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DEVELOPMENT OF AN EMPIRICAL MODEL OF PAVEMENT ROUGHNESS WITH CONDITION SURVEY DATA

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ABSTRACT

This paper presents a model for pavement roughness using empirical survey data that simultaneously corrects for two endogenous explanatory variables: pavement overlay thickness and maintenance and rehabilitation activities performed. These two variables are typically treated as exogenous inputs in previous models, but they are usually not randomly chosen—both are design variables that are selected by pavement engineers based on current and expected field conditions. To account for this endogeneity, two auxiliary models are created to obtain predicted values of these design variables. The predicted values from these models are then used in the roughness progression model to correct for any possible endogeneity bias. The resulting roughness progression model provides more consistent and intuitive parameter estimates than those obtained in previous studies using the same data.

Keywords: roughness progression model, endogeneity correction, empirical pavement modelling

INTRODUCTION

Pavement roughness adversely affects the ride quality of vehicles on a roadway, which can potentially damage valuable goods, abd increase vehicle fuel consumption and operation costs (GEIPOT, 1982; Paterson, 1987; Al-Omari and Darter, 1994). To properly maintain roadway surfaces, Pavement Management Systems (PMS) have been designed that optimally allocate scarce resources for maintenance and rehabilitation (M&R) activities. However, these PMS rely on models of pavement roughness progression to determine where resources are needed most.

Models of pavement roughness have been created based on experimental data (Ozbay and Laub, 2001; Prozzi and Madanat, 2004; Puccinelli and Jackson, 2007) and empirical field data (Way and Eisenberg, 1980; Karan et al, 1983; Paterson, 1987; Kay et al, 1993; Gulen et al, 2001; Prozzi and Madanat, 2003; Madanat et al, 2005). Models of the former type are not preferred due to concerns that experimental tests do not accurately reflect real-world deterioration. However, models of the second type can be inaccurate if not properly specified of if they include endogenous parameters as explanatory variables.

To address this second issue, this paper presents the development of a model of pavement roughness deterioration that corrects for endogeneity in two explanatory variables: 1) thickness of pavement overlays, and 2) maintenance and rehabilitation (M&R) activities. Both of these are design variables selected by pavement engineers based on conditions in the field—e.g., pavement sections expecting the most deterioration often have thicker pavement overlays and more frequent M&R activities performed. This relationship needs to be accounted for or estimates of the model parameters will suffer from endogeneity bias. To account for the endogeneity of these variables the instrumental variables method (for overlay thickness) and the selectivity correction method (for M&R activities) are used. The resulting model of pavement roughness progression should have more consistent parameter estimates than previous models that do not correct for this endogeneity bias.

The rest of this paper is organized as follows. We first describe the empirical dataset used in this study to develop the model for pavement roughness. Then, we explain the source of endogeneity bias and the methodology that will be used to correct for its presence. Next, we present the results of the model development. Finally, we summarize the conclusions.

DATA

This model was created using data from the Washington State Pavement Management System (WSPMS) database. This database consists of pavement condition data collected along each of the state roads from 1983 to 1999. Each road was divided into 0.1 sections and each section was observed multiple times during the duration of the data collection period, resulting in a two-dimensional panel dataset. A total of 352,803 observations were available from 48,484 unique roadway sections. A subset of about 60,000 observations was randomly selected for modeling purposes.

The data included information about the road surface conditions, traffic conditions, environmental conditions, and any maintenance and rehabilitation activities that were performed. These variables included:

- Cumulative traffic loading [in equivalent single axel loads, or ESALs]
- Current year traffic loading [ESALs]
- Base thickness [ft]
- Thickness of last overlay [ft]
- Minimum temperature [°F]
- Maximum temperature [°F]

- Annual precipitation [in]
- Time since last overlay [years]
- Time since last maintenance activity [years]
- Type of M&R activity [AC overlay, BST treatment, Maintenance]
- Roughness (IRI) in previous year [cm/km]
- Change in roughness [cm/km]

METHODOLOGY

A linear regression model was used to describe pavement roughness progression as a function of several of the potential explanatory variables available in the dataset. Specifically, a random effects model with two error terms was used that accounted for the random effects of individual roadway sections (invariant of time) as well as random error terms that occur over time at each location (Washington et al, 2003). The functional form of this model is presented below in Equation 1.

$$y_{it} = \beta_0 + \beta_1 X_{1it} + \beta_2 X_{2it} + ... + \beta_K X_{Kit} + \upsilon_i + \varepsilon_{it} \quad \textbf{(1)}$$

In Equation 1, y_{it} is the change in roughness for section *i* at time *t*, $\beta_1,\ldots,\beta_K$ are the model parameters, and $X_{1it},\ldots,X_{Kit}$ are the explanatory variables. The first error term, υ_i, captures the unobserved heterogeneity (cross sectional variation) between different roadway sections. The second error term, ε_{it}, captures the random error of each section that changes over time. To estimate this model, the two-step generalized least squares (GLS) method was applied (Freedman, 2005).

For pavement roughness, two potential explanatory variables are likely to be endogenous and thus correlated with the error terms: overlay thickness and type of maintenance and rehabilitation activity performed. Both of these are design variables that are selected by pavement engineers based on actual or expected conditions; they are not randomly chosen and cannot be assumed exogenous (Madanat et al, 1995; Madanat and Mishalani, 1998). This endogeneity needs to be accounted or else estimates of the vector of parameters, $\overline{\beta}$, will be biased.

Two methods were used to address this endogeneity. The instrumental variables method was used for the continuous variable overlay thickness (Mannering, 1998). In this method, the endogenous variable is replaced in the GLS model by another that is: 1) highly correlated with it and 2) uncorrelated with the error terms in the GLS model. Such a variable was obtained by estimating an auxiliary model for the endogenous variable using linear regression. This model was a function of several explanatory variables which may or may not be included in the roughness progression model. The predicted values of the endogenous variable were then substituted for the variable in the GLS model since these predicted values were uncorrelated with the error terms. The use of a continuous instrumental variable changes the roughness progression model to the form presented in Equation 2.

$$y_{it} = \beta_0 + \beta_1 X_{1it} + \beta_2 X_{2it} + ... + \beta_{K-1} X_{K-1it} + \beta_K \widehat{X}_{Kit} + \upsilon_i + \varepsilon_{it}, \quad \textbf{(1)}$$

where $\widehat{X}_{Kit}$ is the predicted value of the endogenous variable obtained from the auxiliary model.

The selectivity correction approach was used for the discrete variable M&R activity type (Train, 1986; Mannering and Hensher, 1987). In this method, a discrete choice model was developed to estimate the probabilities of selecting one of several M&R options. The probability of selecting M&R alternative *j*, $\widehat{P}_j$, was then used to add a new explanatory variable in the GLS model known as the selectivity correction term. For a logit discrete choice model (which was used here) with *J* different choices, *J-1* selectivity terms could be added to the GLS model. The inclusion of these terms changes the model to the form presented in Equation 3.

$$y_{it} = \beta_0 + \beta_1 X_{1it} + \beta_2 X_{2it} + ... + \beta_{K-2} X_{K-2it} + \sum_{j=1}^{J-1} \gamma_j \lambda_j + \beta_K \widehat{X}_{Kit} + \upsilon_i + \varepsilon_{it} \quad \textbf{(3)}$$

where $\lambda_j = \left\{ \frac{J-1}{J} \log \widehat{P}_j + \sum_{l=1, l \neq j}^{J} \frac{\log \widehat{P}_l}{J} \left[\frac{\widehat{P}_l}{1-\widehat{P}_l} \right] \right\}$ was calculated using the probabilities from the discrete choice logit model and γ_j were parameters to be estimated.

MODEL DEVELOPMENT

This section applies the methodology described in the previous section to develop auxiliary models for endogeneity correction and the final pavement roughness progression model.

Endogeneity correction of overlay thickness

To use the instrumental variables method, an auxiliary linear regression model was developed to predict the overlay thickness as a function of several explanatory variables. The variables were chosen based on our knowledge of pavement design methods. The objective of this exercise was to develop an empirical model that would produce overlay thicknesses that are close in values to those designed by Washington DOT's pavement engineers. The resulting model is presented in Equation 4.

(log of overlay thickness)$_{it}$ = α_0 + α_1*(current traffic loading)*$_{it}$ + α_2*(log of prev. roughness)*$_{it}$ + α_3*(time since last maint. activity)*$_{it}$ + α_4 *(min. air temp)*$_{it}$ + v_i + e_{it} **(4)**

where v_i and e_{it} are error terms.

Table 1 presents the estimates of the parameters α_0--α_4 using the GLS method. The parameter estimates are both realistic and conform to a priori expectations. Thicker overlays are provided for roadway sections that experience heavier traffic volumes (higher value of current traffic loading) and that are in a more deteriorated state (higher value of previous roughness). Thinner overlays are provided for warmer climates since fewer freeze-thaw cycles would be expected. The time since last maintenance activity was found not to be statistically significant. Therefore, while it was expected that thicker overlays would be provided for roadway sections that have not had recent M&R activities performed, this may not be the case.

Table I – Model estimates for overlay thickness

	Parameter Estimate	T-Statistic	P-Value
Current Year ESALs	4.10E-02	11.05	0.00
Log (Previous Roughness)	6.04E-03	10.83	0.00
Time since last Maintenance	2.46E-05	0.54	0.59
Minimum Temperature	-6.12E-04	-15.37	0.00
Constant	1.37E-01	45.38	0.00
R-squared		0.882	
$\boldsymbol{\sigma_v^2/(\sigma_v^2+\sigma_e^2)}$		0.856	

The model seems to have a very good fit, as evidenced by the high R-squared value (0.882). Additionally, the random-effects model is appropriate, due to the high heterogeneity across pavement sections. σ_v^2 represents the variance of the random disturbance v_i, shown in Equation 4, capturing the unobserved heterogeneity between different roadway sections in the panel data. σ_e^2 represents the variance of the random disturbances e_{it} in Equation 4 and accounts for random errors that occur across time and roadway sections. The ratio of the variance of the error terms between different roadway sections to the total variance ($\sigma_v^2 + \sigma_e^2$) shows that unobserved heterogeneity represents a high fraction of the total unobserved variation in the model (0.856).

Endogeneity correction for M&R activity type

Another auxiliary model, this time multinomal logit (MNL), was developed to predict the probabilities of performing various M&R activities. The objective was to represent empirically the process by which Washington DOT engineers select the M&R treatments to apply to different pavement sections. Four possible activities were available: do-nothing, perform an AC overlay, BST treatment, or routine maintenance. The probability of selecting activity *j* is given by Equation 5.

$$\Pr(i) = \frac{\exp(V_j)}{\sum_{j=1}^{J}\exp(V_j)} \tag{5}$$

where V_j is the utility of alternative *j*. The utilities of the various M&R activities were modeled as a function of several explanatory variables, chosen based on assumptions about M&R decision-making. The resulting model specification is presented in Equation 6.

utility of AC overlay = $\theta_0 + \theta_1$*(log of previous roughness)* + θ_2*(overlay age)* + θ_3*(current year traffic loading)*
utility of BST treatment = $\varphi_0 + \varphi_1$*(log of previous roughness)* + φ_2*(overlay age)* + φ_3*(current year traffic loading)*
utility of maintenance = $\psi_0 + \psi_1$*(log of previous roughness)* + ψ_2*(overlay age)* + ψ_3*(current year traffic loading)* **(6)**

Note that these utilities are relative to the do-nothing alternative.

Table 2 presents the estimates of the parameters θ_0--θ_3, φ_0--φ_3, ψ_0--ψ_3 for the MNL model. Most parameter estimates conform to a priori expectations. Compared to the do-nothing alternative, agencies are more likely to perform M&R activities on more deteriorated pavement sections, and more likely to perform AC overlays and BST treatments on the most deteriorated pavement sections as evidenced by the signs and magnitudes of θ_1, φ_1 and ψ_1. Washington DOT pavement engineers are also more likely to perform AC overlay and maintenance activities for pavement sections that experience heavier traffic loading. The model also confirms that agencies are also less likely to apply a BST treatment on pavement sections with higher traffic loading, since BST treatments are usually selected for lower-traffic segments by Washington DOT engineers (Li et al, 2008).

Table 2 – Model estimates for M&R activity type

	AC Overlay		BST Treatment		Maintenance	
	Parameter Estimate	**P-Value**	**Parameter Estimate**	**P-Value**	**Parameter Estimate**	**P-Value**
Constant	-1.59E+01	0.00	-1.65E+01	0.00	-9.54E+00	0.00
log(Prev Roughness)	2.55E+00	0.00	2.58E+00	0.00	1.76E+00	0.00
Overlay Age	7.42E-04	0.00	---	---	-2.17E-03	0.00
Current Year ESALs	2.62E+00	0.00	-1.18E+01	0.00	1.70E+00	0.00

A higher value of overlay age was found to increase the probability of performing an AC overlay but *decrease* the probability of performing routine maintenance (as compared to doing nothing). While this may initially seem counter-intuitive, it actually makes perfect sense from an agency perspective. As an overlay ages, decision makers may put off routine maintenance for that roadway section because they know a new overlay will be applied in the near future. Therefore, as overlays ages, the probability of doing nothing or performing an AC overlay will increase, but the probability of performing routine maintenance will decrease. Note that overlay age was found to be statistically insignificant for the BST treatment activity.

The MNL model has a goodness-of-fit value (ρ^2) of 0.061. While this is not high, it should be remembered that goodness-of-fit values for discrete models are always much smaller than those of regression models, and most variables are statistically significant. Additionally, a log-likelihood test was performed to determine the model's statistical significance and this had a p-value of 0.00 which means that the model is indeed statistically significant. Therefore, this model was used to determine probabilities of performing different M&R

activities in the endogeneity correction. Using the different probabilities, the correction terms for M&R activities were calculated as shown in Equation 3.

Model for pavement roughness progression

The previous results were included into a linear regression model to predict pavement roughness progression (the increase in roughness between two observations) as a function of several explanatory variables. The explanatory variables were chosen based on knowledge of pavement deterioration and included environmental variables, pavement variables, traffic variables, and the endogeneity corrections. Note that for the M&R correction, we only included the correction term for the AC overlay because BST treatments and routine maintenance are not performed to directly correct for pavement roughness. The model is presented in Equation 7.

$$(\text{change in pavement roughness})_{it} = \beta_0 + \beta_1(\text{previous pavement roughness})_{it} + \beta_2(\text{cumulative traffic loading})_{it} + \beta_3(\text{predicted overlay thickness})_{it} + \beta_4(\text{base thickness})_{it} + \beta_5(\text{min. air temp})_{it} + \beta_6(\text{precipitation in current year})_{it} + \beta_7(\text{overlay age})_{it} + \beta_8(\text{AC overlay correction term})_{it} + \upsilon_i + \varepsilon_{it} \quad \textbf{(7)}$$

where υ_i and ε_{it} are error terms.

Table 3 presents the estimates of the parameters β_0--β_8 using a random effects model and estimated using the GLS method. Overall, the model seems to have a good fit, as evidenced by the moderately high R-squared value (0.413). Further, it is clear that unobserved heterogeneity is present and thus the use of GLS is appropriate, given the value of the error ratio (0.164).

Table 3 – Model estimates for pavement roughness progression

	Parameter Estimate	T-Statistic	P-Value
Previous Roughness	-2.43E-01	-43.96	0.00
Cumulative ESALs	2.42E+00	9.88	0.00
Predicted Overlay Thickness	-4.78E+02	-9.38	0.00
Base Thickness	-5.72E+00	-9.73	0.00
Minimum Temperature	-2.68E+00	-19.81	0.00
Precipitation	1.56E-01	16.04	0.00
Overlay Age	1.52E-02	10.16	0.00
AC Overlay Correction Factor	-1.61E+01	-18.23	0.00
Constant	1.42E+02	10.73	0.00
R-squared	0.413		
$\sigma_\upsilon^2/(\sigma_\upsilon^2+\sigma_\varepsilon^2)$	0.164		

The estimates of the coefficients are realistic and conform to a priori expectations. The model predicts that, all else constant, pavement roughness progression is concave—the change in roughness decreases as pavements become rougher. This concave deterioration pattern has also been observed in the WSPMS data for cracking (Madanat et al, 2010). Pavement roughness progression is also found to increase with cumulative traffic loading, precipitation and overlay age, as expected. Roughness progression decreases for roadway sections with thicker overlays and thicker bases and for higher minimum temperatures.

Model discussion

Predicted values of pavement roughness deterioration can be estimated using (3) and the parameters in Table 3. To examine how well this model predicts the pavement data, cumulative distributions of the predicted and observed values are plotted in Figure 1. Conditional forecasting was applied in which the observed values of the continuous endogenous variable, overlay thickness, were inserted directly into (3). As shown in the figure, the model predicts the data fairly well although there is some over-prediction of large negative values.

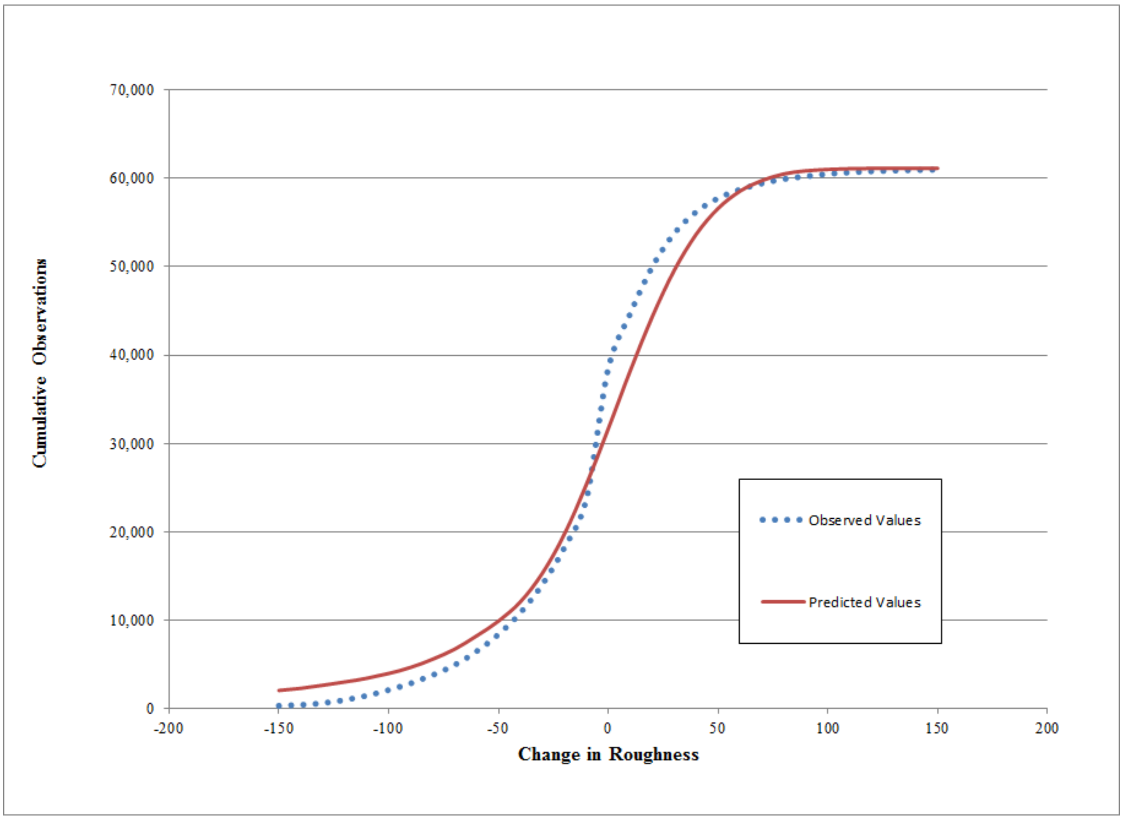

Figure 1 – Cumulative distribution function for observed and predicted values of change in roughness

Typically in a linear regression model, the parameter coefficients reflect the change in the dependent variable due to a unit change in one of the independent variables. However, this model includes endogeneity corrections for maintenance activities that are a nonlinear function of some of the explanatory variables. Therefore, the effect of changing an explanatory variable needs to be examined more closely. Figure 2 shows the effect of changing relevant explanatory variables on pavement roughness progression. Variables were examined at their mean value and ±1 and ±3 standard deviations away from the mean. In some cases, this method resulted in a value that was out of the feasible range for the variable; e.g., negative values for variables that must be positive. For such variables (traffic loadings and base thicknesses) either 0 or the minimum observed value was used instead. For the current year traffic loading, a change in this value resulted in a corresponding change in the cumulative loading variable since the cumulative loading variable includes the current year traffic loading. Note that when one variable was changed, all other variables were kept at their mean value in the dataset.

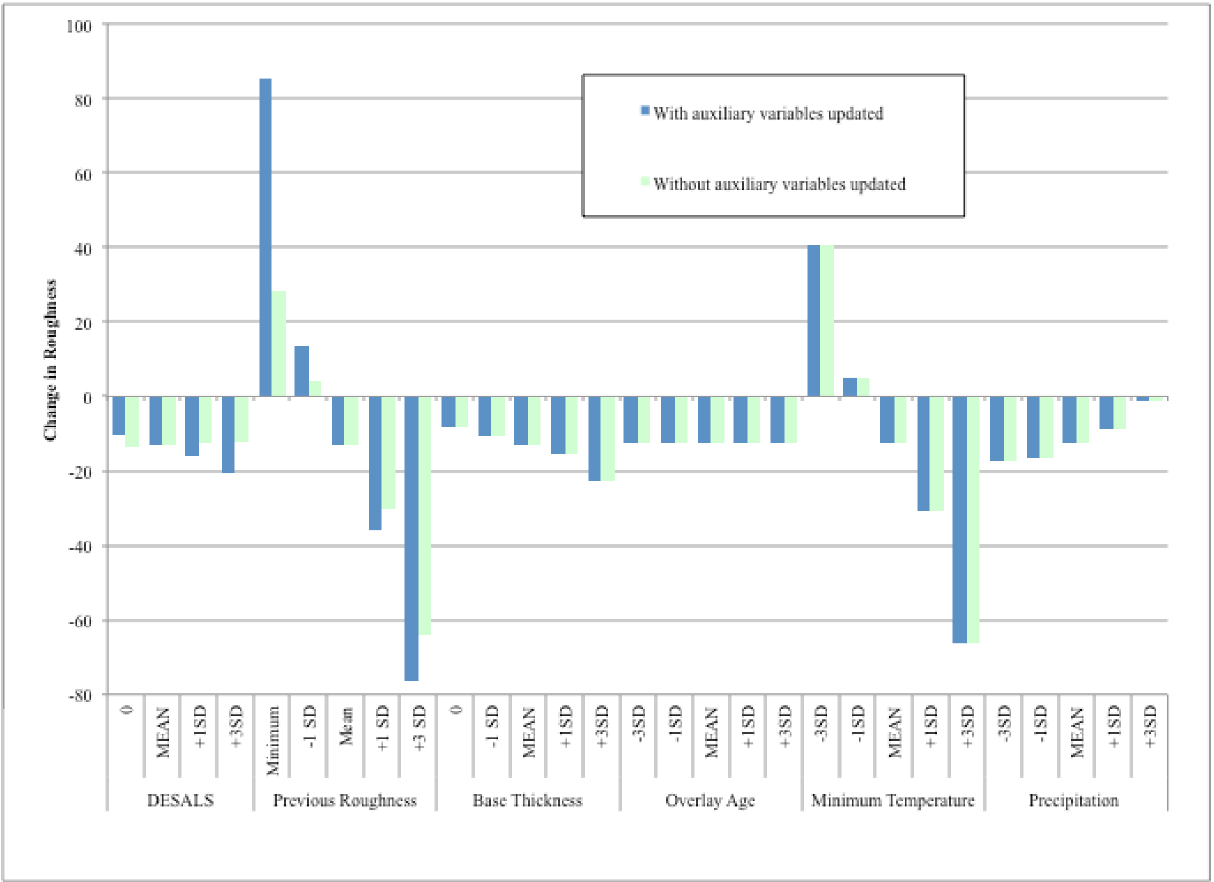

Figure 2 – Effect of different parameters on predicted change in roughness

Figure 2 presents the change in roughness both when changes to independent variables are included in predictions of auxiliary variables and when the auxiliary variables are not updated. The results for some variables (base thickness and precipitation) are exactly the same with and without updates to auxiliary variables because these variables are not included in the auxiliary models. When auxiliary models are updated, we see that the change in roughness decreases with traffic loading—the higher the current year traffic loading, the lower the roughness progression. This may not make intuitive sense until one considers the fact that sections with higher traffic loading would have higher base thicknesses and an increased likelihood of M&R activities being performed. If these two variables are held constant and are not updated, then we see the relationship that we expect—higher current year traffic loadings lead to increased pavement roughness progression. This same relationship is observed for the variable overlay age, although the magnitude of the difference is so small that it does not appear in Figure 2.

For previous roughness, we see that the general trend stays the same both when auxiliary variables are updated or are not updated; however, the magnitude of the roughness progression changes. The magnitude is much greater when auxiliary variables are updated than when they are not. This reflects the fact that pavement sections in good condition would have lower probabilities of M&R activities being performed, which would serve to exacerbate the deterioration of the pavement compared with pavement sections in poor condition.

Based on Figure 2, the variables that cause the highest variation in the change in pavement roughness are previous roughness, minimum temperature, precipitation, annual traffic loading and base thickness (in that order). Overlay age does not seem to have much of an effect on the change in pavement roughness as the predicted change in roughness changes very little for the entire range of overlay age.

The coefficient estimates presented in Table 3 can also be compared with those of a previous pavement roughness progression model (Madanat et al, 2005) to see how correcting for endogeneity changes the influence of different variables when M&R activity probabilities are held constant. This comparison shows that by correcting for endogeneity, temperature and precipitation have a more pronounced impact on roughness progression while overlay age has a less pronounced impact. Perhaps more importantly, the previous model had a negative coefficient for cumulative traffic loading, which surprisingly suggests that pavements deteriorate less quickly under heavy loads. After correcting for endogeneity, the sign of this coefficient is now positive which conforms to a priori expectation about the underlying physical process.

CONCLUSIONS

This paper presents a methodology to account for endogeneity in pavement roughness models by including M&R activities and overlay thickness. Including these endogeneity corrections seems to provide a pavement roughness model with more accurate parameter coefficients than a previous model that does not include these corrections (Madanat et al, 2005). The estimated coefficients all meet a priori expectations and are in accordance with knowledge of pavement deterioration, unlike some of those in the previous model developed with the same dataset. The model seems to predict well for values of change in pavement roughness close to the mean and less well for values far from the mean. The inclusion of endogeneity corrections also sheds insight onto the expected change in pavement roughness when M&R decision-making is included.

The model for M&R activities created as a part of the endogeneity correction also revealed that the probability of routine maintenance of a pavement section decreases with age. This makes sense because agencies are more likely to put off performing routine maintenance on a pavement section (which only slows deterioration) if they know a rehabilitation activity will be applied in the near future. Further work is required to confirm that this type of M&R decision-making behavior is also found in the datasets of other highway agencies.

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