Uncertainty and the transport investment decision

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INTRODUCTION

Uncertainty pervades virtually all aspects of life. It is not to be expected that transportation would be an exception in this respect and, as the title of this conference implies, it is indeed not. Despite the inevitability of uncertainty in the circumstances surrounding transport decisions, it is still a potentially valuable exercise to ask some quite basic questions. How seriously does uncertainty affect transport decisions? To what extent are its potential effects recognised? Is such recognition a matter of both theory and practice? What techniques exist for analysing uncertainty? Are available techniques regularly applied? If not, why not?

The purpose of this paper is to look briefly at these points in the context of transport investment appraisal. It will be argued that there has been a significant failure to take proper account of the potential importance of uncertainty in transport investment appraisal as a whole. Such an omission can certainly affect the correctness of individual decisions and, it will be shown, can unjustifiably favour certain types of investment proposal of a kind, moreover, which may be subject to particularly strong public opposition. It will be further argued that the level of theoretical understanding of problems in this area is reasonably high, and that the principal difficulty lies in the development of operational methods. As a step in this direction, two techniques are described which are appropriate to the preliminary stages of the evaluation of investment strategies in the presence of uncertainty.

THE IMPORTANCE OF UNCERTAINTY IN TRANSPORT INVESTMENT APPRAISAL

What is the correct transport investment strategy in any situation will depend upon future events, or states of nature, which cannot be predicted with certainty. Furthermore, many such decisions involve infrastructure investments which are, by their very nature, particularly inflexible. They are likely to be long-term, suited to a sigle use only and to be fixed both in location and size. It is frequently difficult to adjust the level of their performance once construction is completed. Marginal changes in capacity are likely to be particularly difficult to achieve and demand may not be price-elastic, either naturally or because of constraints of a social, political or institutional nature.

Infrastructure investments in transport, therefore, may have to operate in any one of a number of quite distinct future states of nature in a situation where both supply and demand may exhibit significant inelasticities, especially in the short and medium terms. In such circumstances, the proper recognition of uncertainty at the appraisal stage is of special importance. It is, therefore, rather surprising to learn from the literature that the treatment of uncertainty in the appraisal of transport investment projects is by no means adequate. "A number of decision rules are available but, so far as is known, these have not been used in decision making in the transport sector." (Gwilliam and Mackie (1975), p. 125). Similar comments are made elsewhere, for example, in Meyer and Straszheim (1971), chapter 13 and Heggie (1972), chapter 9.

When the analysis of a topic first moves from the qualitative to the quantitative, it is natural that early techniques should be of a simple kind. They are likely to be deterministic or, if a stochastic element is introduced, it may well not go beyond analysis in terms of expected values, or some simple sensitivity tests. Once quantitative modelling in transport passed the basic stage, the predominant demand, not unreasonably from many points of view, was for models which were wider in scope, both spatially and in terms of the complexities of interdependent behaviour patterns which they sought to analyse. The success with which investment strategies based on such models might cope with sets of circumstances other than the predictions upon which they were founded has been the subject of relatively little attention. Given the computer time demands of many such models and the need for multiple runs of programme suites if different future states of nature were to be considered, the reticence to pay great attention to the analysis of uncertainty is scarcely surprising.

At this juncture, however, when computer technology has reached a stage when quite sophisticated planning models can be run with relative ease, it might well be appropriate to devote future increases in computer power to more thorough consideration of uncertainty, rather than to further modelling advances *per se*. Moreover, it seems that transport is becoming subject to more and more uncertainty, for example through the likely existence of energy shortages within the time horizon of many current investment appraisals, through the increasing doubts about the desirability of living in large conurbations and generally through the increasing importance of transport in the political arena. It may also be added that, when the effects of the provision or non-provision of transport can impose themselves so heavily on certain small sections of the community, for example, countrydwellers, the elderly, etc., then this possibility of gross social inequity if transport fails to react flexibly to changing circumstances must also be fully recognised.

One significant reason, then, for bringing the analysis of uncertainty in transport investment decisions more sharply into focus is the likelihood that variations in the ability of alternative strategies to deal flexibly with a number of different future states of nature are likely to be increasingly important. Uncertainty in demand is also of potential importance. There are a number of theoretical papers in economics which treat problems of this type. For example, Weisbrod (1964) has shown that, where there is uncertainty in the demand for a publicly provided good, there may be an "option value" benefit to the individual in addition to the conventional consumer surplus. Cicchetti and Freeman (1971) have extended this analysis and further work on this still contentious subject has appeared recently, *viz.*, Schmalensee (1972; 1975) and Bohm (1975). Another dimension of the problem has been highlighted by Arrow and Lind (1970). They argue that, in social investment accounting, different rates of discount (by implication, a different evaluation of risk), should be applied depending upon whêther the risks associated with the cost and benefit streams are publicly or privately borne. A further aspect of this question is that, even if this latter policy is adopted, the differential social and spatial effects of a public investment such as one in transport are such as to make important some recognition of the distributional implications of different strategies and how these might vary in different states of nature.

A second reason why the transport analyst should give increased attention to the importance of uncertainty stems from a very interesting point which has arisen recently in the economic literature, namely the treatment of irreversible decisions in project appraisal in the face of uncertainty. This has been discussed by Arrow and Fisher (1974) from the point of view of environmental preservation and, in a more theoretical paper by Henry (1974). The argument which Henry puts forward is that if, in a dynamic decision making environment where knowledge increases over time, irreversible projects with uncertain streams of returns are compared in terms only of their expected values as viewed at the present, there will be a significant tendency for such irreversible decisions to be taken prematurely. Indeed, they may be taken quite unnecessarily in circumstances where an explicit recognition of the full spectrum of possible outcomes would suggest some alternative time path of decisions.

Henry's interest in this theoretical problem was originally sparked by considering the official attitude to the appraisal of a ring road system around Paris which threatened to spoil a number of famous parks on the fringe of the conurbation, in a way which was, for all intents and purposes, irreversible. Clearly, this is not a problem which has affected Paris alone. Many, if not all, major transport improvement schemes tend to involve severe and effectively irreversible dislocation of their immediate environment. Many of the most controversial public enquiries into proposed road extentions in the United Kingdom have had at their heart decisions of this general type. It is thus a matter of some concern when theoretical results such as Henry's come to light. Furthermore, not only does the failure to give proper weight to uncertainty imply that irreversible decisions may have been wrongly taken, but it seems plausible that a similar analysis might reveal that investment schemes which were merely relatively inflexible rather than totally irreversible might similarly be given undue favour by analysis in terms of expected values alone.

PRESENT TECHNIQUES OF INVESTMENT APPRAISAL IN THE PRESENCE OF UNCERTAINTY

Given that transport investment decisions are likely in practice to be taken more and more in circumstances of significant uncertainty, and having mentioned briefly that project appraisal in the presence of uncertainty has received a good deal of attention from theoretical economists, it is appropriate to look briefly at the ways in which uncertainty has typically been taken into account in transport investment appraisal. This question has been reviewed in rather more detail in Pearman (1976), where it is pointed out that, very often, no account of uncertainty is taken at all.

In theoretical discussions of decision making, a fundamental distinction is usually made, following Knight (1921), between decision making under risk and decision making under uncertainty. This distinction was not rigorously adhered to in the first two introductory sections of this paper, but will be from now on. In the former case it is assumed that future states of nature can be identified, and that the probability of occurrence of each alternative can be estimated. In the latter case, it is assumed that no such probability estimates can be made. Objective estimates of probabilities of future states of nature are virtually impossible in the circumstances of most transport investment appraisals. Models of decision making under uncertainty seem, therefore, to be the more relevant of the two theoretical extremes.

For decision making under uncertainty, three principal approaches have been adopted. The first, and most common, is implicit or explicit conservatism. Where doubts are felt about the future, cost elements tend to be biased upward and benefit elements downward in ways which are more or less arbitrary. This will obviously have the desired effect of militating against those strategies which are felt to be less capable of handling an uncertain future. What it fails to do however, is to provide any rational basis for implanting the correct degree of bias against such projects. Some may be cut back far too harshly, others by not enough.

A second approach to appraisal in the presence of uncertainty has been to convert an uncertain environment into one of risk (in the technical sense described earlier) by making some type of objective estimate of the probabilities of the future states of nature, after which an expected value maximisation approach is adopted. Some potential dangers in analysis in terms of expected values alone were mentioned at the end of the previous section, but the approach is a very common one. Its main weakness is that, although the probability estimates may appear objective at one level, they usually have to be calculated from the probabilities of more fundamental events which in turn require *a priori* estimation of probabilities. If these are truly objective, then the problem was never really one of uncertainty in the first place. If not, then the real problem has been obscured rather than solved.

The third category of techniques for handling decisions under uncertainty are termed "complete ignorance" methods, since they assume no knowledge of the probabilities of future states of nature. In the context of road investment appraisal, for example, Quarmby (1967), pp. 1–4, has pointed out the highly volatile nature of the demand component of road use and hence the difficulty of making accurate net benefit assessments over a time horizon of twenty or thirty years. In circumstances such as these where the probabilities of different future states of nature would be virtually impossible to estimate, the application of complete ignorance methods may, on occasions, be appropriate.

Complete ignorance decision making rules – maximax, maximin, the Hurwicz α criterion, minimax regret – are a well established part of most undergraduate courses in economics. Their practical implementation is less common. The reasons for this may be readily demonstrated using the following example. Consider three different investment strategies, A, B and C which will have to operate, it is believed, in one of six future states of nature. The payoffs, measured as net present values (n.p.v.) in millions of pounds of the benefits accruing to each of the eighteen possible strategy/state of nature combinations, are shown in Figure 1.

State of Nature

		1	2	3	4	5	6
	A	6	8	13	17	26	8
Strategy	B	16	10	10	12	20	22
	C	9	11	10	18	16	24

Figure 1

The strategies which would be selected by the four different complete ignorance criteria are as follows:

1) Maximax: Strategy A on the basis of state of nature 5. An optimist's approach, maximax selects the strategy which would give the maximum possible return if the right state of nature occurs.

2) Maximin: Strategy B on the basis of states of nature 2 or 3. A pessimist's approach, maximin selects the strategy which would give the highest minimum payoff, assuming the least favourable state of nature for that strategy were to occur.

3) Hurwicz α Criterion ($\alpha = {}^1/_2$): Strategy C. The Hurwicz criterion is merely a weighted average of the maximin and maximax payoffs. The weighting is arbitrary, reflecting only the importance which the decision maker gives to each extreme. Here, equal weights of one half were chosen for each, leading to scores for the three strategies of:

$A = 16\ ({}^1/_2[6 + 26])$;
$B = 16\ ({}^1/_2[10 + 22])$;
$C = 16{}^1/_2\ ({}^1/_2[9 + 24])$.

4) Minimax Regret: Strategy B on the basis of state of nature 5. For this calculation, an opportunity cost or regret matrix is first calculated, representing, for each state of nature, the absolute value of the difference between the best outcome from any strategy and the actual outcome. The regret matrix is shown in Figure 2.

State of Nature

		1	2	3	4	5	6
	A	10	3	0	1	0	16
Strategy	B	0	1	3	6	6	2
	C	7	0	3	0	10	0

Figure 2

The principal weakness of the first three of these techniques is that their decision is based on only a very small subset of all possible outcomes. For each strategy, all intermediate states of nature are ignored. This is particularly serious if a large number of potential states of nature are identified. Minimax regret partially avoids this criticism by going through the intermediate step of calculating opportunity costs, which will tend to have a neutralising effect, but one which is by no means complete. Established complete ignorance criteria, then, overlook a great deal of potentially valuable information and, furthermore, may very reasonably be criticised on the grounds that they are too crude for application to real problems. There is also no real guidance as to which of the complete ignorance criteria is likely to be the most appropriate. The risk averting maximin model is often favoured for the private sector. The case for risk aversion is less persuasive for the public sector, except insofar as it may help to avoid very poor outcomes for certain sections of the community.

In summary, the evidence on current practice in appraising transport investment projects is this. The most common practice of all seems to be to ignore uncertainty altogether. Also relatively common, and rather more justifiable, is to use expected values, but the first moment of the probability distribution cannot be expected to encapsulate all that might be of relevance in an appraisal. If uncertainty is explicitly recognised, then this recognition is most likely to take the form of using conservative estimates throughout. These are potentially arbitrary. Converting from uncertainty to risk seems likely to give more the illusion of dealing with the problem than actually providing a solution. The complete ignorance methods are theoretically sound, depending upon the attitude to risk of the decision maker, but lack the subtlety required for applied work in the transport sector.

EXTREME EXPECTED PAYOFFS AND VARIANCES

This section describes an approach to decision making which represents a compromise between the classical extremes of decision making under uncertainty and decision making under risk. Further details of the development are available in Cannon and Kmietowicz (1974) and Kmietowicz and Pearman (1976). The technique is based on the assumption that, while it is unreasonable to expect decision makers to calculate precise probabilities of future states of nature and so calculate expected values under risk, it is plausible that they should be able to rank the probabilities of different states of nature. The ranking will reflect both objective information available and the decision maker's subjective views. Moreover, such information must frequently be available and, *ceteris paribus*, to ignore it can only lead to poorer decisions.

The technique to be described provides a simple analytical method of determining the maximum and minimum expected payoffs of any strategy consistent with a probability ranking of states of nature and also the maximum variance of payoff. It is these figures which are then used as a basis for decision making, rather than the crude extrema normally used by the complete ignorance methods. Suppose there exist m alternative investment strategies ($i = 1 \ldots m$) and that the decision maker identifies n ($j = 1 \ldots n$) possible future states of nature in which a given transport investment may have to operate. The states of nature are assumed to be mutually exclusive and exhaustive. Suppose also that a payoff matrix has been constructed with elements X_{ij} which correspond to the n.p.v. of investment strategy i should state of nature j occur. In addition, it is assumed that the decision maker has ranked the n states of nature such that $P_j \geq P_{j+1}$ [$j = 1 \ \ldots (n-1)$].

Given this foundation, minimum and maximum expected payoffs may be computed for each strategy. Dropping the i subscript for simplicity of notation gives the following linear programming problem for each strategy:

Maximise or Minimise $$E(J) = \sum_{j=1}^{n} P_j X_j$$

Subject to $$\sum_{j=1}^{n} P_j = 1 \qquad \text{(I)}$$

$$P_j - P_{j+1} \geq 0 \ [j=1 \ldots (n-1)] \qquad \text{(II)}$$

$$P_j \geq 0 \ [j=1 \ldots n] \qquad \text{(III)}$$

The problem may be greatly simplified by the application of the following transformations:

(i) Let $Q_j = P_j - P_{j+1}$ $\qquad [j = 1 \ldots (n-1)]$

(ii) Let $Y_j = \sum_{k=1}^{j} X_k$ $\qquad [j = 1 \ldots n]$

This leads to the following re-expression of the original problem:

Maximise or Minimise $$E(S) = \sum_{j=1}^{n} Q_j Y_j$$

Subject to $$\sum_{j=1}^{n} j Q_j = 1 \qquad \text{(IV)}$$

$$Q_j \geqslant 0 \qquad \text{(V)}$$

Because it has only one functional constraint, a linear programming problem of this type will have an optimal solution with only one of the decision variables, Q_j, positive and the rest zero. From constraint (IV), if only one Q_j is non-zero, it must equal $^1/j$. Thus it is clear that the objective function will be maximised when Y_j/j is maximised and minimised when Y_j/j is minimised. The extreme expected payoffs of any investment strategy can thus be found by computing the n partial averages.

$$\frac{1}{j} Y_j = \frac{1}{j} \sum_{k=1}^{j} X_k$$

The largest such partial average will be the maximum expected payoff and the smallest will be the minimum.

In choosing between alternative investment strategies, it is probable that, in addition to minimum and maximum expected payoff, the decision maker may wish to have some information about the likely dispersion of outcomes around these mean values. In some cases, it is possible that the decision maker's attitude would favour trading off some loss in expected returns in order to obtain a greater probability of an actual outcome close to that expectation. It follows that any judgement which can be made about likely variance of payoff is potentially valuable. As is shown in Kmietowicz and Pearman (1976), it is possible to calculate minimum and maximum variances in a similar way to that in which extreme expected values were calculated. It transpires that minimum variance will be zero for all strategies and is therefore of no value for purposes of discrimination. Maximum variance, however, varies from strategy to strategy and is found by computing n partial variances

$$\mathrm{Var}(S) = \frac{1}{j} \sum_{k=1}^{j} X_k^2 - \left(\frac{1}{j} \sum_{k=1}^{j} X_k\right)^2$$

for j = 1 ... n. The largest of these partial variances gives the maximum variance.

The computation of the partial averages and variances is straightforward. For example, for Strategy A of the example given in the third section, the first three partial averages and variances are shown below:

$$\bar{X}_A^1 = \frac{1}{1}(6) = 6$$

$$\bar{X}_A^2 = \tfrac{1}{2}(6 + 8) = 7$$

$$\bar{X}_A^3 = \tfrac{1}{3}(6 + 8 + 13) = 9$$

$$V_A^1 = \frac{1}{1}(6^2) - [-(6)]^2 = 0$$

$$V_A^2 = \tfrac{1}{2}(6^2 + 8^2) - [\tfrac{1}{2}(6 + 8)]^2 = 1$$

$$V_A^3 = \tfrac{1}{3}(6^2 + 8^2 + 13^2) - [\tfrac{1}{3}(6 + 8 + 13)]^2 = 8\tfrac{2}{3}$$

Complete results for all strategies are given in Figure 3.

	Partial Average						Partial Variance					
	1	2	3	4	5	6	1	2	3	4	5	6
Strategy A	6	7	9	11	14	13	0	1	8.7	18.5	50.8	47.3
Strategy B	16	13	12	12	13.6	15	0	9	8	6	15	22.3
Strategy C	9	10	10	12	12.8	14.7	0	1	0.7	12.5	12.6	27.9

Figure 3

Similar calculations may also be undertaken in terms of the regret matrix.

If now the conventional complete ignorance criteria are applied to the extreme expected values rather than the crude extrema, it is clear that Strategy B will be favoured by the maximin, maximax and Hurwicz rules. It also has the lowest maximum variance. Such unanimous support would be relatively rare, even using the expected value approach. It serves to emphasise, however, that by taking into account the extra information implicit in the probability ranking, a very different light can be thrown upon a decision making problem.

The techniques just described are potentially valuable when decision making must take place in the face of only partial knowledge about future states of nature. They enable all available information to be used to reduce the range of uncertainty within which the strategy choice decision must be made. In addition to these basic results, various sensitivity tests are available, relating both to potential changes in the decision maker's ranking of the probabilities of future states of nature and to alterations in the values placed on the payoffs, X_{ij}. [See Pearman and Kmietowicz (1976)]. Circumstances in which the use of these methods is likely to be preferable to the alternative of conventional complete ignorance criteria are explained in detail in Pearman (1976). The principal requirements are that there should be a number of separate decisions over which it is possible to trade off payoffs. The relative improvement will be especially high if a large number of potential states of nature is identified. Where these criteria are not obeyed, or where policies such as maximin, minimax regret, etc. are thought to be too simplistic, the expected value techniques can still form a useful alternative to be considered as one of a range of criteria to be entered into a multiattribute decision making process. One of these attributes might well be the maximum variance of payoff described earlier.

A SECOND APPROACH TO THE CALCULATION OF EXTREME EXPECTED VALUES

This section describes a second technique for delimiting a range of expected values for a given strategy. It is appropriate to planning problems specified at about the same level of detail as those discussed in the previous section. Its philosophy, however, is one of estimating the probability of success or failure of a given strategy on the basis of the success or failure of independent component parts of the strategy, and then combining this information with the anticipated outcome of the strategy should it be successful as a whole. The method is most easily demonstrated by example.

Suppose a local authority is considering the introduction of an urban mass transit scheme. For its successful operation, a given minimum level of demand is required. Excess demand can be stifled by appropriate pricing policies. In order to achieve this minimum demand level target, the authority has certain policy instruments at its disposal and there are, additionally, exogenous influences beyond its control. It is assumed that, within the context of the strategy under evaluation, each one of the exogenous variables and instruments can take values which correspond to "success" or "failure". A success may stem from a deliberate policy decision in which case the variable I_j, corresponding to the jth policy instrument, takes the value of one. Failure to implement the right policy from the point of view of this strategy causes I_j to take the value zero. On the basis of the general political environment and the pressure which may be exerted by the need to force I_j to zero or one as a result of policy conflicts in other areas of administration, assume that a probability, P_{Ij} may be estimated giving the chance that $I_j = 1$. Similarly, assume that $E_k = 1$ corresponds to a successful outcome in respect of the kth. exogenous variable and that the corresponding probability is P_{Ek}. It is assumed that the I_j and E_k can reasonably be taken as being statistically independent of each other.

Suppose now that it is possible to identify certain minimum combinations of "success" with the policy instruments and exogenous variables which will achieve the desired level of transit demand while, by implication, all others which do not contain at least one of the minimum combinations as a subset will result in failure to achieve the target. For example, suppose that the six combinations shown in Figure 4 are regarded as the only minima which will guarantee circumstances sufficiently favourable to mass transit for the required target level to be achievable.

Acceptable Combinations

	I	II	III	IV	V	VI		
I_1	X	X				X	Parking	
I_2	X		X				Urban Planning	Policy
I_3	X			X		X	Rate levy for mass transit	instruments
E_1		X				X	Import controls on cars	
E_2		X			X		No home-based mass prodn. of small cars	Exogenous
E_3				X	X		High petrol tax	variables
E_4			X				No alternative fuel to petrol	
E_5				X		X	Government grant to mass transit	

Figure 4

An alternative and illuminating way of presenting the information contained in Figure 4 is to construct a network, as shown in Figure 5. The implication of the network is that, provided there exists at least one path from the start node to the finish node made up of the links which correspond to the policy instruments and nous variables of Figure 4, then it will be poss... achieve the desired level of transit use. Furthermore, using this visual insight, it is straightforward to develop an expression for the probability that the required level of mass transit demand be attainable:

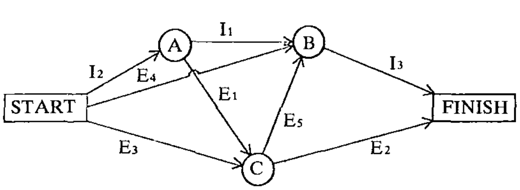

Figure 5

P [Demand level is attainable)

= P [There is at least one path through the network from Start to Finish]

= 1 − P [There is no path through the network from Start to Finish]

= 1−P [(I = 0) Ω (II = 0) Ω (III = 0) Ω (IV = 0) Ω (V = 0) Ω (VI = 0)]

In the final row of this expression, I = 0 implies that the path Start – A – B – Finish is broken by having at least one link missing. Equivalently, this requires $I_1 I_2 I_3 = 0$ where the I_j are binary variables taking the value one if a "success" is recorded for the policy in question and zero otherwise. Interpreting the remainder of the expression similarly leads to the relation:

P [Demand level is attainable]

$$= 1 - P\,[(I_1 I_2 I_3 = 0)\ \Omega\ (I_1 E_1 E_2 = 0)\ \Omega\ (I_2 E_4 = 0)\ \Omega$$

$$(I_3 E_3 E_5 = 0)\ \Omega\ (E_2 E_3 = 0)\ \Omega\ (I_1 I_3 E_1 E_5 = 0)]$$

In principle, the above probability may be calculated, given that the P_{Ij} and P_{Ek} are known. However, the individual parts of the probability statement are strongly interdependent and the computation would be complex, even for a small example such as this one. More promising, therefore, is the possibility of calculating upper and lower bounds on the probability, which is possible by taking advantage of the following theorem concerning binary random variables:

Theorem: if $X_1 \ldots\ldots X_n$ are independent (0, 1) random variables and $Y_i = {}_{j \in J_i} X_j$ $(i = 1 \ldots r)$ where the J_i are any subsets of the integers $1 \ldots n$, then

$$P\,[(Y_1 = 0)\ \Omega\ (Y_2 = 0)\ \Omega\ \ldots\ldots (Y_r\, 6\, 0)] > P(Y_1 = 0).\ P(Y_2 = 0) \ldots P(Y_r = 0)$$

Applying this theorem, an upper bound may immediately be obtained:

P [Demand level is attainable]

$$\leq 1 - P\,(I_1 I_2 I_3 = 0).\ P\,(I_1 E_1 E_2 = 0).\ P\,(I_2 E_4 = 0).\ P\,(I_3 E_3 E_5 = 0).\ P\,(E_2 E_3 = 0).\ P\,(I_1 I_3 E_1 E_5 = 0)$$

$$= 1 - (1 - P_{I_1} P_{I_2} P_{I_3}) (1 - P_{I_1} P_{E_1} P_{E_2}) (1 - P_{I_2} P_{E_4})$$

$$(1 - P_{I_3} P_{E_3} P_{E_5}) (1 - P_{E_2} P_{E_3}) (1 - P_{I_1} P_{I_3} P_{E_1} P_{E_5})$$

Returning to the network representation of the problem, it can be seen that the required level of demand can be regarded as attainable provided that at least one of the policy instruments of exogenous variables in each cut of the network is recorded as a "success" (see Wagner (1975), p. 597 for the definition of a cut in this context). In this example, this implies

P [Demand level is attainable]

= P [(at least one of $I_1, E_4, E_3 = 1$) Ω (at least one of $I_2, E_4, E_1, E_3 = 1$) Ω

(at least one of $I_1, E_4, E_5, E_2 = 1$) Ω (at least one of $I_2, E_4, E_5, E_2 = 1$) Ω

(at least one of $E_2, I_3 = 1$) Ω (at least one of $E_1, E_3, I_3 = 1$)]

$$= P[(1-(1-I_1)(1-E_4)(1-E_3) = 1)\,\Omega\,(1-(1-I_2)(1-E_4)(1-E_1)(1-E_3) = 1)\,\Omega$$

$$(1-(1-I_1)(1-E_4)(1-E_5)(1-E_2) = 1)\,\Omega\,(1-(1-I_2)(1-E_4)(1-E_5)(1-E_2) = 1)\,\Omega$$

$$(1-(1-E_2)(1-I_3) = 1)\,\Omega\,(1-(1-E_1)(1-E_3)(1-I_3) = 1]$$

$$= P[((1-I_1)(1-E_4)(1-E_3) = 0)\,\Omega\,((1-I_2)(1-E_4)(1-E_1)(1-E_3) = 0)\,\Omega$$

$$((1-I_1)(1-E_4)(1-E_5)(1-E_2) = 0)\,\Omega\,((1-I_2)(1-E_4)(1-E_5)(1-E_2) = 0)\,\Omega$$
$$((1-E_2)(1-I_3) = 0)\,\Omega\,((1-E_1)(1-E_3)(1-I_3) = 0)]$$

Now the $(1 - E_k)$ and $(1 - I_j)$ are independent binary variables, so the previously quoted theorem may be applied again:

P [Demand level is attainable]

$$\geq P[(1-I_1)(1-E_4)(1-E_3) = 0].\,P[(1-I_2)(1-E_4)(1-E_1)(1-E_3) = 0].$$

$$P[(1-I_1)(1-E_4)(1-E_5)(1-E_2) = 0].\,P[(1-I_2)(1-E_4)(1-E_5)(1-E_2) = 0].$$

$$P[(1-E_2)(1-I_3) = 0].\,P[(1-E_1)(1-E_3)(1-I_3) = 0]$$

$$= [1-(1-P_{I_1})(1-P_{E_4})(1-P_{E_3})]\,[1-(1-P_{I_2})(1-P_{E_4})(1-P_{E_1})(1-P_{E_3})]$$

$$[1-(1-P_{I_1})(1-P_{E_4})(1-P_{E_5})(1-P_{E_2})]\,[1-P_{I_2})(1-P_{E_4})(1-P_{E_5})(1-P_{E_2})]$$

$$[1-(1-P_{E_2})(1-P_{I_3})]\,[1-(1-P_{E_1})(1-P_{E_3})(1-P_{I_3})]$$

This expression constitutes a lower bound on the probability that the required demand level is reached.

By way of numerical illustration, suppose that the demand level specified is 200,000 trips per day, that each exogenous variable has a probability of 0.3 as being recorded as a success and that the equivalent figure for the policy instruments is 0.7. By substituting into the expressions given for the upper and lower bounds, it may be shown that a demand level of 200,000 can be met with a probability lying between 41.50% and 60.36%.

This basic analysis may readily be extended in at least two directions. Firstly, the probability limits may be subjected to sensitivity tests in response to marginal changes in various P_{Ij} and P_{Ek} terms. Secondly, supposing that opinions differ as to the demand level required adequately to support an urban mass transit system, it would be possible to re-run the whole analysis with a different target figure, different combinations of required policy decisions and exogenous outcomes (indeed different variables, if necessary) and so obtain a series of probability limits, which might be illustrated graphically in the manner shown in Figure 6.

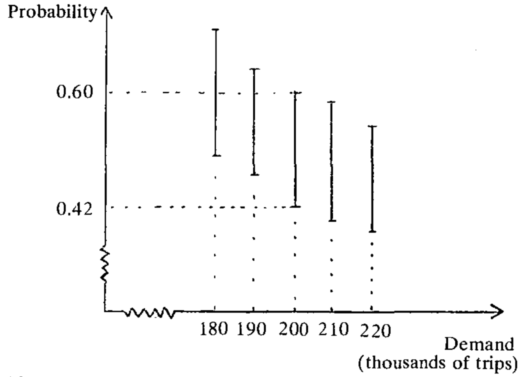

Figure 6

It should be noted that the reason the minimum probability levels associated with the five demand levels 180...220 sum to more than one is basically that the corresponding events are not mutually exclusive.

CONCLUSIONS

This paper has sought to achieve two main goals, one largely by assertion, the other by demonstration. The first concerns the attitude towards uncertainty typically found in the transport sector. Uncertainty has not been adequately taken into consideration in the past. Now, however, not only is there good reason to believe that its presence is likely to be felt more and more often in transport decisions, but also there is increasing evidence from the theoretical literature, that proper recognition of uncertainty will not only affect individual decisions, but, more importantly, may have a real qualitative effect on the type of investments which are undertaken. In particular, flexibility in the face of uncertainty may well have a significant value in its own right which ought to appear in the cost-benefit analyses which normally form the basis for major transport investment decisions. In addition to potential bias within the transport sector, there also exists the possibility of inter-sectoral distortions if, say, health service investments are less prone to uncertainty than projects in transport.

The second goal of the paper has been to demonstrate the existence of a sizeable gulf between theoretical understanding of uncertainty and the practical tools for taking it into account. Although both the proposed methods discussed in the later part of the paper are very simplistic, they do demonstrate that workable analytical tools can be devised appropriate to at least some types of problem in the transport sector. Equally, their limitations should serve to emphasize how great is the need for further research effort on this particular interface between theory and practice.

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