis period by period on the number of agents that actually change the household type, and also the new cluster sizes that generate variations of urban distribution. Note that even in a city with slow relocation processes, if the households change their socioeconomic characteristics over time, then the city will have different population distributions, given the agents' endogenous dynamics.

Some numerical simulations will be presented in the conference based on classical models of residential segregation that seek to explain the presented modeling and its effects in making location decisions

5 CONCLUSIONS AND FINAL DISCUSSION

This paper presents two microeconomic formulations of a household choice model of residential location that incorporates the effects of past experiences and expected future change in the household life cycle.

In a first formulation, a static microeconomic model with endogenous learning is formulated where households give higher utility to real estate with previous positive experiences and less valuation to property in which they had negative experiences. For this, the memory of a real estate is analyzed by the gained utility in former periods. Given the process of utility maximization decision-making with endogenous learning, the willingness to pay formulation when an agent changes category between periods is obtained, generating an additional differentiating element among households based on their history. It is proposed a stochastic formulation assuming logit distributions of behavior to obtain urban distributions based on a myopic learning. Some numerical simulations will be presented in the conference based on classical models of residential segregation that seek to explain the presented modeling and its effects in making location decisions.

On the other hand, the microeconomic modeling of residential location with memory is an interesting contribution in the social assessment of private or public projects, because two agents of an urban system with identical conditions (residential location and household type) in a period t can perceive the utility differently at to their current residential location because of the past experiences' reference. In this sense, a social planner should include within the analysis not only the perceived utility in the period t, but also how the property characteristics, where every home is located, improve or worsen with respect to previous periods. Moreover, the formulation increases the amount of household categories because it includes within the characteristics or descriptor attributes of the cluster all the locations background, generating a great variety of attributes within the agent's classification. Then, each household will be described by their socioeconomic characteristics in the period t and the characteristics in previous periods that define the perceived utilities in their anterior locations. Thus, the proposed formulation provides a useful tool in micro-simulation models, where each agent is analyzed in detail, but in large-scale problems it would be a bit difficult to obtain general behavior rules and even less to analyze asymptotic or long-term states. As a general conclusion, this work includes a theoretical analysis of the effects of agents learning in the aggregated urban configuration and in the individual choice of residential relocation, which have been studied and evidenced in previous econometric studies.

Furthermore, the use of household change's expectations in relocation decisions is incorporated in the discrete choice microeconomic formulation the by mean of transition probabilities among household clusters in the life cycle and the imitation's hypothesis of such agents under the consideration that they behave rationally. This postulation considers that imitable or expected agents' tastes are known. Based on a microeconomic consumer formulation, a multi-objective bid function is obtained which includes an expected income per unit of time and a utility consistent with the behavior of the agents that are potentially imitables. For both the expectations and learning model, it is necessary to include an income restriction per period, since such valuations can generate infeasible willingness to pay because they are greater than the net income of each household in that period. The microeconomic formulation and the willingness to pay of the model with imitation are extensible to other contexts, such as social networks or fuzzy clustering due to the interaction among different agents and the effect of being able to incorporate their likes (valuations) in their own valuation.

As a final conclusion of the theory developed in this paper, it can be analyzed the relocation process by means of endogenous forces or the households' own generating attraction to relocation or location in already known goods (memory effect) by the agents given by past experiences, assuming that the obtained utility level was positive or change forces given by expectations in the life cycle, such as the possibility of a better income or changes in long-term activities (work, education). In that sense, each household's bid can be seen as a valuation given by the agents to their own experiences and other ones' valuations related or imitable under the assumption that they make rational decisions.

In this way, it is proposed a consumer's problem associated with the choice by the real estate i in a period t by an agent (h, g, j) where h is the cluster in the period t, g is the cluster in the period $t-1$ and j is the location in $t-1$ and with a set of expectations described by the probability $P(f^{t+1}|h^t)$ described as:

$$\max_i \max_x \theta_h^t U_h^t(x, Z_i^t) + \theta_h^{t+1} \sum_f P(f^t|h^{t-1}) V_f(Z_i^t, I_f^t - r_i^t) + \theta_h^{t-1} m_{gj}^{t-1}$$

For simplicity it might be assumed that the agent h analyzes the current utility that an agent f receives for the good i without including the learning or experience valuation of that agent because of two reasons: the first is that it is very difficult to know the valuation given by other agents to past experiences, and on the other hand, given the formulation of the learning model, it would be difficult to include all these behaviors in the valuation of a single agent, because it would become a high dimensionality problem, since it would not only include its own history but the others' one. However, since the history process helps to generate or form valuations of every agent and under the assumption that the transition probability matrix among clusters is a sparse matrix (slow processes in the life cycle change), it might be assumed that under the imitation process the history valuations of the imitable agents are captured.

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