Equilibrium between travel demand system supply and urban structure

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INTRODUCTION

W hile techniques for urban travel-demand modeling have reached high levels of sophistication, basic understanding of the travel process still lags behind. This paper tries to close the gap, by suggesting an approach which, though is in some disagreement with current models, appears to unify the isolated components of travel under a unified process, consistent with basic economic concepts. More specifically, it seems to both describe and explain the interactions between travel demand, transportation system supply and urban structure.

This paper is based on on-going research conducted for the Urban Projects Department of the World Bank, and the US Department of Transportation, FHWA, ². It should, however, be noted that

(i) the results are preliminary, and

(ii) the views expressed in this paper are those of the author and not necessarily those of either of the above two organizations.

Since the subjects of urban travel are too numerous and complex to be covered in one paper, the presentation in this paper is short, telegraphic in style, with the belief that a perspective view of the concepts is preferable at this stage to a detailed, micro scrutinization of only a limited number of issues.

The first part of the paper describes the Unified Mechanism of Travel (UMOT), while the second part sug-

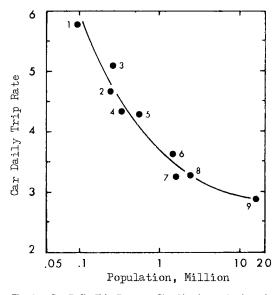


Fig. l - Car Daily Trip Rate vs. City Size in a selection of Cities in the US.

gests several possible implications of the UMOT to the understanding of, and models for, urban travel.

Travel demand models should preferably be based on behavioral travel phenomena which tend to be stable both cross-sectionally, within and between cities, and over time. For instance, trips, which are currently regarded as the building blocks of travel demand, do not meet such a criterion; consider a car driver who used to make 6 trips per day in his small hometown, but after moving to a large city reduced it to 3 trips per day; did his travel demand change? Did his mobility decrease?

Figure 1 shows how the trip rate changes with city size, as one parameter of many, in a selection of cities in one country (App.1).

It may, therefore, be inferred that models which are based on trips are not fully transferable between cities and, as such, have to be carefully calibrated to local conditions in each city separately. But if they are not fully transferable between two cities during the same period, how can we be certain that they are transferable over time for one city, especially when this city is planned to undergo substantial expansion in the future? While this problem may not be serious in relatively stable cities in developed countries, it could become critical in rapidly expanding cities in developing countries, some of which double their population every decade.

The problem facing us, therefore, is to find the stable behavioral travel phenomena upon which demand models can be based with a high level of assurance and reliability.

Another subject discussed in this paper is the a-priori assumption that travel demand, system supply and urban structure are in equilibrium. This approach is mainly dictated by the structure of the current models, which have to be calibrated by the observed daily trips. Furthermore, an equilibrium condition must always be reached at the conclusion of the converging iterations, whether of trip distributions or traffic assignments. Thus, travel demand is portrayed on a daily basis, always under equilibrium conditions with supply, and latent demand is reflected, if at all, implicity only.

It will be suggested here that travel demand, in the general sense (including latent demand), may be in disequilibrium with system supply, and that this disequilibrium is one of the major forces that can change urban structure.

It was already suggested by many researchers that the money and time allocated by travelers (or their households) for travel act as constraints within which travel benefits (utilities) are maximized ³,⁴. While the concept was basically correct, its application suffered from two difficulties: (i) the daily money and time constraints are mostly applied in the early stages of model formulation, to appear later in the combined 'generalized-cost' as relative measures affecting single trips only, and (ii) such models still need calibration on the same trips that they are expected to produce as outputs, thus they tend to be self-fulfilling.

Although the UMOT starts with the same basic concepts of travel money and time budgets, it applies them in a different way, thus arriving at conclusions which diverge from the current beliefs and techniques. It considers the travelers - and not their trips - as the building blocks, the indivisible units, of travel demand models. Furthermore, it is shown that only then are the two travel budgets found to be stable both cross-sectionally and over time, as detailed in the following sections.

THE TRAVEL TIME BUDGET

Since the resources of time and money available to travelers are limited, they have to allocate them sparingly to their daily activities, in accordance with their perceived values of each.

The available evidence indicated that the daily doorto-door travel time per traveler, including access time, tends to be stable, at about 1.1 hrs., both crosssectionally and over time, even when speeds increase by over 30 percent, as can be seen in Table 1. This daily travel time per traveler is defined as the travel time budget, or TT-budget for short.

 Table 1 - The TT-budget and other travel characteristics of car and transit travelers in a selection of cities in the US

City	Year -	CAR TRAVELERS				TRANSIT TRAVELERS			
		TT,hr.	v,kph.	R	D,km.	TT,hr.	v,kph.	R	D,km.
Washington Washington Twin Cities Twin Cities Whole US	1955 1968 1958 1970 1970	1.09 1.11 1.14 1.13 1.06	18.8 23.3 21.5 28.5 47.4	3.07 3.16 3.62 3.84 3.33	20.48 25.91 24.48 32.26 50.47	(1.27) (1.42) 1.05 1.15 0.99	10.7 10.0 12.0 12.1 24.6	2.31 2.12 2.12 2.09 2.03	13.60 14.35 12.58 13.87 24.33

Hence, while travelers may save time on single trips when speeds increase, they appear to trade it off for more travel. In Washington, D.C. and Twin Cities most of the time savings went into longer trip distances, which are not represented at all in the travel demand models.

It may be added that trip rates were hardly affected, which suggests that models based on trip rates are not sensitive to system supply, nor to urban size and structure. Furthermore, the apparent stability of trip rates over time - a strong argument for their use for travel demand, forecasting - actually does not reflect travel demand in the general sense. For instance, a 33 percent increase in speed in Twin Cities during 1958-1970 resulted in a corresponding increase in the daily travel distance, but in only a marginal increase in the trip rate.

It should also be noted that if travel speeds fall below a critical level, of about 11 kph., travelers appear to spend additional time in order to make the minimum number of just 2 trips per day, as was the case in Washington, D.C. in both 1955 and 1968. Namely, under US conditions it seems that only when the daily travel time is above 1.1 hrs., can be there the real savings in time if speeds increase.

^{*} In conclusion, although the TT-budget may differ widely between different travelers from day to day, it appears to be a stable behavioral phenomenon, at least as a controlling total, thus it can be used as a basis for describing and explaining travel demand and patterns.

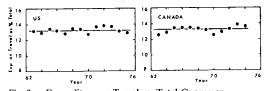


Fig. 2 - Expenditure on Travel vs. Total Consumer Expenditure, US 1963-1975 and Canada 1963-1974

THE TRAVEL MONEY BUDGET

The concept of the travel money budget, TM-budget for short, is similar to the TT-budget, in the sense that the money resources available to a household are limited and, therefore, have to be divided between different activities in accordance with their perceived value. The TM-budget tends to be stable over time, similar to the TT-budget, as can be seen in Figure 2. (The proportions may be different between countries, depending on local definitions and living habits).

The TM-budget tends to be a stable proportion of income also when analyzed cross-sectionally. Figure 3 shows the daily expenditures on travel by households who made all their travel by cars versus their annual income, in Washington, D.C. and Twin Cities.

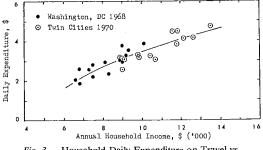


Fig. 3 - Household Daily Expenditure on Travel vs. Annual Income (when all travel was by car only)

The relationship can be expressed, within the range of observations, by:

Exp./_{HH}, = -28.95 + 3.52 In Inc.; ($r^2 = 0.807$); (1) It then becomes evident that the expenditure on travel, as a proportion of income, is very stable at about 10.5 percent *at all income levels*, as can be seen in Figure 4. This figure also shows the same trend in the whole UK (although at a different level, depending on the definitions).

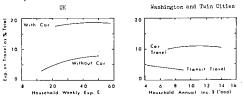


Fig. 4 - Household Expenditure on Travel vs. Total Household Expenditure, UK 1972; and vs. Household Income, Washington D.C. 1968 and Twin Cities 1970

Figure 4 also shows the proportions of expenditure on travel by households that made all their travel by transit, where it becomes evident that a wide gap exists between the two categories of households, a crucial gap that will be discussed later.

In conclusion, it appears that travelers are constrained in their daily travel by their individual TT-budgets and by their households' TM-budget.

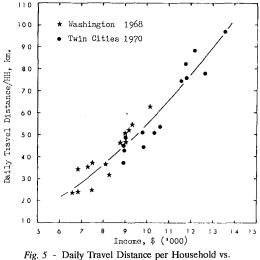
APPLICATION OF THE TT-BUDGET FOR TRAVEL DEMAND ESTIMATION

Before going on any further, let us first examine the meaning and possible application of the TM-budget for travel demand estimation. The following exercise is based on data from the studies in Washington, D.C. 1968 and Twin Cities 1970.

Figure 5 shows the daily travel distance per household, by district, versus income for households who made all their travel by car. The relationship can be expressed by:

D/_{HH}, km. = 1.8(10 - 6)Inc. 1.869; (r² = 0.885); (2)

(Data on a disaggregated basis result in even a better relationship, $r^2 = 0.943$).



Annual Income (all travel by car only)

It can also be shown that the daily door-to-door travel speed of households is strongly related to the income level, and can be expressed for the above case by: v, kph. = -2.72 + 0.0029 Inc. (r² = 0.833); (3)

The car travel cost per unit distance depends on the speed of travel, as can be seen in Figure 6 for the travel conditions in the US in 1967-68 for a standard size car. This figure is based on the conventional operating costs by speed, while the standing costs are related to the car TT-budget, and within the range of speeds in urban areas it can be expressed by:

 $c = 1.494 v^{-0.75}$; (4)

where c is in US ¢ and the speed is in kph.

Table 2 summarizes the estimation of travel distance by income groups for households who made all their travel by car in Washington, D.C. 1968 and Twin Cities 1970, according to the following steps:

- 1) Income group, by district;
- 2) The road travel speed, derived form Eq. 3 and

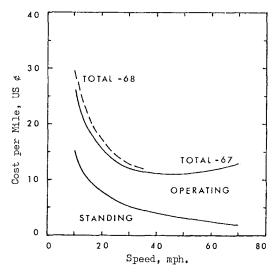


Fig. 6 - The cost per mile vs. Speed for standardsize Car in the US, 1967-1968

multiplied by 1.58, the factor which transfers door-todoor speeds to network speeds;

3) The cost per unit distance at the above speed, according to Eq. 4;

4) Income per weekday = Annual Income/312 days;
5) The TM-budget per household, at 10.5 percent of the daily income;

6) The daily travel distance per household, derived as the quotient of TM over c and multiplied by 1.5, the car average occupancy rate;

7) The observed daily travel distance per household, as derived from Eq. 2.

Table 2 - Estimated vs. observed daily travel distance bycar per household, by income, Washington D.C. 1968 andTwin Cities 1970

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Inc.	vxl. 58	c	Inc/day	TM	Dxl. 5	Dobs.
7,000	27.78	0.123	22.44	2.36	28.7	28.15
8,000	32.36	0.110	25.64	2.69	36.7	36.13
9,000	36.94	0.100	28.85	3.03	45.4	45.02
10,000	41.52	0.091	32.05	3.37	55.5	54.82
11,000	46.10	0.084	35.26	3.70	66.1	65.51
12,000	50.69	0.079	38.46	4.04	76.7	77.08
13,000	55.27	0.074	41.67	4.38	88.7	89.52
14,000	59.85	0.069	44.87	4.71	102.4	102.82

The comparison between the estimated and the observed values is shown in Figure 7, where the curve represents the best-fit line of the observed values, as in Figure 5, while the dots represent the estimated values for each discrete income group.

It should be noted that the above exercise is based on average factors for the whole area, such as the TMbudget at 10.5 percent, and 1.58 and 1.5 mentioned in steps (2) and (6) respectively. Nonetheless, the match between the estimated and the observed values can be regarded as fully satisfactory.

Two conclusions may be inferred at this stage:

1) The daily travel distance appears to be a better representation of travel demand than the trip rate because:

(i) it is a direct derivation from the TM-budget, which is a stable behavioral phenomenon;

(ii) it interacts with system supply through speed; and

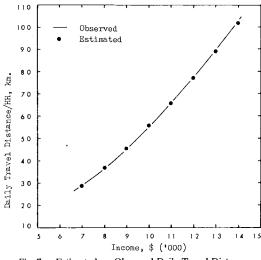


Fig. 7 - Estimated vs. Observed Daily Travel Distance per household vs. Household Annual Income, Washington, D.C. 1968 and Twin Cities 1970

(iii) it does not need calibration and, therefore, is fully transferable both within and between cities;

2) Income alone is a sufficient descriptor of the amount of travel generated by households and their travelers, when related to the cost of travel.

It is of interst to note that while the daily travel distance is the *final* output from the current lengthy models, it can now be derived as the *first phase* directly from the TM-budget and the speeds available from system supply. Furthermore, the same results shown in Figure 7 can also be derived from the TT-budget. The question on how does the TM-budget interact with the TT-budget is discussed in the following section.

THE FUNDAMENTAL TRAVEL EQUATION The fundamental travel demand equation is: $\frac{M}{T} = \overline{v}\overline{c} \qquad ; \qquad (5)$

where M is the TM-budget, T is the TT-budget, \bar{v} is the daily mean speed and \bar{c} is the cost per unit distance traveled at this speed.

While the left hand side represents travel demand, as expressed by the travelers willingness to allocate money and time for travel, the right hand size represents the product that they would like to purchase from the system supply, in terms of the system's performance and the price of using it.

When substituting Eq. 4 for c in Eq. 5 and applying the fundamental travel equation to the available observations from Washington, DC 1968 and Twin Cities 1970, it becomes evident that households at increasing incomes tend to travel at increasing speeds, as can be seen in Figure 8.

These observations are consistent with the hypothesis that households would tend to locate their residence and travel to such places that will enable them to maximize their daily travel distance (namely, maximize their spatial opportunities) within the constraints of the two travel budgets and the speeds obtainable from the transport system.

The stability of the two travel budgets also suggests that there is a negligible amount of substitution between the money and the time budgets on a daily basis ⁵. Thus, while the current understanding of travel behavior is based on the observation of single trips, where some travelers are found to exchange money for saved travel time, it becomes evident that they often do so in order to travel more during the day.

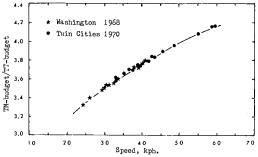


Fig. 8 - The Demand for Speed with increasing Household Incomes, as expressed by their Two Travel Budgets

It may, therefore, be concluded that the 'generalized cost' concept, where the two stable travel budgets are combined with full substitution between them, does not seem to be helpful when applied on a daily basis. In other words, applying the generalized cost to single trips and then aggregating all the trips over the day, could lead to erroneous results. (This brings into mind the known problem of 'the one and the many' where the rules that govern the behavior of the sum of trips could be on a different level from the rules that govern the behavior of single trips).

Another interesting aspect of the fundamental travel equation is whether the right hand side of it, representing the product that the household would like to purchase from the system supply, can actually be realized. Namely, must travel demand always be in equilibrium with system supply? ('Equilibrium' is defined as the state in which both budgets are just satisfied).

Let us consider first a household that increases its income. At low incomes it will be able to purchase only the low speeds, that can be supplied by transit. However, as its income increases, it will search actively for the higher speeds that can be supplied only by cars and, hence, the household will tend to purchase a car after crossing a certain income threshold. This process can explain the rapid increase in motorization after incomes cross such thresholds (which depend on the cost of owning and operating a car).

Referring back to Figures 6 and 5, it becomes evident that there is a very strong incentive to travel at higher speeds, since a stable proportion of a TM-budget at increasing incomes results in accelerated increases in the daily travel distance per household. Namely, the daily travel distance per household vs. income is significanly elastic (at 1.87, as seen in Eq. 2).

Households that do not, or cannot purchase a car and have to rely on buses only, are then found to exhaust their TT-budget on the slow buses much before they expend even half of the TM-budget; they are in a 'disequilibrium' condition, in the sense that they would be prepared to spend more on travel than they actually do.

At this stage one might raise the argument that such a household may prefer to allocate less money for travel, thus remaining in an equilibrium condition between their travel demand and system supply. There are, however, several indirect indications to suggest that generally this is not so. The first indication, of course, is the rapid increase of motorization with income. The second is that households owning and not owning cars tend to expend the same proportion of their income on housing, at least in the UK, although there is a wide gap between their expenditures on travel. For instance, the same basic data upon which the diagram for the UK in Figure 4 is based, also supplies information on household expenditures on housing, and as can be seen in Figure 9, there is no marked difference between the two groups along the major range of incomes ⁶.

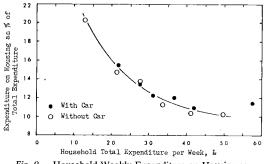


Fig. 9 - Household Weekly Expenditure on Housing as Percentage of Total Weekly Expenditure, by Car Availability, All UK 1971

In conclusion, it is suggested that travelers, within their own TT-budget and their household's TM-budget, tend to maximize their daily spatial opportunities. If, however, the system supply and/or urban structure do not allow them the freedom of choice in the short run, their travel demand is in disequilibrium with system supply and urban structure, thus generating forces that may change urban structure in the long run.

All current models are based on the concepts and techniques of equilibrium between travel demand and system supply. This may be so in the short run, on a daily basis. If, however, travel demand is based on the two behavioral travel budgets, it becomes more fruitful to measure the amount of disequilibrium, and dissatisfaction, as an aid to forecasting changes in motorization and/or urban structure.

One indication should, however, be emphasized again: the door-to-door travel speed appears to be the key to a better transportation system, whether car or transit. This is especially true for transit, where travel is found to be inelastic to fares.

THE TRAVEL COMPONENTS

Almost all current models regard most of the travel components in isolation. For instance, trip rates are generated as the first phase, while trip distances are produced as incidental outputs from the distribution of trips between zones at a later and independent phase. Even when all phases are conducted simultaneously, they are still independent, in the sense that they are rigidly calibrated to observations with no explicit feed back mechanism between the various travel components, such as between the trip rate and the trip distance. Another example is the treatment of trip purposes, where a trip generation equation is calibrated for each purpose separately.

As already mentioned above, it appears that travelers strive to maximize their daily travel distance within their TT-budgets and their household's TM-budgets. Each traveler then has to decide how to allocate his daily travel distance between the trip rate and the trip distance. It was shown above that travelers tend to use their 'saved' time for longer trips rather than for more trips. In other words, they tend to value their spatial opportunities over more trips as long as the trip rate is still above a critical value.

Travelers are found to rank their trip purposes by their perceived values. Hence, all trip purposes are interlinked within their daily trip rate. The proportions of trip purposes by the trip rate, for both car and transit travelers in Twin Cities 1958 and 1970, can be seen in Figure 10.

It is also indicated that the perceived value of the trip rate increases with the trip rate at decreasing rates, thus following the expected decreasing marginal utility 7. Namely, the addition of one trip to a daily trip rate of 2 is valued more than the addition of one trip to a daily trip rate of 6.

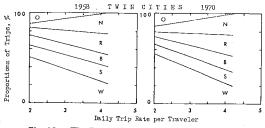


 Fig. 10 - The Proportions of Trip Purposes vs. the Daily Trip Rate per Traveler, in the Car and Transit Modes
 W - HB to Work; S - HB to Shopping; B - HB to Business;
 R - HB to Recreation/Social; O - HB to Other;

N - Non Home Based.

The daily trip is inversely related to both the trip distance and the trip time, as can be seen in Table 3.

 Table 3 - Car travel characteristics in two cities in the UK

Characteristic	Hull	London	
Year	1967	1961	
Population	344,890	8,826,620	
Car trip rate	6.25	3.27	
Car trip distance, km.	4,2	7.9	
Daily travel distance	26.3	25.8	
Trip time, min.	6.9	13,7	
Daily car TT-budget, hr.	0.72	0.75	

As can be seen, although the daily travel distance and the daily travel time are practically identical in the two cities, the trip rates are entirely different, and related inversely to both the trip distance and the trip time.

Since the inverse relationship holds true both within and between cities, it is also suggested that the trip time frequency distributions can be tied up with all other travel components, as an integral part of travel behavior, without having to calibrate them in each city separately.

There is strong evidence that the difference between the mean distances of the population and the job spatial distributions to the city center approximates very closely the daily mean trip distance, thus suggesting a link between travel characteristics and urban structure, and explaning changes in urban structure, such as the change of mono-nucleated cities into multi-nucleated ones⁸. Expansion of a city, dispersion of households by income, and the shift of jobs from the center outwards, can all be explained as part and parcel of the behavioral mechanism of travel.

When only two travel modes are considered, such as cars and transit, modal splits by travel distance can be solved directly by the equations of the two travel budgets, of time and of money. However, when more than two modes are considered (such as the addition of a rapid transit system) the travel utility functions have to be established first. It is of interest to note that the formulation of the two travel budgets allows the derivation of the structure of the utility functions directly from observations, without having to assume them a-priori.

Preliminary estimates of modal splits with 2 and

3-mode conditions have produced very encouraging results.

In conclusion, all travel components, such as the daily travel distance, trip rate, trip distance, trip time, proportions of trip purposes and modal splits, can be unified in a consistent way within one mechanism, with interactions between them. The same mechanism also appears to unify the interactions between travel demand, system supply and urban structure.

POSSIBLE IMPLICATIONS OF THE UMOT TO TRAVEL MODELING

Current models, both aggregated and disaggregated, start with trip generation and conclude with the daily travel distance of travelers and vehicles. It now appears that the models should be turned upside down, by starting with the daily travel distance, as constrained by the two travel budgets, and conclude with the trip rates.

It is indicated that no calibrations should be made to the same outputs that the models are expected to produce, and certainly not for each travel component separately. If the model is to be a behavioral one, all the isolated travel characteristics observed should be explained by one behavioral mechanism.

It is indicated that the 'generalized cost' should be split back into its two constituents, especially when the daily travel is considered.

Accessibility indices are currently calibrated and allocated to zones on the basis of incidentally observed travel conditions. It is indicated that a preferable procedure would be to define 'travel demand' descriptors on the one hand, and 'system supply' and 'urban supply' descriptors on the other hand. The spatial distribution of travel (and the observed 'accessibility' indices) would then be the result of the interaction between demand and supply, at least in the short run, on the basis that travelers strive to maximize their daily travel distance within their budget constraints. Thus, the distribution and assignment phases can be combined into one process, with feed back sensitivity with all other travel components.

Mobility should be expressed by the daily travel distance weighted by the travelers' perceived values and preferences of trip distance vs. trip rate. Thus, travelling, say, 30 km. per day, at either 6 trips of 5 km, each or 3 trips of 10 km. each, could result in different levels and values of mobility.

An approach based on the two separate travel budgets allow the analyst to evaluate the possible effects of policy options on travel, such as changes in the components of travel cost (e.g., operating or standing costs), road pricing, free transit, car pooling, and so on. The important part to note is that different options will have different effects on different income groups. For example, and referring back to Figure 6, it becomes evident that increasing either the standing costs or the operating costs will have different effects at different speeds, thus affecting households at different income levels in different ways.

The economic evaluation of urban transport improvements is assisted by the UMOT process in that design year conditions can be assessed 'with' and 'without' proposed investments or policy changes under realistic conditions. The concept of the TM-budget to estimate travel demand, including latent demand, is particularly useful for the analysis of the travel of specified income groups such as those with low incomes - which might be of special concern.

IN CONCLUSION

In closing, it should be noted that although this paper presents examples and relationships from cities in developed countries, the same principles seem also to apply in cities of developing countries, only under much more adverse conditions. For instance, there is an urgent need to define the thresholds under which travelers cannot afford even a bus fare. Furthermore, in fast developing cities the poor live farthest from jobs, thus aggravating their conditions and forcing them to expend higher proportions of both money and time on travel for less spatial opportunities.

As can be seen from the indications in this paper, and they are only indications at this stage, much search and research are still needed before we can close the gap between the highly sophisticated techniques currently available for analyzing each travel component separately and the basic understanding of travel behavior.

It may come as a surprise to many that simple questions about travel are still unanswered. For instance: How do we measure the benefits of mobility? Is mobility beneficial, to be encouraged? Or is it a luxury that should be suppressed? And if mobility increases with increasing incomes, will a decrease in mobility also decrease income? Namely, what is cause and what is effect in travel behavior? Surprisingly enough, there is no operational model that can answer such questions. And they are crucial questions especially now, when we are faced with continuously increasing shortages of energy and available funds for transportation improvements.

Hence, the message of this paper is that more attention should be given now to the neglected basic issues of travel demand, and its interactions with system supply, urban structure, and economic development.

ACKNOWLEDGEMENTS

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Appendix 1 - (Car Daily Trip Ra	te vs. Population Size
in a se	lection of US Citic	es (Figure 1)

City	Year	Population	Trip Rate
Monroe	1965	96,530	5.79
Baton Rouge	1965	245,076	4.66
Peoria	1964	260,826	5.10
Orlando	1965	355,619	4.33
Springfield	1965	532,188	4.30
Cincinnati	1965	1,391,869	3.63
Baltimore	1962	1,607,980	3.26
Washington, DC	1968	2.562.025	3.28
Tri-State	1964	16,303,000	2.89
	Monroe Baton Rouge Peoria Orlando Springfield Cincinnati Baltimore Washington, DC	Monroe1965Baton Rouge1965Peoria1964Orlando1965Springfield1965Cincinnati1965Baltimore1968Washington, DC1968	Monroe 1965 96,530 Baton Rouge 1965 245,076 Peoria 1964 260,826 Orlando 1965 355,619 Springfield 1965 532,188 Cincinnati 1965 1,391,869 Baltimore 1965 1,607,980 Washington, DC 1968 2,562,025

FOOTNOTES

1. The UMOT - a Policy Sensitive Model. In preparation for the Urban Projects Department, the World Bank, Washington, D.C.

2. Travel over time. In preparation for the US Department of Transportation, FHWA, Washington, D.C.

3. Entropy in urban and regional modelling. A. G. Wilson. Lion Ltd., London, 1970.

4. A critique of entropy and gravity in travel forecasting. M. J. Beckmann and T. F. Golob. Proceedings of the Fifth International Symposium on the Theory of Traffic Flow and Transportation. American Elsevier Publishing Co., Inc., 1972.

5. This may reflect the relatively low charges payable for the use of roads in cities. Evidence on the effect of the Singapore Area Road Pricing scheme is not yet available.

6. Family Expenditure Survey for 1971, Tables 28-29, HMSO, UK.

7. Travel characteristics in cities of developing and developed countries. World Bank Staff Working Paper No. 230, March 1976, pages 56-61.

8. The effects of transportation systems on the spatial distribution of population and jobs. Y. Zahavi, Joint National Meeting of ORSA/TIMS, 1976.