Vertical product differentiation in multi-product duopolistic aviation markets

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Abstract

The aviation industry, just as many other industries, is characterised by a large degree of product differentiation between airlines as well as a large variety in fare types per airline. In this article we study how consumer heterogeneity and strategic interaction between firms lead to vertically differentiated equilibrium patterns in terms of number of supplied variants, quality, and fare per variant. We apply the standard model of vertical product differentiation within a random utility framework. We conclude that if demand is deterministic, interlaced equilibriums do not exists. In point of fact, both the low- and high-quality carrier do not have strong incentives to supply an extra product variant at the lower end of the market. Accounting for unobserved random utility increases the range of possible equilibriums in terms of ordering of quality variants.

Keywords: Random utility models, Market segmentation, Multi-product firms, Yield management

1 Introduction

We generalise the model of vertical product differentiation for multi-product duopolistic competition, as discussed by Cheng et al. (2011), into a random utility framework accounting for both observed and unobserved random consumer heterogeneity. Unobserved heterogeneity is introduced by a random term in the consumers' utility function. Observed heterogeneity is caused by the dispersion in willingness to pay for quality. The duopolistic firms compete in number of variants, qualities, and prices. We analyse under which conditions subgame perfect Nash equilibria exist with firms supplying a product line in which at least two variants supplied by a single firm differ in prices and qualities.

Studies of vertical product differentiation in a random utility framework and endogenous price- and quality setting only report symmetric equilibria. In these equilibria, all the product variants supplied by a single firm have the same price- and quality setting (Anderson et al. (1992) and Anderson and de Palma (1992)). Clearly, these symmetric equilibrium patterns are not anticipated to be found in reality. Therefore, the appropriateness of the random utility model in modelling endogenous product differentiation – see, for example, Anderson et al. (1989 and 1992), Anderson and de Palma (2001), Berry (1994), and Berry et al. (1995) – may depend on whether the only type of equilibria obtainable within the random utility framework are these symmetric ones.

The aviation industry is a prime example of an industry with vertically differentiated multi-product duopolies. In Europe, multiple origin-destination pairs are served by two carriers only. For instance, a consumer flying from Manchester to Amsterdam, Copenhagen, or Dublin currently can choose between two carriers in each market. The same holds for consumers travelling from Edinburgh to Amsterdam or Dublin. In these markets a traditional legacy carrier, like Air France-KLM or Aer Lingus, competes with a low-cost carrier, like easyJet or Ryanair. The legacy carrier supplies different fare types characterised by different prices and qualities, whereas the competing low-cost carrier supplies one fare type with a single low price- and quality setting. The typical single-product strategy of low-cost carriers on the one hand, and the multi-product strategy of legacy carriers on the other, suggests a relation between number of variants supplied and the price- and quality setting. This article analyses profitability of multi-product strategies compared with single-product ones for both the low- and high-quality firm. The analysis takes into account the different relative levels of unobserved heterogeneity the firms may face.

Although the literature on vertical product differentiation is well established, aviation specific literature on this topic is still rather limited.[1] Botimer and Belobaba (1999) are the first to explicitly mention the trade-offs that carriers supplying multiple product

[1] Proussaloglou and Koppelman (1999), Algers and Beser (2001), and Balcombe et al. (2009) are the only empirical studies that address consumer behaviour regarding intrafirm product differentiation in the aviation industry.

variants face. Their model, however, does not include quality, unobserved heterogeneity, or strategic interaction between firms. More recently, Lin and Sibdari (2009) use a random utility framework in a duopolistic market setting to conclude that a Nash equilibrium in prices exists.

Compared with the study by Cheng et al. (2011), we take into account unobserved idiosyncratic consumer preferences and the possibility of entangled product variants implying interlaced equilibria. In comparison with Lin and Sibdari (2009), we include quality as a second strategic variable. The results show that in the random utility model of vertical product differentiation the nature of resulting equilibrium patterns – fully differentiated or not – crucially depends on the level of unobserved heterogeneity relative to the level of observed heterogeneity. Without unobserved demand heterogeneity, the incentive to supply an extra variant is only present if this extra variant serves the high-end of the market. This finding implies that interlaced equilibria are highly unlikely to exist in a deterministic setting. Taking into account unobserved heterogeneity, however, restores the existence of interlaced equilibria as observed in, for example, the aviation industry.

The next section introduces the model and shows how to define the subgame perfect Nash equilibrium in number of variants, qualities, and prices. Section 3 discusses the random utility framework and specifies the nested logit demand model. Section 4 changes the focus to the numerical analysis with a discussion about possible market configurations. In Section 5, we present the numerical results and analyse the subgame perfect Nash equilibrium in qualities and prices for a set of market configurations and thus number of variants. Section 6 provides a discussion and conclusion.

2 Model

2.1 Background

Throughout this article, we assume that two firms, firms A and B, compete by supplying multiple product variants. Each variant may differ in the observed characteristics price and quality. Each individual consumer maximises his or her utility by choosing whether to buy the product and which particular variant. The indirect utility function of consumer *t* is defined as:

$$\begin{cases} V_t = z - \alpha p_{ij} + \theta_t q_{ij} + \varepsilon_{ij,t} & \text{if consumer } t \text{ purchases one of the variants } ij, \\ V_t = z + \varepsilon_t & \text{otherwise,} \end{cases} \tag{1}$$

with subscript i indicating the product variant and subscript j the firm. The indirect utility function consists of a systematic and stochastic part. The generic parameter z captures the utility derived from other goods and equals the utility obtained from not buying the product at all. The systematic utility is further determined by the generic marginal utility of income, α, the marginal utility of quality, θ_t, the price, and the quality of the specific variant. The marginal utility of income and quality determine the willingness to pay for a unit increase in quality: θ_t / α. The willingness to pay differs across consumers following a uniform distribution over the interval $\left[\underline{\theta},\overline{\theta}\right]$ and density normalised to 1. This definition of observed heterogeneity across consumers is in line with the literature (see, for example, Mussa and Rosen (1978), Gabszewicz et al. (1986), and Cheng et al. (2011)).

The stochastic part represents the unobserved heterogeneity across consumers. It is included in Eq. (1) via the individual and product variant specific error term ε_t. The specification in Eq. (1) is flexible and allows for specifying consumer behaviour to be consistent with a random utility framework. Furthermore, it also offers the possibility to model the deterministic demand structure as used by Mussa and Rosen (1978) and Cheng et al. (2011).

2.2 Market coverage

The last line of Eq. (1) specifies that consumers may choose not to buy a product at all. Hence, firms do not have to cover the whole market. To avoid full market coverage, it is crucial to assume that the lowest prevailing willingness to pay across consumers is not too high with respect to both the outside alternative and the highest prevailing willingness to pay. The outside alternative is necessary for the existence of a differentiated multi-product equilibrium.

Consider the case in which no outside alternative exists. This implies that each consumer buys a product variant no matter what. For a single-product vertically differentiated duopoly, it is obvious that both firms will maximise product differentiation under all circumstances to avoid Bertrand price competition. The incentive to maximise differentiation still holds if one of the two firms, let's say firm B, supplies a second product variant. Assume that firm B supplies the lowest quality variants and starts increasing the quality of the second variant. Firm A already supplies the highest quality variant and will not change its quality setting. As a result, the second variant of firm B, located more close to the variant of firm A, increases price competition with firm A. This causes the mark-up of all variants in the market to decrease. Because of the full market coverage, the loss in mark-up cannot be offset by an increase in market share.[2] Bonnisseau and Lahmandi-Ayed (2006) show this mechanism in detail. They illustrate that if firm B would set the quality of its second variant too close to firm A's quality, firm A may locate its variant in between the two variants of firm B thereby earning a positive profit and reducing firm B's profits even further.

[2] If firm A would supply a second variant instead, the low-quality firm B would not change its quality either: by lowering its quality the firm cannot attract new consumers because the market is already fully covered.

The example above is a general result of inelastic demand. In this case the inelastic demand is clearly dictated by the assumption that firms need to cover the whole market. The choice between buying and not-buying a product variant is inelastic with respect to the observable characteristics of the product variants. Regarding multi-product differentiation, two possible reasons for inelastic demand are mentioned in previous studies. First, ignoring the availability of the non-buying option – as done in Champsaur and Rochet (1989) and Bonnisseau and Lahmandi-Ayed (2006) – implies inelastic demand.[3] Second, if the lowest prevailing willingness to pay across consumers is too high relative to the attractiveness of the outside alternative, no consumer considers not-buying. Demand, therefore, will be inelastic and the market fully covered (Cheng et al., 2011). By including unobserved consumer heterogeneity, a third possible reason emerges. If unobserved heterogeneity across consumers between buying and not-buying is large, the demand becomes inelastic for the price- and quality setting of each variant. Although the market is not fully covered, more precisely the market is divided in half, demand becomes inelastic and the incentives for product differentiation evaporate.

2.3 Number of variants-then-quality-then-price game

We define $\mathbf{p}$ as the vector of prices, and $\mathbf{q}$ as the vector of qualities of all product variants, so $\mathbf{p}=[p_{1A},\ldots,p_{n_AA},p_{1B},\ldots p_{n_BB}]$ and $\mathbf{q}=[q_{1A},\ldots q_{n_AA},q_{1B},\ldots,q_{n_BB}]$, where A and B are the j-subscripts referring to the two firms. Furthermore, let $\mathbf{r}$ be the vector containing the number of product variants per firm: $\mathbf{r}=[n_A,n_B]$. Hence, the profit function of firm j is as follows:

$$\pi_j(\mathbf{p};\mathbf{q};\mathbf{r})=\sum_{i=1}^{n_{j(\mathbf{r})}}\left(p_{ij}-c(q_{ij})\right)x_{ij}-n_{j(\mathbf{r})}F. \tag{2}$$

The demand for each product variant x_{ij} is based on the indirect utility function in Eq. (1) and will be discussed in Section 3.

The firm incurs two types of costs. First, a generic fixed cost for each variant, F, applies. Due to these fixed costs, n_A or n_B will not go to infinity.[4] Second, the marginal cost of quality are equal to: $c(q_{ij})=(a+bq_{ij})q_{ij}$. This specification assumes – like Mussa and Rosen (1978), Champsaur and Rochet (1989), and Cheng et al. (2011) – that marginal improvements in quality become increasingly costly. The combination of a utility function that is linear in the willingness to pay for quality and a cost function that is convex in quality ensures that none of the firms sets infinitely high qualities.[5]

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[3] They assume that the demand for the lowest quality variant is determined via $\underline{\theta}$, instead of the firm deciding which consumers to serve.

[4] In contrast, Cheng et al. (2011) omit these costs and consequently only look at the case in which the number of supplied variants goes to infinity.

[5] For reasons of exposition, the parameters a and b are set equal to 0 and 1/2 respectively.

The duopolists play a three-stage game in which they optimise their profits by designing their product line. In the first stage the firms decide, simultaneously, how many variants they supply, whereas in the second stage the simultaneous quality setting of all product variants takes place. In the last stage, the firms determine prices given the afore chosen qualities and number of variants. In each of the subgames, subgame perfect Nash equilibria can be established, taking into account the consequences in later subgames.

The full game is solved by backward induction. Let's define $\mathbf{p}_A = [p_{1A},\ldots,p_{n_A A}]$ and the other firm-specific vectors in the same manner. Given the vector of varieties $\mathbf{r}$ and the vectors of qualities $\mathbf{q}_A$ and $\mathbf{q}_B$ in the first and second stage respectively, the corresponding price-subgame is solved by $p^*_{1A}(\mathbf{q};\mathbf{r}),\ldots,p^*_{n_A A}(\mathbf{q};\mathbf{r}),\quad p^*_{1B}(\mathbf{q};\mathbf{r}),\ldots p^*_{n_B B}(\mathbf{q};\mathbf{r})$ ensuring that:

$$\pi_j\left(\mathbf{p}^*_j;\mathbf{p}^*_{-j};\mathbf{q};\mathbf{r}\right) \geq \pi_j\left(\mathbf{p}_j;\mathbf{p}^*_{-j};\mathbf{q};\mathbf{r}\right). \tag{3}$$

$\mathbf{p}^*(\mathbf{q};\mathbf{r})$ is the resulting optimal price vector at which profits need to be evaluated in the quality stage. Denote this profit function for each firm as $\bar{\pi}(\mathbf{q};\mathbf{r}) \equiv \pi\left(\mathbf{p}^*;\mathbf{q};\mathbf{r}\right)$. The second stage quality-subgame equilibrium is then characterised by $q^*_{1A}(\mathbf{r}),\ldots,q^*_{n_A A}(\mathbf{r}),q^*_{1B}(\mathbf{r}),\ldots q^*_{n_B B}(\mathbf{r})$, satisfying the following condition:

$$\bar{\pi}_j\left(\mathbf{q}^*_j;\mathbf{q}^*_{-j};\mathbf{r}\right) \geq \bar{\pi}_j\left(\mathbf{q}_j;\mathbf{q}^*_{-j};\mathbf{r}\right). \tag{4}$$

The resulting optimal quality vector, $\mathbf{q}^*(\mathbf{r})$, is used to evaluate profits in the final stage. The profit function for each firm in the last stage is: $\hat{\pi}(\mathbf{r}) \equiv \bar{\pi}\left(\mathbf{q}^*;\mathbf{r}\right) \equiv \pi\left(\mathbf{p}^*;\mathbf{q}^*;\mathbf{r}\right)$. The third stage subgame equilibrium can then be characterised by n^*_A, n^*_B, satisfying the following condition:

$$\hat{\pi}_j\left(n^*_j, n^*_{-j}\right) \geq \hat{\pi}_j\left(n_j, n^*_{-j}\right). \tag{5}$$

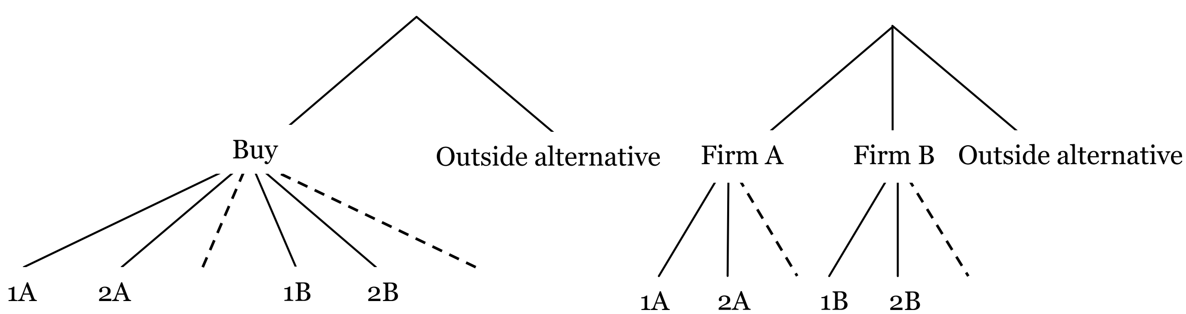
Therefore, the subgame perfect Nash equilibrium is characterised by $\mathbf{r}^*$, $\mathbf{q}^*(\mathbf{r})$ and $\mathbf{p}^*(\mathbf{q};\mathbf{r})$. If $\mathbf{p}^*$ is evaluated at $\mathbf{q}^*(\mathbf{r})$ and $\mathbf{q}^*$ subsequently is evaluated at $\mathbf{r}^*$, we have found the corresponding equilibrium path.

3 Consumer behaviour

Following Anderson and de Palma (1992), we use a nested logit model to analyse consumer demand. Two possible specifications of such a nested logit model are depicted in Figure 1. The structure depicted on the left assumes that all product variants, irrespective of the firm supplying them, are in one nest, whereas the other structure on the right depicts a nested model with firm specific nests. Product variants within the same nest are more similar in unobserved attributes compared with variants from other nests. The set-up on the left has therefore the straightforward interpretation that all variants, irrespective of the firm supplying them, are more similar in unobserved attributes compared with the outside alternative. In the structure depicted on the right, this distinction is made at the firm level. The question how the actual nesting structure looks like is eventually an empirical question.[6]

For the current analysis, it is important how the nested structure applies to both single and multi-product firms under varying levels of unobserved demand heterogeneity across consumers. A first analysis revealed that modelling the variant of the single-product firm in a separate nest, hence as a degenerated alternative, implies that for nearly all plausible levels of unobserved consumer heterogeneity the price- and quality setting of both firms hardly impact consumers' choice between firms. As a result, the price- and quality setting of the single-product firm becomes irrelevant. In contrast, the multi-product firm still needs to take into account the impact of the price- and quality setting on all variants within the firm specific nest. The nesting structure depicted on the left in Figure 1 does not suffer from this drawback. Therefore, the remainder of this article applies the nested logit structure with all product variants in one nest.

Figure 1 Possible demand structures (nested logit) for multi-product duopoly.



[6] One may also specify a three-level nested logit model: product vs. outside alternative, firm, product variant. For reasons of tractability, however, this analysis focuses on the two-level nested logit model. The standard multinomial logit model cannot be used because one cannot differentiate between the effect of stochasticity on the buying decision, i.e. the elasticity of the total demand, and on the product variant choice.

The model as depicted in Figure 1 requires to split the individual and product variant specific error term, as shown in Eq. (1), into two components: $\varepsilon_{ij,t} = \varepsilon_{ijk,t} + \varepsilon_{k,t}$. Here the subscript k indicates the nest and thus whether the consumer actually buys a product variant. The error component $\varepsilon_{k,t}$ represents the individual unobserved random utility in whether buying any product variant, whereas $\varepsilon_{ijk,t}$ represents the individual unobserved random utility for the combination of choosing product variant ij and buying the product.

The two error terms are assumed not to be correlated. Furthermore, they are assumed to be independent and identically distributed with scale parameters μ_1 and μ_2 respectively. The variance of the corresponding density function equals $\mu^2\left(\pi^2/6\right)$. The scale parameters determine the variance of the error terms and thereby the importance of the unobserved component of utility. If, in the limit, both μ_1 and μ_2 approach zero, the random utility model reduces to a deterministic model ignoring heterogeneity in unobserved attributes.

Some additional assumptions on the scale parameters are needed. First, the unobserved heterogeneity between alternatives within a nest may not be larger than the unobserved heterogeneity between alternatives from different nests, this requires $\mu_1 \geq \mu_2$. Second, the level of unobserved heterogeneity between the elemental alternative and the nest cannot differ for degenerated alternatives. So, for these alternatives one needs to impose $\mu_1 = \mu_2$.

A higher value of the variance, whether through an increase in μ_1, μ_2, or both, means that the systematic part of the utility, $z - \alpha p_{ij} + \theta_t q_{ij}$, becomes a less important factor in determining whether and which variant a consumer buys. Stated alternatively, an increase in the variance via μ_2 makes the demand for a particular variant less sensitive to its own price-quality setting whereas an increase in the variance via μ_1 makes the demand for whether or not buying a product at all less sensitive to the maximum expected utility of buying the product.

Based on the nesting structure on the left of Figure 1, total demand for product variant ij becomes:

$$x_{ij} = \sum_{t=1} \mathrm{P}_{ij,t}, \tag{6}$$

with $\mathrm{P}_{ij,t} = \mathrm{P}_{ij|k,t}\mathrm{P}_{k,t}$ being the probabilities following a nested logit specification. So, each consumer t has an expected demand equal to the probability of choosing product variant ij conditional on choosing nest k (buying a product variant or not) multiplied by the probability of choosing nest k. The total demand for product variant ij is the summation over all consumers. The conditional probability is defined as:

$$\mathrm{P}_{ij|k,t} = \frac{\exp\left(\left(z - \alpha p_{ij} + \theta_t q_{ij}\right)/\mu_2\right)}{\sum_{i=1}^{n_A}\exp\left(\left(z - \alpha p_{iA} + \theta_t q_{iA}\right)/\mu_2\right) + \sum_{i=1}^{n_B}\exp\left(\left(z - \alpha p_{iB} + \theta_t q_{iB}\right)/\mu_2\right)}, \tag{7}$$

The probability of nest k equals:

$$\mathrm{P}_{k,t} = \frac{\exp\left(S_{k,t}/\mu_1\right)}{\exp\left(S_{k,t}/\mu_1\right) + \exp\left(z/\mu_1\right)}, \tag{8}$$

with the logsum, $S_{k,t}$, defined as:

$$S_{k,t} = \mu_2 \ln\left(\sum_{i=1}^{n_A}\exp\left(\left(z - \alpha p_{iA} + \theta_t q_{iA}\right)/\mu_2\right) + \sum_{i=1}^{n_B}\exp\left(\left(z - \alpha p_{iB} + \theta_t q_{iB}\right)/\mu_2\right)\right). \tag{9}$$

The non-linear nature of the random utility framework prevents insightful analytical results from being available. In order to analyse the patterns of product differentiation in a multi-product duopoly and study the incentives to supply extra variants, we discuss in the remainder of this article an extensive numerical analysis. We solve the price setting stage, using sequential quadratic programming, for every set of possible qualities in a pre-defined grid of qualities given a pre-determined number of variants. Appendix A provides further details about the search algorithm. The pre-determined number of variants are based on actual observed market configurations that are described in the next section.

4 Market configurations: Low-cost carrier versus legacy carrier

Here, the possible market configurations are introduced. A particular market configuration is defined as the combination of the number of product variants supplied by each firm and the vertical order (price or quality based) of all variants. Table 1 gives an example of the current market configuration in the Manchester–Amsterdam and Manchester–Dublin aviation markets based on price.

Table 1 Fares per fare type for a return flight, including a weekend stay, as published on the airlines' websites for bookings 6 weeks in advance (in Euros).

Manchester–Amsterdam		Manchester–Dublin	
Air France-KLM	**easyJet**	**Aer Lingus**	**Ryanair**
95 (lowest fare)	75 (lowest fare)	80 (lowest fare)	35 (lowest fare)
120 (economy)	210 (flexi fare)	120 (plus)	
355 (economy flexible)		670 (flex)	
545 (economy fully flexible)			
745 (business fully flexible)			

In the Manchester–Amsterdam market Air France-KLM competes with low-cost carrier easyJet, whereas in the Manchester–Dublin market Aer Lingus competes with Ryanair.[7] With 180 scheduled flights in June 2012, Air France-KLM captures a market share – measured in number of flights – of 77 per cent. The market shares in the Manchester–Dublin market are more equal, in fact Aer Lingus and Ryanair split the market with 100 and 104 scheduled flights in June 2012.[8]

Table 1 clearly shows a vertically differentiated product, with prices ranging from €75 to €745 in the Manchester–Amsterdam market and from €35 to €670 in the Manchester–Dublin market. The quality differences per fare type differ per carrier. For Air France-KLM, the differences in fare types represent the levels of flexibility the consumer has in cancelling and changing the flight and not the actual in-flight quality level. The business fully flexible fare type is an exception. The 'flexi fare' of easyJet provides both less restrictions on the ticket and a higher in-flight quality, like priority boarding and luggage handling.

Table 1 reveals that the lowest available fares offered by the legacy carriers are more expensive compared with the ones offered by the low-cost carriers. Furthermore, low-cost carriers supply less number of variants than the legacy carriers. From this perspective, the launch and rollout of 'flexi fares' by easyJet in June 2011 – a high-quality variant – may be considered as a remarkable strategy. In the numerical analysis, we will address the possible incentives for a low-quality firm to supply an extra high-quality variant.

The numerical analysis translates the actual market configurations, as shown above, into three distinctive scenarios. In the first scenario, both firms are single-product firms. The high-quality firm, let's say firm A, becomes a multi-product firm in the second scenario, whereas the low-quality firm remains a single-product firm. This second scenario resembles the duopolistic competition between Aer Lingus and Ryanair in the Manchester–Dublin market. In the third scenario, both the low- and high-quality firm supply multiple product variants with a maximum of two for the low-quality firm. This scenario reflects easyJet's strategy. The first and second scenario show whether, and if so, under which circumstances, a multi-product strategy is more profitable for the high-quality firm. The second and third scenario repeat this analysis for the low-quality firm.

Based on the second and third scenario, all the product variants in the market can be (vertically) ordered into two different ways. First, each firm produces only all low- or all high-quality product variants. As a result, firm A and B each supply one variant which is in direct competition with the variant of the competing firm, i.e. the so-called fighting variant (Cheng et al., 2011). Second, the firm's specific product variants may be (perfectly) interlaced, resulting in multiple fighting variants in the product line of both

[7] Due to the recent hostile takeover bids by Ryanair for Aer Lingus, competition between these two carriers in specific market attracts the attention from media and policy makers.
[8] Figures are taken from the UK Punctuality Statistics, accessible from www.caa.co.uk/punctuality.

firms. In contrast to Cheng et al. (2011), here the latter market configuration is not a priori excluded.

For all three scenarios, i.e. number of product variants supplied, all possible patterns of vertical product differentiation are analysed. The resulting equilibrium may be symmetric or not. The equilibrium is symmetric if and only if prices and qualities of all product variants in the market are equal, otherwise the equilibrium is asymmetric. The asymmetric equilibrium may be fully differentiated or not. In case prices and qualities of all product variants supplied by a singly firm vary, the equilibrium is fully differentiated. The equilibrium is asymmetric but not fully differentiated otherwise.

5 Numerical results

5.1 Patterns of product differentiation

In absence of unobserved heterogeneity, the equilibrium, if it exists, always has a fully differentiated pattern of product differentiation for multi-product duopolies. Differentiation between the two firms intensifies if the choice to buy any variant at all becomes less responsive to the price- and quality setting. An increase in unobserved heterogeneity between buying and not-buying, via μ_1, results into less elastic total demand. As mentioned in Section 2.2, in the extreme case that the decision to buy at all is unresponsive to the price- and quality setting, both firms maximise the quality difference like in a fully covered market. In this extreme case, each multi-product firm supplies symmetric variants at the low- or high-end of the market. At a critical level of heterogeneity in unobserved attributes between buying and not-buying, each duopolistic multi-product firm switches from supplying equal product variants to supplying differentiated variants. Whether or not the product variants are differentiated between the two firms, depends on the level of heterogeneity in unobserved attributes regarding the elemental alternative: μ_2.

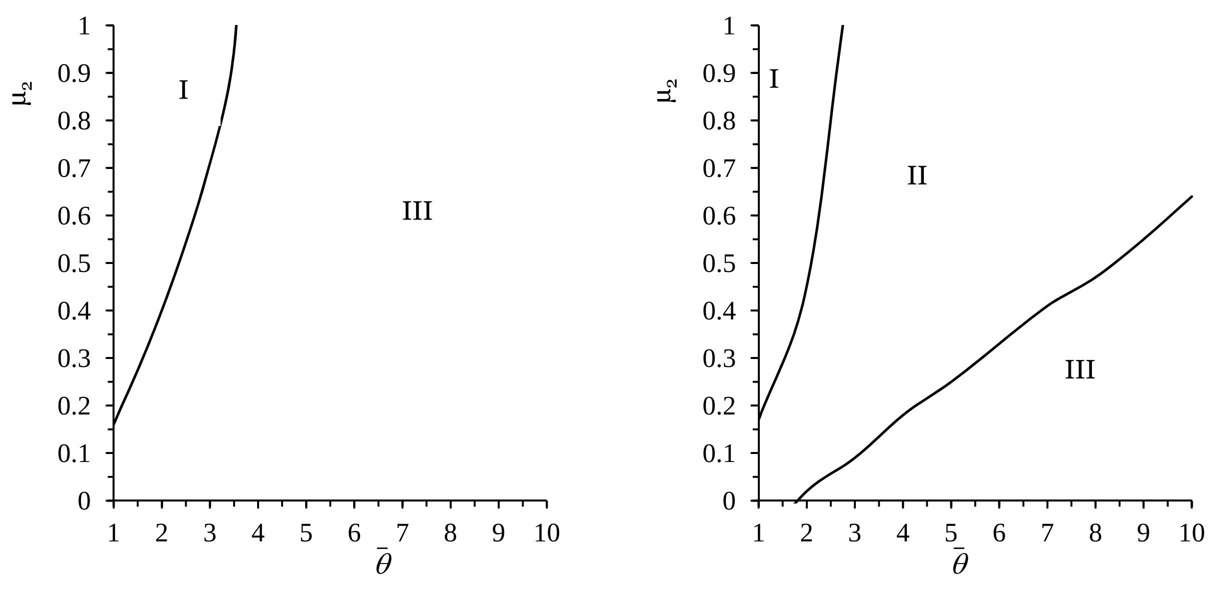
The effect of increasing the unobserved heterogeneity regarding the elemental alternatives is shown in Figure 2.[9] The figure shows the patterns of product differentiation, as function of μ_2 and $\overline{\theta}$, for a single-product duopoly (left panel) and a multi-product duopoly with one firm supplying a single variant and the other firm supplying two variants (right panel). The figure clearly shows that for relatively low levels of observed demand heterogeneity, $\overline{\theta}$, combined with relatively high levels of unobserved demand heterogeneity, μ_2, the equilibrium becomes symmetric (area I in both panels). Differentiation becomes less attractive when consumers are more similar in observed behaviour and less predictable in general. Appendix B shows that if

[9] We set both μ_1 and α equal to 1 and $\underline{\theta}$ equal to 0.

observed demand heterogeneity is low relative to unobserved demand heterogeneity (as in area I), profits are maximised by supplying symmetric variants. Additionally, each firm sets the quality of the product variant in such a way that the marginal costs of quality, $c'(q_i)=q_i$, equal the willingness to pay for quality of the average consumer, $(\partial V_{av}/\partial q_i)/(\partial V_{av}/\partial p_i)=\theta_{av}/\alpha$.[10]

In case of a multi-product duopoly, the area for which a symmetric equilibrium is obtained is smaller than for a single-product duopoly. However, the area for which a fully differentiated asymmetric equilibrium (area III) exists also decreases with the number of product variants supplied in the market. This contradiction is caused by the existence of an intermediate area, area II. In this area, the high-quality firm offers two equal product variants, but different from the variant of the low-quality firm: the so-called asymmetric but not fully differentiated equilibrium. In the remainder of this numerical analysis, most attention will be given to fully differentiated equilibria, as located in area III, because these are the type of equilibria observed in the aviation industry.

Figure 2 Combinations of observed, $\bar{\theta}$, and unobserved, μ_2, heterogeneity for single-product duopoly (left panel) and multi-product duopoly (right panel) yielding different patterns of differentiation: I = symmetric equilibrium, II = asymmetric equilibrium, not fully differentiated, III = asymmetric equilibrium, fully differentiated.



[10] The firms attain the highest mark-up if the quality variants are optimally chosen.

5.2 Multi-product strategies and profitability

Before analysing the impact of unobserved heterogeneity, first profitability of a multi-product strategy for the high-quality firm in the absence of unobserved heterogeneity needs to be analysed. Table 2 shows the effect of a multi-product strategy of the high-quality firm, firm A, on the profitability of both firms given a single-product strategy of firm B. The table shows, in different rows, the Nash equilibria for different orderings of product variants in terms of their quality in the absence of demand heterogeneity in unobserved attributes.[11]

The results in Table 2 provide three main insights. First, the subgame perfect Nash equilibrium for a given number of product variants supplied per firm is not unique. This can be readily verified by looking at equilibria 3.I, 3.II, and 3.III in Table 2; given that in total three product variants are supplied, at least three subgame perfect Nash equilibria exist. More precisely, we observe unique subgame perfect Nash equilibria for each market configuration, i.e. for each number of variants and specific ordering of variants such as 3.I or 3.II. Because the quality stage does not have a unique subgame perfect Nash equilibrium, one cannot solve for the number of variants in the first stage of the model.

Table 2 Market configurations and profitability with deterministic demand: high-quality firm's multi-product strategy. Configurations with inactive variants are denoted in between brackets.

Label	Ordering (in quality) *high* — *low*	Profit *firm A*	Profit *firm B*
2.I	$q_{1A},\ q_{1B}$	$0.0328\overline{\theta}^2 - F$	$0.0243\overline{\theta}^2 - F$
3.I	$q_{1A},\ q_{2A},\ q_{1B}$	$0.0366\overline{\theta}^2 - 2F$	$0.0213\overline{\theta}^2 - F$
3.II	($q_{1A},\ q_{1B},\ q_{2A}$	$0.0305\overline{\theta}^2 - 2F$	$0.0224\overline{\theta}^2 - F$)
3.III	$q_{1B},\ q_{1A},\ q_{2A}$	$0.0264\overline{\theta}^2 - 2F$	$0.0293\overline{\theta}^2 - F$
4.I	$q_{1A},\ q_{2A},\ q_{3A},\ q_{1B}$	$0.0373\overline{\theta}^2 - 3F$	$0.0208\overline{\theta}^2 - F$
4.II	($q_{1A},\ q_{2A},\ q_{1B},\ q_{3A}$	$0.0362\overline{\theta}^2 - 3F$	$0.0212\overline{\theta}^2 - F$)
N.I	$q_{1A},\ q_{2A},\ \ldots,\ q_{n_A A},\ q_{1B}$	$0.0377\overline{\theta}^2 - n_A F$	$0.0205\overline{\theta}^2 - F$

Second, although the subgame perfect Nash equilibrium is not unique in the number of variants, the number of potential candidate equilibria may be more restricted than suggested at first sight in Table 2. Starting from a single-product strategy of both firms, equilibrium 2.I, we observe that firm A only achieves higher profits following a multi-product strategy if the extra variant is positioned at the high-end of the market, i.e. equilibrium 3.I. Supplying a variant with lower quality compared to firm B's variant, equilibrium 3.II, generates multiple 'fighting brands', intensifies competition with firm

[11] The rows are labelled with the first number indicating the number of variants in the market and a roman numeral as index.

B's variant, and causes the market share and mark-up of the own high-quality variant(s) to decrease. As a result, the high-quality firm maximises the differentiation in quality between the own low-quality variant and the variant of the low-quality firm. The high-quality firm achieves this by setting the lowest possible quality, i.e. zero, for this variant. Due to the outside alternative, the high-quality firm cannot capture any mark-up with the low-quality variant. Essentiality, this low-quality variant becomes inactive in equilibrium 3.II. Hence, it is highly unlikely that these equilibria arise. This limits the number of candidate equilibria.[12] The unlikely equilibria are denoted in between brackets in Table 2, Table 3 and Table 4.

Finally, Table 2 shows that the marginal profitability of adding an extra variant diminishes quickly. The maximum profit, ignoring fixed costs, equals $0.0377\overline{\theta}^2$ and is achieved with approximately eight variants. This finding may explain the limited number of product variants one observes in, for example, the aviation industry. In particular if there are (constant) fixed costs related to introducing new variants, decreasing marginal profitability limits the maximum number of variants.[13] From Table 2, it becomes clear that the low-quality firm has lower profits due to the lower mark-ups that can be achieved from serving consumers with a lower willingness to pay.[14] As a result, the marginal profitability of supplying an extra product variant is lower for the low-quality firm. This implies that with equal fixed costs, the low-quality firm would supply less product variants compared to the high-quality firm. Thus, fixed costs may explain both the limited number of product variants and the fact that low-cost carriers provide less variants compared with legacy carriers.

Let's now focus on the incentives of a multi-product strategy for the low-quality firm. The rollout of 'flexi fare' by easyJet, as discussed in Section 4, suggests that an incentive to provide a high-quality variant should be present. The model results as depicted in Table 3, however, tell another story. Table 3 shows that given a two-variant or three-variant strategy of the high-quality firm, the low-quality firm maximises profits by supplying a second low-quality variant, equilibrium 4.I and equilibrium 5.I. If the quality of the second variant is a high-quality variant, located in between variants of the high-quality firm, like in equilibrium 4.II, the profit maximising quality and price of the lowest-quality variant, q_{2B}, become zero. In other words, under the assumption of deterministic demand, the low-quality firm will not supply a second product variant at

[12] Given the quality setting of the low-quality firm, profits for the high-quality firm are slightly higher (< 1 per cent) supplying the inactive variant compared to removing this variant from the product line. However, allowing for fixed costs per variant may easily alter this conclusion in favour of not supplying this low-quality variant at all.

[13] This is valid as long as the number of variants of the low-quality firm is fixed and in absence of fixed costs per variant. In this absence, the firms face the classic prisoner's dilemma and will supply an infinite number of variants.

[14] This finding is in contrast to what is observed in the aviation industry where low-cost carriers attain higher earnings. However, as explained in Borenstein (2011), legacy carriers struggle in closing the cost gap with low-cost carriers and attaining a profitable margin. In the current analysis, generic cost functions for all firms are assumed. Including a more realistic cost setting would complicate the analysis, whereas the qualitative insights regarding differentiation keep unaltered.

the high-end of the market. The same intuition as before with the high-quality firm not supplying low-quality variants applies. The standard model for vertical product differentiation and multi-product firms, hence, produces results that are in sharp contrast to the actual strategic behaviour of easyJet.[15] In the absence of unobserved demand heterogeneity, both the low- and high-quality firm have no incentive for supplying an extra product variant that faces direct competition with the competitor's product variant(s). As a result, the high-quality firm does not undercut the quality of the low-quality firm, whereas the low-quality firm does not supply variants with higher qualities than the high-quality firm does. In other words, interlaced equilibria – in which all product variants have positive prices – do not exist given deterministic demand. Taking into account demand heterogeneity in unobserved attributes restores the existence of interlaced equilibria. In contrast to the deterministic case, the unaltered principle of maximum differentiation between the competing lowest quality product variants of both firms, now results in a quality of the lowest variant in the market approaching the minimum level, like in the deterministic model, but having a positive price and hence a positive mark-up. The effect of taking into account unobserved heterogeneity can be illustrated using the earlier mentioned rollout of 'flexi fare' by easyJet as an example. Table 4 shows the profits for both firms in case of stochastic demand – with $\bar{\theta} = 10$, $\mu_1 = 1$, and $\mu_2 = 0.2$ – and deterministic demand as shown earlier in Table 3. Table 5 shows the detailed price- and quality setting per product variant in each equilibrium. With unobserved heterogeneity, two subgame perfect Nash equilibria arise in case the low-quality firm decides to supply an extra variant. In both equilibria, the profits of the low-quality firm are higher compared with supplying a single low-quality variant only, whereas the profits of the high-quality firm are lower.

Table 3.5 shows that the two equilibria, 5.I and 5.IV, are fully differentiated. The lowest quality variant in the market has the lowest quality possible, close or equal to zero, but with a positive price and therefore mark-up. In addition, equilibrium 5.IV is an example of an interlaced equilibrium. For the chosen parameter setting, a perfect interlaced equilibrium in which all variants are entangled by the competitor's variant, like q_{1A}, q_{1B}, q_{2A}, q_{3A}, q_{2B} or q_{1A}, q_{2A}, q_{1B}, q_{3A}, q_{2B}, does not exist. The reason is that in these configurations the low-quality firm can unilaterally increase its profits by placing its low-quality variant close to its high-quality variant.[16] As a result, an equilibrium completely matching easyJet's strategy is not found, even when accounting for unobserved demand heterogeneity across consumers.

[15] A possible reason for this contrast may lie in the fact that in reality the low-quality firm may believe only to affect competition in the high-end of the market, whereas in theory the whole product line of both firms is affected. For example, Botimer and Belobaba (1999) already indicated that airline yield management models tend to treat the demand for each fare product as completely separate.

[16] Due to the flexibility of the random utility formulation of the model, one does not have to impose the ordering of qualities beforehand. Therefore, one can invalidate equilibria defined prior as subgame perfect Nash equilibria in the deterministic model. This shows that imposing the ordering of qualities beforehand may result in incorrect statements about the existence of subgame perfect Nash equilibria.

Table 3 Market configurations and profitability with deterministic demand: low-quality firm's multi-product strategies. Configurations with inactive variants are denoted in between brackets.

Label	**Ordering (in quality)** high — low	**Profit** firm A	firm B
3.I	$q_{1A},\ q_{2A},\ q_{1B}$	$0.0366\bar{\theta}^2 - 2F$	$0.0213\bar{\theta}^2 - F$
4.I	$q_{1A},\ q_{2A},\ q_{1B},\ q_{2B}$	$0.0326\bar{\theta}^2 - 2F$	$0.0237\bar{\theta}^2 - 2F$
4.II	($q_{1A},\ q_{1B},\ q_{2A},\ q_{2B}$	$0.0289\bar{\theta}^2 - 2F$	$0.0220\bar{\theta}^2 - 2F$)
4.III	$q_{1A},\ q_{2A},\ q_{3A},\ q_{1B}$	$0.0373\bar{\theta}^2 - 3F$	$0.0208\bar{\theta}^2 - F$
5.I	$q_{1A},\ q_{2A},\ q_{3A},\ q_{1B},\ q_{2B}$	$0.0333\bar{\theta}^2 - 3F$	$0.0232\bar{\theta}^2 - 2F$
5.II	($q_{1A},\ q_{2A},\ q_{1B},\ q_{3A},\ q_{2B}$	$0.0362\bar{\theta}^2 - 3F$	$0.0212\bar{\theta}^2 - 2F$)
5.III	($q_{1A},\ q_{1B},\ q_{2A},\ q_{3A},\ q_{2B}$	$0.0296\bar{\theta}^2 - 3F$	$0.0247\bar{\theta}^2 - 2F$)

Table 4 Profitability of multi-product strategy low-quality firm for stochastic – with $\bar{\theta}=10,\ \mu_1=1,$ and $\mu_2=0.2$ – and deterministic demand. Configurations with inactive variants are denoted in between brackets.

Label	**Ordering (in quality)** high — low	**Profit stochastic demand** firm A	firm B	**Profit deterministic demand** firm A	firm B
4.I	$q_{1A},\ q_{2A},\ q_{3A},\ q_{1B}$	$3.77-3F$	$2.08-2F$	$3.73-3F$	$2.08-2F$
5.I	$q_{1A},\ q_{2A},\ q_{3A},\ q_{1B},\ q_{2B}$	$3.38-3F$	$2.29-2F$	$3.33-3F$	$2.32-2F$
5.II	$q_{1A},\ q_{2A},\ q_{1B},\ q_{3A},\ q_{2B}$	NA	NA	($3.62-3F$	$2.12-2F$)
5.III	$q_{1A},\ q_{1B},\ q_{2A},\ q_{3A},\ q_{2B}$	NA	NA	($2.96-3F$	$2.47-2F$)
5.IV	$q_{1A},\ q_{2A},\ q_{1B},\ q_{2B},\ q_{3A}$	$3.33-3F$	$2.30-2F$	($3.33-3F$	$2.33-2F$)

Table 5 Equilibrium price- and quality setting for selected market configurations for stochastic demand with $\bar{\theta}=10,$ $\mu_1=1,$ and $\mu_2=0.2.$

Label	**Ordering (in quality)** high — low	**Qualities** firm A q_{1A}	q_{2A}	q_{3A}	firm B q_{1B}	q_{2B}	**Prices** firm A p_{1A}	p_{2A}	p_{3A}	firm B p_{1B}	p_{2B}
4.I	$q_{1A},\ q_{2A},\ q_{3A},\ q_{1B}$	9.4	8.5	7.3	3.6		57.6	49.1	38.3	12.7	
5.I	$q_{1A},\ q_{2A},\ q_{3A},\ q_{1B},\ q_{2B}$	9.4	8.8	7.6	4.0	1.7	56.5	50.8	39.8	14.5	5.4
5.IV	$q_{1A},\ q_{2A},\ q_{1B},\ q_{2B},\ q_{3A}$	9.3	7.8	0	4.1	2.2	55.3	41.3	1.1	15.0	7.1

6 Conclusion

This article models the number of variants-then-quality-then-price competition for multi-product firms accounting for demand heterogeneity in observed and unobserved attributes. The random utility framework offers the flexibility to model demand for different relative levels of observed and unobserved demand heterogeneity across consumers. Here, we focuse on duopolistic markets. This focus introduces strategic interaction within the number of variants-then-quality-then-price game.

In most industries, firms supply multiple product variants with different price- and quality settings. For example, in the aviation industry, low-cost carriers supply less number of variants, often just one, with a low price- and quality setting, whereas legacy carriers have an extended table of fares to choose from.

Although at first sight the positive relationship between product differentiation and profits may look straightforward, prior studies do not confirm the profitability of a multi-product strategy in duopolistic competition. The only exception, the article by Cheng et al. (2011), shows the existence of a subgame perfect Nash equilibrium under the restriction that each firm only supplies low- or high-quality product variants. Although Cheng et al. (2011) hint at the existence of interlaced equilibria, the results show that both the low- and high-quality firm do not have the incentive to provide an extra product variant at the lower end of the market. This limits the likelihood for interlaced equilibria to exist. In point of fact, our findings suggest that if there are sufficiently high fixed costs related to introducing new variants and demand heterogeneity is only based on heterogeneity in observable attributes, the interlaced equilibrium does not exist.

If the interlaced equilibrium does not exist, the subgame perfect Nash equilibrium is unique given the number of variants supplied by each firm. Including unobserved heterogeneity restores the existence of interlaced equilibria. Whereas with deterministic demand the price, and therefore the mark-up, for the lowest quality variant in interlaced equilibria is zero, unobserved heterogeneity provides the firms with the possibility to set a positive price and obtain a positive mark-up for this particular variant. As a result, including unobserved heterogeneity results in multiple subgame perfect Nash equilibria per number of variants supplied. However, for every market configuration, i.e. number and ordering of variants, the subgame perfect Nash equilibrium, if it exists for the particular configuration, is unique.

The numerical simulation in this article shows the predicted equilibrium patterns of vertical product differentiation for different levels of unobserved heterogeneity. The strategy of easyJet to supply a high-quality, fully flexible fare in addition to its low-quality variant serves as an illustration. Without unobserved heterogeneity, i.e. the classic deterministic model, there is no subgame perfect Nash equilibrium describing the actual strategy of easyJet. Taking into account unobserved consumer heterogeneity results in multi-product equilibria that resemble the actual patterns of product differentiation observed in duopolistic aviation markets. The scope of market configurations that can be studied using the standard model of vertical product differentiation enlarges taking into account unobserved heterogeneity.

Appendix A: Numerical method

In order to approximate the best response functions of the two firms in each market configuration, an algorithm is used in which the price setting stage in Eq. (3) is solved for every set of qualities of the product variants. The number of variants is assumed to be given, but the ordering of the qualities per firm is endogenous. Since potentially the number of sets is infinite, one needs to make the quality dimension discrete. The discrete set of qualities forms the grid for which the pricing stage is solved. We first define the coarse grid by letting the qualities range from $\underline{\theta}$ to $\overline{\theta}$ with step size $\left(\overline{\theta}-\underline{\theta}\right)/10$ and subsequently adapt the grid by adding evaluation points where the reaction functions intersect (i.e. the candidate equilibrium).[17]

The subgame perfect Nash equilibrium in prices, for each set in the grid, is determined in an iterative manner. First, the profit maximisation problem of firm A given a fixed pricing strategy of firm B is solved. Second, the resulting optimal pricing strategy for firm A is introduced into the profit function of firm B. Third, firm B's profits are maximised with respect to firm B's prices.[18] This iterative process continues until the change in both object values, i.e. the profits of firm A and B, is smaller than 0.1 per cent.[19] We use sequential quadratic programming to maximise the non-linear profit functions in each iteration.

After having solved the pricing stage for each quality setting in the grid, one can determine A's best response in qualities for each quality setting of B, and vice versa, taking into account the price setting in the subsequent stage. To check whether the resulting outcome is indeed a profit maximising equilibrium, the determinant of the Hessian matrix – including the second order conditions – for both firms is calculated for every outcome.

[17] Note that if the market is not fully covered and there is no consumer heterogeneity, i.e. there is a single representative consumer t^*, the supplied quality of the highest quality variant will never exceed $q^* = \theta_{t^*} / \alpha$. Similarly, in case of consumer heterogeneity, the supplied quality of the highest quality variant will not exceed the highest willingness to pay. Therefore, the chosen range covers the relevant quality strategy space of both firms.

[18] The fixed pricing strategy of firm B in the first iteration is randomly assigned. After using multiple starting values, one may conclude that the subgame perfect Nash equilibrium is robust against different starting values.

[19] If after a reasonable number of iterations, in our case around 20, convergence is not reached, the subgame perfect Nash equilibrium in prices does not exists for the particular market configuration.

Appendix B: High level of unobserved heterogeneity and symmetry

Here it is shown that when, due to a high level of unobserved heterogeneity, the choice probability, i.e. the market share, is unresponsive to changes in the systematic part of the utility, the best strategy of each firm, given any number of product variants, is to supply product variants that are identical in quality. Furthermore, each firm sets the quality of the product variant in such a way that the marginal costs of quality, $c'(q_i)=q_i$, equals the willingness to pay for quality of the average consumer, $\frac{\partial V_{av} / \partial q_i}{\partial V_{av} / \partial p_i} = \frac{\theta_{av}}{\alpha}$. The mark-up per product variant increases in the number of own product variants supplied and therefore differs between firms if the firms supply an unequal number of variants.

The quality-then-price equilibrium is determined based on the profit function depicted in Eq. (2), ignoring the fixed costs per product variant:

$$\pi_j(\mathbf{p};\mathbf{q};\mathbf{r}) = \sum_{i=1}^{n_j} \left(p_{ij} - c(q_{ij}) \right) x_{ij}, \tag{B.1}$$

with demand x_{ij} as defined in Eq. (7) to Eq. (9). In the first stage, all first order conditions with respect to prices of all product variants need to be determined and solved simultaneously for the prices. In the second stage, all first order conditions with respect to qualities are determined and solved for the qualities evaluated at optimal prices. With a sufficient high level of unobserved heterogeneity, as a result from an increase in the scale parameters μ_1 and μ_2, the average probabilities of any particular variant or the probability of buying the product at all are equal for each product variant supplied by a single firm. Therefore, *after* taking the first order conditions with respect to prices and qualities, one can substitute $\mathrm{P}_{k,t} = \mathrm{P}_{k,-t} = \mathrm{P}_k$ and $\mathrm{P}_{ij|k,t} = \mathrm{P}_{ij|k,-t} = \mathrm{P}_{ij|k}$. Note that $\mathrm{P}_{ij|k} = \mathrm{P}_{-ij|k}$ for all product variants supplied by a single firm. Under these conditions, the first order condition with respect to prices becomes:

$$\frac{\partial \pi_j(\mathbf{p};\mathbf{q};\mathbf{r})}{\partial p_{ij}} = \mathrm{P}_{ij|k}\mathrm{P}_k\left(1 - \frac{\alpha}{\mu_2}(\psi_{ij})\right) + \alpha \mathrm{P}_{ij|k}^2 \mathrm{P}_k \left(\frac{1}{\mu_2} - \frac{1-\mathrm{P}_k}{\mu_1}\right) \sum_{i=1}^{n_j} \psi_{ij} \stackrel{!}{=} 0, \tag{B.2}$$

with $\psi_{ij} = p_{ij} - \frac{q_{ij}^2}{2}$. Solving Eq. (B.2) for ψ_{ij} yields the mark-up:

$$\psi_{ij} = \frac{\mu_1 \mu_2}{\mu_1 - n_j \mathrm{P}_{ij|k}\left(\mu_1 - \mu_2(1-\mathrm{P}_k)\right)}. \tag{B.3}$$

The denominator in Eq. (B.3) is decreasing in n_j, hence the mark-up is increasing in n_j. The first order condition with respect to qualities equals:

$$\frac{\partial \pi_j(\mathbf{q};\mathbf{r})}{\partial q_{ij}} = \mathrm{P}_{ij|k}\mathrm{P}_k\left(\left(\frac{1}{\mu_2}(\psi_{ij}) - \mathrm{P}_{ij|k}\left(\frac{1}{\mu_2} - \frac{1-\mathrm{P}_k}{\mu_1}\right)\left(\sum_{i=1}^{n_j}\psi_{ij}\right)\right)\frac{1}{T}\sum_{t=1}^{T}\theta_t - q_{ij}\right) \overset{!}{=} 0, \tag{B.4}$$

with $t \in T$. It is now possible to substitute Eq. (B.3) into Eq. (B.4) in order to evaluate the first order condition at optimal prices. Solving for q_i^* yields:

$$q_i^* = \frac{1}{\alpha}\frac{1}{T}\sum_{t=1}^{T}\theta_t. \tag{B.5}$$

The numerator in Eq. (B.5) equals the quality sensitivity of the average consumer. So, it follows that $q_i^* = \frac{1}{\alpha}\frac{1}{T}\sum_{t=1}^{T}\theta_t = \frac{\theta_{av}}{\alpha}$ if the unobserved heterogeneity is at the level that market shares are not responsive anymore to the systematic part of the utility function.

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