EMME:
A planning method for multi-modal urban transportation systems

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INTRODUCTION

The purpose of this paper is to present a new multi-modal urban transportation planning method, EMME (Equilibre Multi-Modal/Multi-Modal Equilibrium), that is based on and is consistent with a general theory of equilibrium in transportation systems. The requirement that is often asked of transportation systems analysis methods is that they be policy responsive, in the sense that a considerable number of the variables considered are explicit policy variables: changes in these variables bring about changes in the distribution of transportation demands and flows between origins and destinations, over specified links, between modes or by time of day. EMME has been designed to respond, as much as is possible within the current state of the art of transportation systems modelling, to changes in these variables and is oriented to short term and medium term planning applications. The implementation of EMME exploits the considerable progress that has been achieved recently in the development of efficient algorithms for the computation of equilibrium flows in transportation networks and on advanced concepts for computer based systems and data bank design. Full advantage has been taken of the possibilities offered by the current computing technology, to facilitate the interaction of the user with the possibilities offered by EMME; the input data is provided in a form related to the originating activity and not to the internal file design and the specification of planning scenarios is achieved in a way that requires minimal user effort.

The paper is organized in the following way: first we describe the general notions of equilibrium in transportation systems, provide a spatially disaggregated interpretation of these notions and then indicate the specific modelling choices made in EMME that are consistent with this general framework; we discuss next, the innovations that are offered by EMME in modelling the road and transit networks, in the integration and use of travel demand functions and in the algorithms that are used to compute the equilibrium demands and flows; then we describe the computer systems design principles on which EMME is based and give an overall description of the design; last but not least we show how EMME may serve in the analysis of land use allocation and changes in the supply of transportation services provided.

EQUILIBRIUM IN TRANSPORTATION SYSTEMS

We begin our presentation with the formulation of the notion of equilibrium in transportation systems, such as exposed by Manheim [6] and Beckmann [3]. (Our presentation is inspired by Manheim's work although many of the details are different). The transportation system consists of the following main components:

T, the transportation infrastructure and its control or regulatory measures,

A, the socio-economic activities in the area spanned by T,

D, the transportation demands on T generated by A, and

L, the levels of service offered on T to demands D.

The fundamental relations that link the components of the system are:

D = G (A, L): the demands generated by the activities A depend on the level of service (or performance) L of the transportation infrastructure; as the service level decreases, the demand decreases.

L = C (T,D): the level of service depends on the transportation infrastructure T and on the realized demand D; as the demand increases, the level of service decreases and its perceived cost increases.

The simplest and most general definition of equilibrium is that *equilibrium is a steady state that is reached when the demand for transportation gives rise to a service level that maintains that demand.* The common pictorial representation of the equilibrium state is achieved by supposing that the above relations are *continuous functions* for given A and T, and their intersection defines the equilibrium demand D_E and service level L_E.
Sie Figure 1 below.

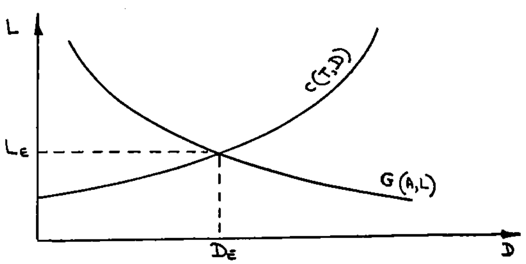

Figure 1

For a fixed A, we denote the restriction of the demand function $\tilde{G}(L)$ and similarly, for a fixed T, the restriction of the service (performance) function is $\tilde{C}(D)$. Then, the equilibrium state is (D_E, L_E) that satisfy

$$L_E = \tilde{C}(D_E) = \tilde{G}^{-1}(D_E) \tag{1}$$

If the transportation infrastructure and its control measures is modified such that $T \rightarrow T^1$ or the activities change from $A \rightarrow A^1$, the demand and service level functions are displaced. The *basis of all impact analysis* is the prediction of changes in the demands $D_E \rightarrow D_E{}^1$ and service levels $L_E \rightarrow L_E{}^1$, which is the new equilibrium state. See Figure 2 below.

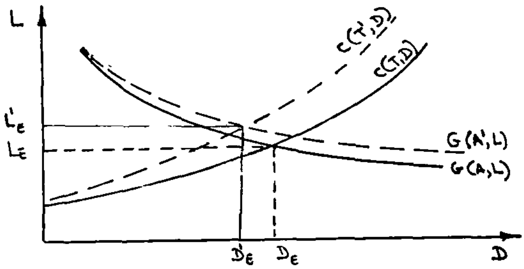

Figure 2

Before presenting a spatialized version of this notion of equilibrium we remark that both D and L are vectors, that is the demand functions may be specified by population subgroups, by time of day, by mode, etc. and the service levels may characterize each mode and may be travel time, distance, cost, comfort, etc. In this exposition our notation does not explicitly recognize this; rather than use vector notation and refer to demand vectors and service level vectors, we use simply demand and service level. We shall however make this distinction in describing the details of EMME.

The simple paradigm of equilibrium in a transportation system of Figure 1 is a reasonable representation for the aggregate of an urban area or for the analysis of a well defined corridor between 2 centers, where the distribution of activities in space and the details of the network of T are not relevant. The explicit consideration of the spatial distribution of the socio-economic activities A and of the network aspects of T makes the definition of equilibrium more tedious although the basis notion of equilibrium *remains intact.* We consider, as usual, that the urban area is subdivided into zones, a zone representing a (hopefully) homogenous subgroup of activities that generate and attract demands for transportation. The socio-economic activities A are subdivided by zone: A_p is the set of activities that generate travel demands at origin zone p and A_q is the set of activities that attract travel at destination q. Thus $A = (\bigcup_p A_p) \cup (\bigcup_q A_q)$

The service levels that are relevant are L_{pq}, that is between origins p and destinations q. The demand for travel between p and q is given by a demand function $D_{pq} = G_{pq}(A_p, A_q, L_{pq})$. Similarly T is the collection of T_a, the characteristics of all the links of the transportation network(s) and their control or regulatory measures. The service function L is specified for each link as a function of the *flows* on each link, F_a, induced by (or derived from) the travel demands D_{pq} : $L_a = c_a(T_a, F_a)$. The definition of the equilibrium travel demands and service levels is more tedious, as mentioned earlier, since the relation between L_{pq} and L_a must be determined in an unambiguous way. In order to achieve this it is also necessary to relate the origin destination demands D_{pq} to the link flows F_a.

The following discussion applies simultaneously to all origin destination pairs, however for simplicity of exposition we focus on a single origin destination pair (p, q). See Figure 3.

The travel demands D_{pq} use several paths, not entirely distinct, between p and q and induce path flows $H_{k,pq}$, and path service levels $L_{k,pq}$. The values of the path flows are a function of the demands and the path service levels

$$H_{k,pq} = \Phi(D_{pq}; L_{k,pq}, \text{ all } k), \text{ all } (p, q) \qquad (2)$$

and the flows *are conserved,* that is

$$D_{pq} = \sum_k H_{k,pq} \qquad (3)$$

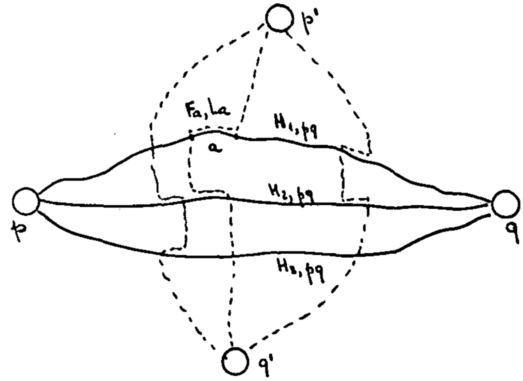

Figure 3

Also, the origin to destination service level is a function of the path flows and service levels

$$L_{pq} = \Psi(H_{k,pq}, \text{ all } k; L_{k,pq}, \text{ all } k) \qquad (4)$$

Each of the paths is composed of several links a of T. For a given path, the arcs are identified by an indicator function $\delta_{ak,pq}$ which equals 1 if link is used by the path k,pq and 0 otherwise. The service level $L_{k,pq}$ is then a function of the service (performance) levels associated with each link of the path, that is

$$L_{k,pq} = \Omega(L_a; \delta_{ak,pq}), \qquad (5)$$

and the service level of a link a depends on its induced flow F_a

$$L_a = c_a(T_a; F_a), \text{ all a} \qquad (6)$$

The last relation which is necessary for the definition of the equilibrium demands and service levels, and consequently the equilibrium flows, is the way in which the link flows depend on the demand D_{pq}, that is

$$F_a = \sum_p \sum_q \sum_k \delta_{ak,pq} \cdot H_{k,pq}, \quad \text{all a} \qquad (7)$$

By considering the chain of relations and equations (2)-(7) it is easy to see that the service level L_{pq} depends on all the demands D_{pq}, that is

$$L_{pq} = C(T_a, \text{ all a}; D_{pq}, \text{ all } (p, q)), \text{ all } (p,q) \qquad (8)$$

where C may be interpreted as the transformations, analytic or algorithmic, that are necessary to achieve (2)-(7). Thus, the equilibrium state is *the steady state that is reached when the demands for transportation give rise to link and path flows and corresponding service levels that maintain those demands.*

This is the definition given by Beckman [3]. More precisely, for fixed A_p, A_q, T_a, the equilibrium demands and service levels are

$$L^E_{pq} = \tilde{C}(D^E_{pq}, \text{ all } (p,q)) = \tilde{G}^{-1}_{pq}(D^E_{pq}), \text{ all } (p,q) \qquad (9)$$

the equilibrium flows F^E_a, $H^E_{k,pq}$

are the flows that are induced by D^E_{pq}

as described by (2), (3), (7) and the link service levels L_a given by (6) correspond to $L_{k,pq}$ given by (5) and origin-destination service levels L_{pq} given by (4).

Particular choices of the functions Φ, Ψ and Ω result in different specific models of the equilibrium in the transportation system and the possibility of numerically computing the equilibrium state depends on the mathematical structure of the resulting models.

Before proceeding to describe the specific modelling choices imbedded in EMME, we touch briefly on the extension of this equilibrium notion to consider systematic changes in T. We postulate a supply relation $T = S(D_E, L_E)$, that is, the changes in the transportation infrastructure offered to the users D is dependent on the equilibrium state. In addition, public agencies and private enterprise are subject to constraints of available resources such as capital, and identify objectives, such as profit, that influence the nature of the above relation. The important concept is that *supplier's actions are related to the equilibrium state,* and, in a spatial context, to the equilibrium flows and service levels.

Similarly, $A = R\ (D_E, L_E)$ is the relation that we postulate to describe the changes in the level and distribution of activities.

EMME: THE EQUILIBRIUM MODEL AND THE SOLUTION ALGORITHM

The transportation infrastructure modelled in EMME consists of a road network, T^{au}, and a transit network, T^{tr}. The road network consists as usual of nodes and links a, which carry the flow of private cars and transit vehicles. The transit network consists of nodes, access and transfer links and transit line segments s. Certain links and segments of the two networks can be considered to coincide in the sense that all transit lines that use the road network share the use of the road links with the private cars. The interaction between the two networks, a major feature of EMME, is denoted by setting the indicator function $\Delta_{a,s} = 1$ whenever a line segment s uses link a and 0 otherwise. The road link service function depends on the link volumes F_a^{au} and the number of transit vehicles that use link a, F_a^{tr}. Thus

$$L_a = c_a\ (T_a^{au} ; F_a^{au} ; F_a^{tr} \text{ if } \Delta_{a,s} = 1) \quad (10)$$

The link service function may include both time and distance. If distance is not relevant then the link service functions are simply link volume delay functions. The transit segment service functions do not depend on the transit passenger volumes, F_s^{tr}, but depend on the corresponding service level on the road link if $\Delta_{a,s} = 1$

$$L_s \doteq c_s\ (T_s^{tr} ; L_a \text{ , if } \Delta_{a,s} = 1) \quad (11)$$

See Figures 4 and 5.

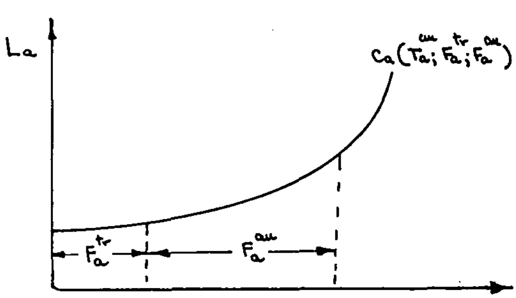

Figure 4

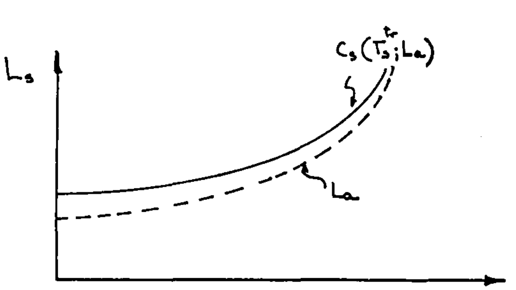

Figure 5

In EMME, the relations Φ and Ψ are subsumed by Wardrop's *user-optimized* principle [10], which states that the travel demands between origins and destinations is distributed among the utilized paths in such a way that no traveller can reduce his journey time by choosing a different path on the road network. As is well known, the implication is that all the utilized paths have equal service levels, that is

$$L_{k,pq}^{au} = L_{pq}^{au} \quad \text{if} \quad H_{k,pq} > 0 \text{ , all } (p,q) \quad (12)$$

and all paths that do not carry flow have higher service levels

$$L_{k,pq}^{au} \geq L_{pq}^{au} \quad \text{if} \quad H_{k,pq} = 0 \text{ , all } (p,q) \text{ .} \quad (13)$$

The relation Ω is additive, that is

$$L_{pq}^{au} = \sum_a \delta_{ak,pq} \cdot L_a \text{ , all } (p,q) \quad (14)$$

On the transit nework, a similar *user optimized* flow distribution is used, that is

$$L_{k,pq}^{tr} = L_{pq}^{tr} = \sum_s \delta_{sk,pq} \cdot L_s \text{ , if } H_{k,pq}^{tr} > 0 \text{ , all } (p,q) \quad (15)$$
$$L_{k,pq}^{tr} \geq L_{pq}^{tr} \quad \text{if} \quad H_{k,pq}^{tr} = 0$$

where $\delta_{sk,pq}$ is the indicator function which equals 1 if transit link or segment s is used in the transit path k from p to q.

The service times on the transit network identify separately, access and egress time, transfer time and in-vehicle travel time. Thus a path length is described by the above identified components and not by the total travel time.

The demand functions that EMME can accept are *zonal aggregate* functions of the general form

$$D_{pq}^{au} = G^{au}\ (A_p ; A_q ; L_{pq}^{au} ; L_{pq}^{tr})$$
$$\text{and} \quad D_{pq}^{tr} = G^{tr}\ (A_p ; A_q ; L_{pq}^{au} ; L_{pq}^{tr}) \quad (16)$$

The socio-economic variables A_p, A_q are fixed for the purposes of computing the equilibrium demands, flows and service levels. Therefore, we are only interested in the restrictions

$$D_{pq}^{au} = \tilde{G}_{pq}^{au}\ (L_{pq}^{au} , L_{pq}^{tr})$$
$$\text{and} \quad D_{pq}^{tr} = \tilde{G}_{pq}^{tr}\ (L_{pq}^{au} , L_{pq}^{tr}) \quad (17)$$

which are prepared by an internal manipulation imbedded in EMME.

It is important to remark that consistent with (17) are the so-called *direct demand* functions which include a trip generation effect, such as indicated in Figure 6 or simply *modal split* functions, where the demand for travel by all modes is fixed, say $\bar{D}_{pq} = D_{pq}^{tr} + D_{pq}^{au}$,
and the demand changes are only between modes, such as indicated in Figure 7.

Another important consideration is that these demand functions must be calibrated using service levels L_{pq}^{au}, L_{pq}^{tr} that are consistent with the modelling of the transportation network. If for instance, the travel time components are given different weights and these weights are calibrated using measures obtained from the network models, it is essential that these be consistent in the sense that the same L_{pq}^{tr} are found on the network using the results of the calibration. It is also worthwhile to note that it is not possible to exploit directly for EMME the recent advances in *disaggregate* demand modelling and calibra-

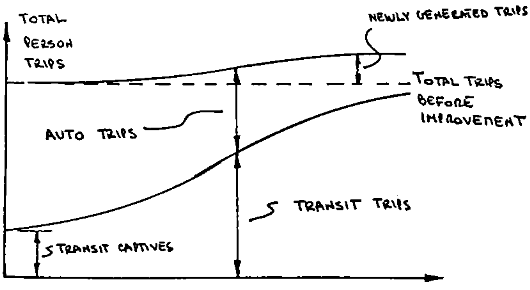

Figure 6

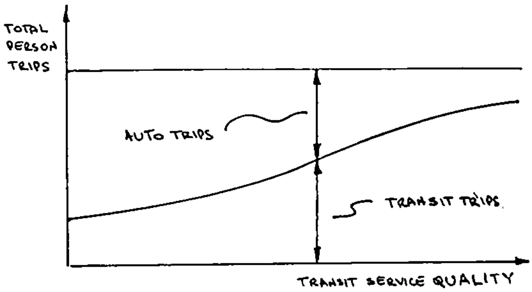

Figure 7

tion since the demand functions used are, as indicated earlier, zonal aggregate.

The resulting mathematical model is analysed in detail and the solution algorithm which is imbedded in EMME is described in [4]. The resulting model may be solved by a quasi-decomposition of the problem for each mode and results in a sequence of shortest path computations on the transit network and equilibrium demands, flows and service level computation on the road network. The oveall steps of the computational procedure, which yields a unique solution, is as follows:

Step 0: Select initial estimate of transit service levels $\bar{L}_s^{tr}$ (travel times) for all segments s such that $\Delta_{as} = 1$

Step 1: Compute shortest paths in the transit network and obtain transit service levels $\bar{L}_{pq}^{tr}$ for all (p, q)

Step 2: Solve an equilibrium problem on the road network using the demand functions $\tilde{G}_{pq}^{au}(\bar{L}_{pq}^{au}, \bar{L}_{pq}^{tr})$ for all (p, q) and obtain the equilibrium demands, flows and service levels, $\bar{L}_a$.

Step 3: If the current transit service levels $\bar{L}_s^{tr}$ are significantly different than those given by relations (11)

$\bar{L}_s^{tr} = c_s(\bar{L}_a, \text{ if } \Delta_{a,s}=1)$,

then replace $\bar{L}_s^{tr} \leftarrow \bar{L}_s^{tr}$ and return to Step 1. Otherwise continue to Step 4.

Step 4: Determine the transit demands

$$D_{pq}^{tr} = \tilde{G}_{pq}^{tr}(\bar{L}_{pq}^{au}, \bar{L}_{pq}^{tr}), \text{ all } (p,q)$$

and allocate these trips along the current shortest paths in the transit network.

Although we have shied away from using the term *assigment* due to the connotation that it has the fourth step in the classical sequence of generation, distribution, modal split and assignment, we shall refer, for simplicity, to the above procedure as a *bimodal assignment.*

EMME: THE NETWORK ALGORITHMS

Although EMME is evidently not as extensive a planning method as UTPS, that permits the execution of all the steps in the classical urban transportation planning format, it offers the following possibilities:

Given a fixed road demand $\bar{D}_{pq}^{au}$ compute the link and path flows and service levels according to Wardrop's user optimized principle. The computation is known (loosely) as the *equilibrium traffic assignment with fixed demand.* The algorithm imbedded in EMME is a further refinement [5] of the adaptation of the Frank and Wolfe algorithm for this problem realized in TRAFFIC [8], that was used extensively by us and other users.

Given a fixed transit demand $\bar{D}_{pq}^{tr}$ compute the link and path flows and service levels. This computation is a variation on the "all-or-nothing" transit assignment that considers diversion on common line segments, as realized in TRANSCOM [1], that was also used extensively before.

Given demand functions $\tilde{G}_{pq}^{au}$ compute the equilibrium demands, link and path flows and service levels according to Wardrop's user optimized principle. This computation is carried out by the adaptation of the Frank and Wolfe algorithm for this problem as described by Nguyen [7] and represents *the first time* that such an algorithm has been imbedded in an operational computer-based package.

Given demand functions $\tilde{G}_{pq}^{tr}$ compute shifts in the transit demands, link and path flows and service levels. This computation is necessary if a change in the transit services offered change the service levels significantly and induce a change in the transit demands.

Carry out the full bi-modal assignment outlined in the previous section.

It is important to remark that the algorithms imbedded in EMME, although rather sophisticated, may be used for planning purposes due to the development of the recent generation of computers that permits to complete seemingly cumbersome computations in reasonable total computing times. We estimate that the execution of the full bi-modal assignment for urban transportation networks of the order of 5000 links should take about 15-20 minutes of elapsed time on a CDC CYBER 74, or a cost of the order of $300.

EMME: CODING THE ROAD AND TRANSIT NETWORKS

One of the major features and innovations imbedded in EMME is an efficient interface between the road and transit networks. Particular attention was given to the way in which the data required by EMME is prepared by the user in order to ease the dialogue between the planner and the computer-based procedures.

The innovation of the coding procedure is that the road and transit networks may be coded independently if the user does not wish to use the interface feature and then, the minimal information necessary to achieve the interface is provided, if the interface feature is desired. The details of the coding procedure are described in [1]. We summarize in this paper the main concept used. We suppose that a road network is coded in a conventional way and is available. See Figure 8. Three types of transit networks are distinguished in the coding:

Type 1 network (low level of detail): represents the minimum data required to compute the transit flows and service levels given a fixed O/D matrix of transit demands. This type of network is used to represent only passenger movements. The only nodes used in this type network are line terminus, transfer nodes and access nodes. This type network is coded on the basic road network using road nodes, when they exist where required. If no road node exists but one is required, a new "transit" node is used. See Figure 9.

Type 2 network (intermediate level of detail): is required to perform the computation of transit flows and service levels with the EMME bi-modal equilibrium al-

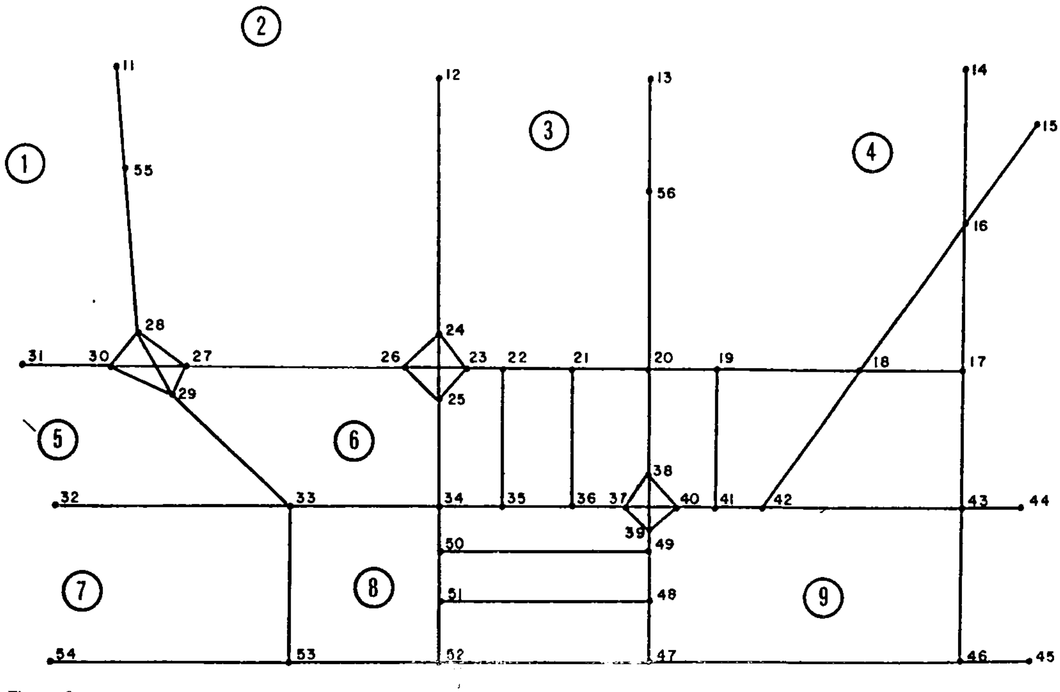

Figure 8

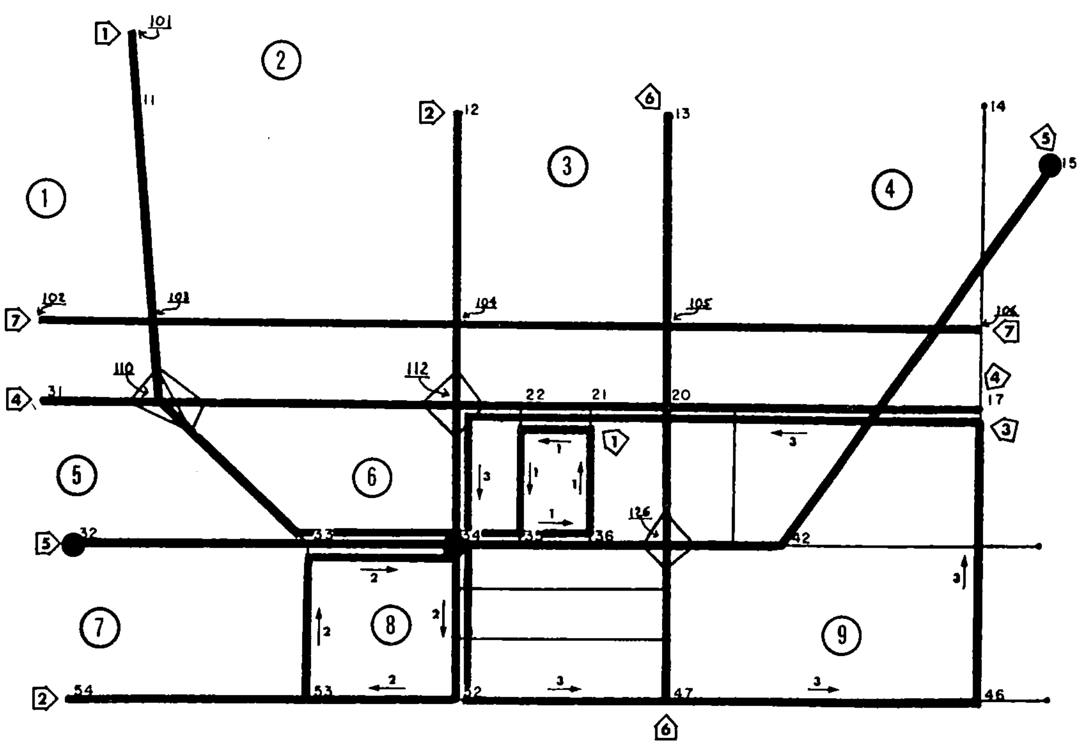

Figure 9

gorithm. It is obtained by inserting intermediate road nodes on the type 1 network, while ignoring the detail of "turning" road nodes at an intersection. A transit node, that corresponds to the expanded intersection, is coded to represent the intersection.

To permit interface with the coded road network, the user must define a correspondence table between transit nodes and road nodes for expanded intersection. Finally the user provides the access and egress links for the transit network. See Figure 10.

Then, the interface module in EMME generates internally a Type 3 network (high level of detail) which describes completely the transit vehicle movements by using coded road links. The module uses as input the type 2 network and the associated node correspondence table and each transit line is transformed into a sequence of road links. A byproduct of the interface is the possibility of determining transit vehicle travel time from the travel time of the road link as indicated earlier.

Thus the coding of the transit network may be viewed as a useful exercise which gives the planner a global image of the complete transit network and its interrelationship with the road network, as well as the nitty-gritty details of modelling access and egress links which are specific to the transit network.

EMME: THE COMPUTER SYSTEMS DESIGN

The computer based implementation of EMME is designed to construct a data base containing the description of the urban road and transit networks, the socio-economic activities and the coefficients of the demand functions. Data from the data base is then transformed into internal files that are used by EMME's assignment algorithms to predict the equilibrium demands for each mode, the link and path flows and service levels on each network.

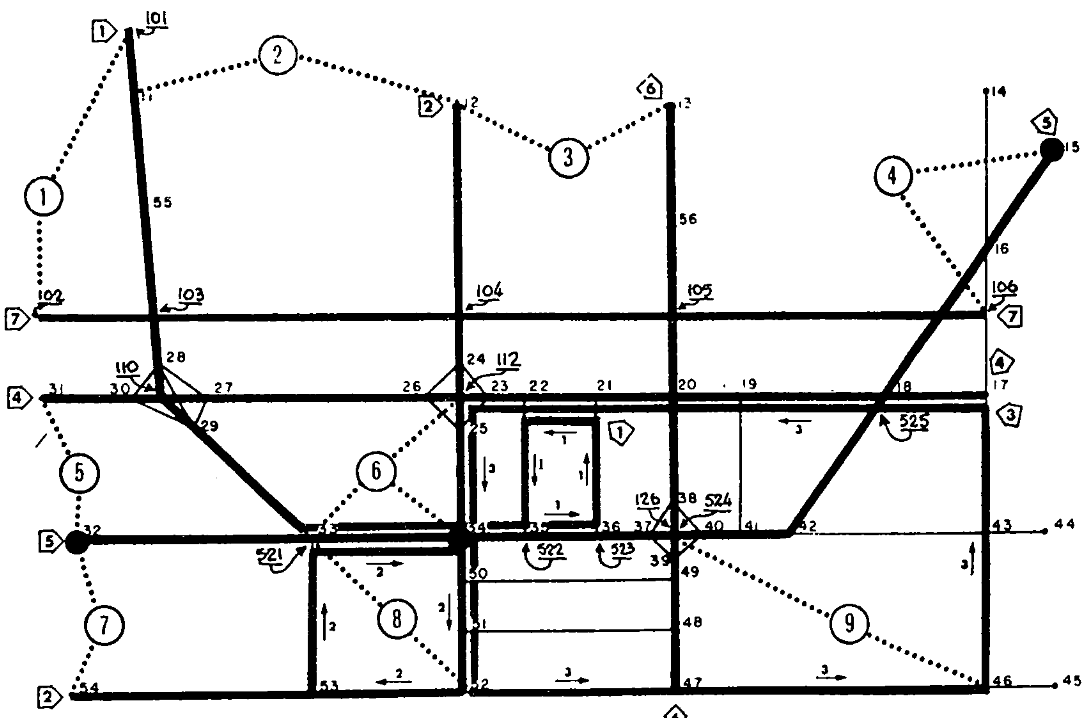

Figure 10

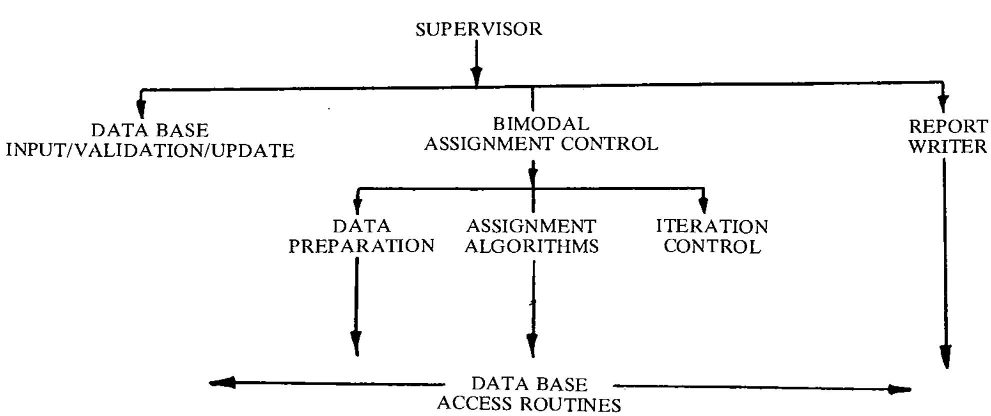

Figure 11 - EMME: System Hierarchy

EMME accepts a simple command language which permits the user to specify data base manipulations and request the execution of any one of the network algorithms outlined earlier. The commands are decoded by a supervisor which calls the relevant sections of the system.

The structure of the system is hierarchical, as shown in Figure 11. Within the hierarchy, the design is modular so that each section consists of a simple control procedure which calls different modules to perform the necessary functions. *Well defined files provide the interface between the modules.*

The design objective for the data base include efficient use of its random file structure and the ability to store and apply temporary modifications so that the user can easily investigate, for example, several different network configurations.

The Data Base Section include modules to read and validate the data provided by the user. It also includes modules to transform the node numbering system defined by the user to a form which enables the network algorithms to operate efficiently. This renumbering is however not apparent to the EMME user.

The Bi-modal Assignment Control Section of the system calls three types of modules: data preparation modules, network algorithm (assignment) modules and iteration control modules. The data preparation modules perform demand function manipulations and transform the demand depending for instance on the auto occupancy factors. The assignment modules with their associated subroutines perform the road traffic assignment and transit assignment. During the iterative bi-modal process, communication between the two assignment modules is defined by files such as travel time by auto on each link, travel time by transit on each line segment, automobile and transit service levels (impedances) for each origin-destination pair, etc... The iteration control modules adjust the values associated with the transit network, such as transit frequency and capacity, based on the resulting equilibrium demands, flows and service levels, and determines if another bi-modal assignment iteration is necessary. When an assignment terminates, the predicted equilibrium demands, flows and service levels for each mode are saved on files which can be accessed by the Report Writer. The Report Writer modules produce reports on the predicted values. The user may also use the files containing the predicted values and the data base to produce any reports that he requires.

EMME: A POLICY RESPONSIVE PLANNING METHOD

As remarked earlier, one of the principal aims of EMME is to aid in the efficient evaluation of alternative policy options. Most of the variables that are important for policy evaluation are likely to be independent explanatory variables in the demand functions (variables A_p, A_q, L_{pq}) and the others are contemplated network modifications that indirectly induce changes in the service levels L_{pq}. Thus, a well specified and properly calibrated demand model is of prime importance. If such a *good* demand function is available and the road and transit networks are modelled at the *adequate* level of detail a variety of changes in the *supply* of transportation may be evaluated. Some of these are:

– the effect of changes in public transit services such as line routing, line frequency, fares, introduction of express lines, exclusive bus lanes, etc...

– the effect of road pricing and changes in parking space availability and parking costs

– the effect of changes in the road network such as changes in one way orientation of streets, closing or opening of streets, etc.

Conceptually, this amounts to a large extent to recomputation of the equilibrium state (D_E, L_E) when actions dictated by relation(s) of the type $T = S(D_E, L_E)$ are contemplated. Thus, viewed in this way, EMME offers the possibility of interfacing supply decisions and, if available, explicit supply functions for transportation. So far, the supply decisions or functions have not been studied with the same intensity as demand functions. Future findings in this area will permit the generalization of EMME to include this component.

In a completely analogous way, EMME may be used to interface with decisions or systematic procedures that modify the socio-economic activities A according to relation(s) $R(D_E, L_E)$, as outlined earlier. The challenge for the development of these expanded planning methods is one of sheer size: it remains to determine at what level of detail it is possible to apply and use such methods efficiently.

With relatively minor modification, the elastic demand network equilibrium computation of EMME may be used to estimate origin-destination matrices for *"windows"* in the network based on the method recently proposed by Nguyen [9]. This will enable the more detailed study of *subareas* without excessive data needs, requiring only observed times on the network and the link service level functions.

Finally, we would like to mention that the road and transit components of EMME have already been used successfully to evaluate changes in the supply of transportation services. The road component has been used to test the feasibility of introducing an intermediate capacity transit system in a Canadian city and the transit component has been used by the Montreal Urban Community Transportation Commission (MUCTC) to evaluate five scenarios of transit network development in the Montreal region.

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