TOPIC 16
TRAVEL SUPPLY-DEMAND
MODELLING

A NETWORK DESIGN MODEL THROUGH CONGESTED MULTIMODAL TRAFFIC ASSIGNMENT

BRUNO MONTELLA
Dipartimento di Ingegneria dei Trasporti
Università degli Studi di Napoli FEDERICO II
Via Claudio 21, 80125 Napoli, ITALY

MASSIMO DI GANGI
Dipartimento di Ingegneria dei Trasporti
Università degli Studi di Napoli FEDERICO II
Via Claudio 21, 80125 Napoli, ITALY

GENNARO N. BIFULCO
Dipartimento di Ingegneria dei Trasporti
Università degli Studi di Napoli FEDERICO II
Via Claudio 21, 80125 Napoli, ITALY

Abstract

In this paper a transit-network design procedure which incorporates a multimodal and multiuser assignment model is presented. Consistency is achieved between mode and path choices. An heuristic design procedure is adopted for both private and transit networks. Networks performances are evaluated by using proper indicators.

INTRODUCTION

This paper deals with the problem of searching a procedure for the optimal configuration of a private and public on-wheels transport system in an urban area.

There are at present no closed form optimal search procedures, although a "good" configuration may be found by using interactive simulation techniques. In this respect, an interesting procedure is currently being developed. Its initial results, both theoretical and computational, are reported in the paper.

The approach of such a problem implies several theoretical aspects relative to multimodal assignment and operative aspects relative to the assessment of various configuration hypotheses. In order to restrict the length of the paper, theoretical aspects are covered in brief, with a more detailed analysis envisaged for further publications. Computational aspects were developed on test networks and the obtained results are reported.

The schematic structure of the procedure is that described in Figure 1.

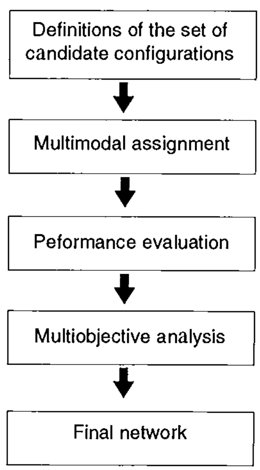

Figure 1 General scheme of the simulation model

In the following sections of the paper, problems connected with the definition of supply models will be discussed and, in particular, attention will be paid to the methodology for identifying private and public network graphs and to the generation of alternative network configurations. The problem also will be taken of multimodal assignment to transport networks, highlighting theoretical aspects which are peculiar to the problem in hand. A set of service indicators will be presented for the multimodal network and their use in the evaluation methodology will be covered.

The authors' original contribution consists in the definition of a methodology to produce several network configurations to be evaluated, consistently with the Italian norms in traffic management and with effective logical rules. An original multimodal and multiuser stochastic assignment model is also presented and its mathematical properties and applicability investigated.

SUPPLY MODEL DEFINITION

The problem proposed is that of determining, among all the possible schemes, a limited number of transport network configurations from which to choose by means of multiobjective analysis. As we are unable to propose some universally valid schemes, given the great variety of urban agglomeration configurations and transport demand characteristics, a logical procedure is put forward for use in generating some different supply models to be examined.

Constraints on road geometry and/or functionality contained in this paper are referred to rules and regulations currently operating in Italy, nevertheless the definition process is an original contribution of the authors and it can be adapted only considering other kind of constraints.

Identification of the existing road network graph

Let the set of all the urban network roads be S. However the graph does not show either roads unsuitable for traffic (SNC set) or roads which, for clear reasons of capacity and connection, are definitely for a very local service (SSL set). Their representation in the graph is only necessary if network configurations for local roads are explicitly studied or in an adapted form, if the parking supply is explicitly modelled.

A set of arcs can thus be identified which comprises part of the principal urban network (motorway links, through roads and local roads) given by SRPU = S – SNC – SSL.

Identification of constrained links

In this phase, those links belonging to the SRPU set are identified which, for various reasons, have a limited number of possible alternatives.

We may thus split the SRPU network set into four subsets:

- $SRPU_1$ links which must be one-way;
- $SRPU_2$ links which must be two-way;
- $SRPU_P$ links which must be pedestrianized;
- $SRPU_{NV}$ non-constrained links;
- $SRPU_0$ motorway links.

Thus, $SRPU = SRPU_1 \cup SRPU_2 \cup SRPU_P \cup SRPU_{NV} \cup SRPU_0$ while the intersection between pairs of the various subsystems have to be empty.

Network scheme for through roads

Through roads must have at least two lanes in each direction for private transport (any line of public transport must run on a further lane added to them). Besides, such roads have the function of serving longer trips and of allowing high travelling speeds (70 km/h speed limit). According to these considerations, a system of through-road links (PSS) may be defined as follows:

$$PSS = SRPU_{NV} \cup SRPU_2^4 - SRPU_0 - S_{ped} - S_{cen} - S_{tor}$$

where

$SRPU_2^4$ roads which must be two-way and have at least 4 lanes ($SRPU_2^4 \subseteq SRPU_2$);

S_{ped} roads crossed by heavy pedestrian traffic;

S_{cen} roads in central or commercial zones and not protected from interference from other traffic flows;

S_{tor} roads which are very tortuous or steeply sloping.

The set of roads thus obtained may, in turn, be divided into three subsets:

- PSS_2 two-lane links: they must be one-way to allow for through traffic
- PSS_3 three-lane links: they may be through roads only in one direction;
- PSS_4 links with four or more lanes: these are unlikely to be useful if one way; they are almost always two-way.

Then it also holds that $PSS = PSS_2 \cap PSS_3 \cap PSS_4$

Aggregation procedure of the primary graph

Once the PSS set has been defined and the motorway links added to it, O/D pairs are identified for which the distance is greater than a pre-established limit (eg 2 km); the centroid nodes are connected to a first attempt network consisting only of the through roads and the main ones. Network connection has to be verified. If this is not the case, it is made connected by necessarily using two-lane links which were not considered in the PSS set (because of location, pedestrian crossing flows or tortuousness).

Once the peak-hour O/D matrix is known and a matrix of distances between centroids is constructed, the following stages are carried out:

- all roads are initially assumed two-way, as compatible with size;
- the O/D pairs are placed in descending order of demand, and demand is loaded (only for design purposes) to the minimum path until capacity is reached. If a link belongs to PSS_3 set or PSS_4 set, link capacity in the opposite direction is forced to zero adding the capacity to the remaining direction. The rest of the demand will be loaded to properly calculated sub-optimum paths.

This procedure ensure that the great part of the "qualified" demand may be (not necessarily has to be) assigned to the network by means of high performance links. Should the network be already saturated (or not connected for any O/D pair), the remaining demand is assigned to the local road network. From an examination of the loading results, "really" through-road links may be defined as those for which the flow is equivalent to at least 70% of capacity or which are used for the whole flow between an O/D pair. The links used in the above network make up a set which we call SS.

Network scheme for local roads

Once the network configurations have been defined for through roads, possible schemes of the local road network are constructed to supplement the previous ones. The set of probable local links (PSQ) is defined as that comprising all the links of the SRPU except main and through-road links: $PSQ = SRPU - SRPU_0 - SS$.

The PSQ set may, in turn, be split into the following subsets:

- PSQ_1 roads which must be one way by necessity (one lane);
- PSQ_2 two-lane roads;
- PSQ_3 three-lane roads;
- PSQ_4 roads with 4 or more lanes.

Sets PSQ_3 and PSQ_4 are in most cases empty, insofar as the links which they should consist of, have generally already been used in the previous stage. We thus proceed to connect all the centroids with the network comprising the main road arcs, through-road arcs and the arcs of set PSQ (all are initially assumed as two-way except PSQ_1).

At this stage, there are two ways forward:

- We proceed as for through roads, assigning O/D flows to the minimum paths. In this case, first the long-distance flows not assigned to the previous stage are assigned; subsequently, the others are assigned in descending flow order.

- We hypothesize a scheme entirely of one-way roads, which in most cases maximizes system performance. A connected network is not always achieved in this way, which makes it preferable to have a scheme studied according to the existing infrastructures.

Both the procedures give, in general, different configuration possibilities according to the different choices of the planner that yields to several schemes to be evaluated.

Having carried out the flow loading, the "really" local links may be identified as those that have reached a ≥ 0.5 degree of saturation or, though with a degree of saturation < 0.5, are two-way. The set of arcs used for local roads will be indicated by SQ. Remaining arcs (not belonging to the main, SS and SQ sets) are discarded from the network, if in this case the graph is already connected.

Public transport network schemes

The first network scheme is that oriented on transport demand as with that for private transport. The second network scheme under consideration is that based on circular lines. The third scheme is still largely based on the same assumptions as the network scheme with circular lines, as regards the suitability of encircling the urban centre with several paths. However, in this case, semi-rings are used, that is every line reaches the centre starting from a peripheral terminal and, going round it for a more or less extensive stretch, departs towards the second peripheral terminal.

MULTIMODAL ASSIGNMENT

The difficulties arising from modelling multimodal assignment may be summarised into (Abdulaal and LeBlanc 1982):

- the reciprocal interference between various transport modes which share the same physical transport infrastructure and which thus contribute jointly to congestion;
- consistently modelling two different levels of mobility choices, modal and path choice, making allowances for the reciprocal effect which the two choices exert on each other through the mechanism of congestion.

In the case of a multimodal network, some of the physical infrastructures represented with links may be affected by several flows, as they are representative of several transport modes. For each of these flows, the link travel cost is different but, in general, it varies jointly with the variations of all the flows on the same link. The hypothesis of cost function separability is thus irrevocably invalidated, also in the case where the reciprocal influence between physically separate links is not considered. Furthermore, it may be easily imagined that the influence of a transport mode on link flow may in general be different from that of another mode on the same link: the symmetry of the Jacobian matrix of cost functions is thus fairly difficult to ascertain. Failure to verify the properties of cost function separability and non-symmetry of the Jacobian cost function matrix invalidates the sufficient conditions for equilibrium uniqueness and the formulation of this problem as one of equivalent optimization.

Moreover the problem arises of describing the users' contribution to the presence of various types of vehicle according to chosen mode and, secondly, the mechanism by which such vehicles of different types affect congestion which in turn affect users' mode choice.

Of the contributions in the literature which aim to integrate the problem of equilibrium assignment with modal choice, one of the most effectively applied in practice was put forward by Florian (1977) who proposed a deterministic equilibrium model for path choice by car and an All or Nothing (A/N) assignment procedure relative to one or more lines and/or modes of public transport. The above procedure, which heuristically summarizes two different levels of consistency, the equilibrium of the private transport system and its consistency with public transport performance, if converging, leads to a consistent solution of private and collective costs and respective flows, still supposing that collective costs are a function of private costs. Even though this procedure ensures consistency between public transport costs and private costs (albeit with the simplificatory hypothesis that the former depend one-to-one on the latter) and, in some way, the consistency of modal choice with transport costs at equilibrium, there are still some parts worth exploring, above all tied to the fact that neither the convergence of the procedure itself nor the uniqueness of the achieved solution is demonstrated.

Others deterministic approach based on the variational inequality or on other mathematical programming have been presented more recently over the years, but very few contributions have be done to the stochastic multimodal assignment.

In the following an original multimodal (and multiuser) stochastic model is proposed.

Let r be the generic network physical link, on which there may be various user categories and different transport modes. A set of flow/mode indexes may be associated to the link in question (M_r) such that $f_l(l \in M_r)$ represents the set of flows associated to network link r.

If n_a stands for the number of links/modes present on the network, flows relative to every category of users may be ordered in vectors: f^i of size n_a x 1.

A transport cost corresponds to each flow $f^i{}_l$ associated to the generic link r, which measures the disutility perceived by users constituting flow $f^i{}_l$ in crossing the link. Such a cost is hypothesized as consisting of two aliquots, one constant ($c^i{}_{0l}$) and the other dependent on the flow conditions along the link, on which the effects of congestion are felt. The travel cost of link r relative to the generic flow $f^i{}_l(l \in M_r)$ is expressed as:

$$c^i_l = c^i_{0l} + b^i.t_l \tag{1}$$

where t_l is the part of cost dependent on congestion and independent of category. Such a part contributes to the cost of each category proportionately to a parameter b^i, which differs for each category but, within the categories, is constant on all links. Note that, with this notations, it is allowed that on the same physical link r various transport modes may be described by different impedance functions, referred to different modal link $l \in M_r$.

Equation (1) may be expressed vectorially as:

$$\underline{c}^i = \underline{c}^i_0 + b^i.\underline{t} \forall i \tag{2}$$

The *standardized flow* z_l, the linear combination of all the flows associated to link r, is assumed as the independent variable of the cost function:

$$z_l = \Sigma_i \alpha^i \sum_{m \in Mr} \psi_{lm}.f^i_m \quad \forall l \in Mr \tag{3}$$

where ψ_{lm} represents the relative contribution which a generic mode, with index l of association to link r, makes to flow conditions for the mode relative to the index of association m to the same link r (with $\psi_{lm} = 0$ if l and m are not both associated to the same link r) and $\alpha^i \geq 0$ represents the weight attributed to the various user categories in the composition of standardized flow.

The vector of standardized flows may be expressed in matrix form as:

$$\underline{z} = \underline{\Psi}.\Sigma_i \alpha^i \underline{f}^i \tag{4}$$

Matrix $\underline{\Psi}$ is a diagonal block matrix, each block relating to the transport mode associated to the same physical link.

Using the above notations, the impedance functions may be expressed vectorially:

$$\underline{t} = \underline{t}(\underline{z}) = \underline{t}(\underline{\Psi}.\Sigma_i \alpha^i \underline{f}^i) \tag{5}$$

$$\underline{c}^i = \underline{c}_0^i + b^i.\underline{t}(\underline{\Psi}.\Sigma_i\alpha^i\underline{f}^i) \tag{6}$$

Let path be defined as any travelling consistent possibility, indicated by k. Let C_k^i be the transport cost on path k for the generic category. We introduce quantities a_{lk} equivalent to 1 or 0 according to whether mode/link l is part of path k or not. Thus, let $\underline{A}$ be the link/mode-path incidence matrix, of size (n_a x n_p), with n_p the number of possible paths on the network. It must possess consistency conditions on the succession of links and/or modes which comprise the paths and it allows path costs to be expressed as a function of arc costs:

$$\underline{C}^i = \underline{A}^T.\underline{c}^i \quad \forall i \tag{7}$$

where it is hypothesized that the path cost is the sum of the link/mode costs that comprise it, that every transport alternative is in principle available for any user category, and with C^i as the path cost vector for the generic category i.

Let F_k^i be the user flow of the generic category i which uses path k for making its own trips. The following relation may be expressed:

$$\underline{f}^i = \underline{A}.\underline{F}^i \quad \forall i \tag{8}$$

which expresses the fact that flow f_l^i associated to a certain mode/link is the sum of flows (of the same user category) of all the paths which share the route of the link with the transport mode.

Using the above notations, we may write:

$$\underline{C}^i = \underline{A}^T.\underline{c}^i = \underline{A}^T.\left(\underline{c}_0^i + b^i.\underline{t}(\underline{\Psi}.\Sigma_i\alpha^i\underline{f}^i)\right) \quad \forall i \tag{9}$$

which expresses the path cost vector as a function of path flows. For ease of notation, it may be further expressed as:

$$\underline{W} = \Sigma_i\alpha^i\underline{F}^i \tag{10}$$

$$\underline{\Gamma}(\underline{W}) = \underline{A}^T.\underline{t}(\underline{\Psi}.\underline{A}.\underline{W}) \tag{11}$$

Hence:

$$\underline{C}^i = \underline{A}^T.\underline{c}_0^i + b^i.\underline{\Gamma}(\underline{W}) = \underline{C}_0^i + b^i.\underline{\Gamma}(\underline{W}) \quad \forall i \tag{12}$$

The above formulas, together with the functional description of the cost functions for the various link types present in the supply model, exhaust the formal description of the supply model.

Let d_{od}^i be the number of users of the generic category i who must undertake a trip between the centroid pair (o,d). Let $\underline{d}^i$ be a diagonal matrix of size ($n_p \times n_p$) whose generic element of the main diagonal d_k^i is the trip demand between the pair (o,d) joined by the k-th path. Clearly, $\underline{d}^i$ is formed by blocks relative to paths which join the same pair (o,d) and hence whose elements are equal.

Let $\underline{P}^i$ be a vector of size (n_p x 1) whose generic element p_k^i is the probability of a user of the generic category i choosing to undertake his own trip on path k from all those that join the pair (o,d), that is from all those of the set of alternatives K_{od}^i.

According to the random utility theory, let us consider the vector $\underline{V}^i$ of systematic utilities of each category. Suppose it consists of $\underline{V}^i=\underline{V}_0^i-\underline{C}^i$, where $\underline{V}_0^i$ indicates the (constant) vector of the systematic utility aliquot of paths which does not depend on service level variables of the transport system (for example, linked to specific attributes of alternative paths). The generic element V_k^i is the systematic utility of the k-th path.

Let the demand function then be defined in the form:

$$\underline{\Phi}^i(\underline{C}^i) = \underline{d}^i.\underline{P}^i(\underline{V}_0^i - \underline{C}^i) \quad \forall i \tag{14}$$

and thus let:

$$\underline{F}^i = \underline{\Phi}^i(\underline{C}^i) \quad \forall i \tag{15}$$

It is noteworthy that no hypothesis has been made on the operative specification of the choice model (Probit, Logit...).

The globality of demand function and supply model expresses the circular dependence, non-separable by category, between path costs and path flows. Thus, the path flow vector is indicated as equilibrium vector, which resolves such dependence:

$$\underline{F}^{i*} : \underline{F}^{i*} = \underline{\Phi}^i(\underline{C}_0^i + b^i.\underline{\Gamma}(\underline{W} = \Sigma_i \alpha^i \underline{F}^{i*})) \quad \forall i \tag{16}$$

Note that the equilibrium flow vector must be sought within a precise feasibility set which satisfies conditions of flow non-negativity and demand satisfaction:

$$S_{F^i} = \left\{ \underline{F}^i : \underline{F}^i \geq \underline{0}, \sum_{k \text{ with same o,d}} F_k^i = d_{od}^i \right\} \quad \forall i \tag{17}$$

The feasibility sets of the other flow vectors defined are consequently derived:

$$S_{f^i} = \left\{ \underline{f}^i = \underline{A}^T.\underline{F}^i \quad \underline{F}^i \in S_{F^i} \right\} \quad \forall i \tag{18}$$

$$S_W = \left\{ \underline{W} = \Sigma_i \alpha^i \underline{F}^i \quad \underline{F}^i \in S_{F^i} \right\} \tag{19}$$

$$S_z = \left\{ \underline{z} = \underline{\Psi}.\Sigma_i \alpha^i \underline{f}^i \quad \underline{f}^i \in S_{f^i} \right\} \tag{20}$$

It may be demonstrated that the above defined feasibility sets are closed, limited and convex. Moreover, if:

- the supply function $\underline{\Gamma}(\underline{W})$ is continuous in set S_W
- the demand functions $\underline{\Phi}^i(\underline{C}^i)$ are continuous in set $\underline{\Gamma}(S_{\underline{W}})$ (image of S_W through Γ)
- and the following conditions also occur:

a) $\forall i \; (\underline{C}_1^i - \underline{C}_2^i)^T.\left[\underline{\Phi}^i(\underline{C}_1^i) - \underline{\Phi}^i(\underline{C}_2^i)\right] < 0 \qquad \forall \underline{C}_1^i, \underline{C}_2^i$

b) $(\underline{\Gamma}(\Sigma_i \alpha^i \underline{F}_1^i) - \underline{\Gamma}(\Sigma_i \alpha^i \underline{F}_2^i))^T.\left[\Sigma_i \alpha^i \underline{F}_1^i - \Sigma_i \alpha^i \underline{F}_2^i\right] \geq 0 \qquad \forall \underline{F}_1^i, \underline{F}_2^i \quad (\forall i)$

then the equilibrium problem may be proofed (by contradiction) to admit only one solution.

Condition a) occurs for Logit and Probit random utility models, furthermore, condition b) is satisfied if the following condition occurs:

b′) $\underline{y}^T.\mathrm{Jac}\left[\underline{\Gamma}(\underline{W})\right].\underline{y} \geq 0 \qquad \forall \underline{y}, \quad \forall \underline{W} \in S_W$

in turn satisfied if the following has occurred:

b″) $\underline{x}^T.\mathrm{Jac}\left[\underline{\Gamma}(\underline{W})\right].\underline{\Psi}.\underline{y} \geq 0 \qquad \forall \underline{x}$

It is possible to express the hypotheses under which condition b″) may be satisfied and thus equilibrium uniqueness is ensured.

Pre-Hypothesis (0): separability with regard to generalized flows

If this hypothesis is satisfied, the Jacobian matrix $\mathrm{Jac}[\underline{t}(\underline{z})]$ is diagonal. Moreover, if the congestion functions are monotonically increasing, the elements of the Jacobian matrix are non-negative. As a result, $\mathrm{Jac}[\underline{t}(\underline{z})]$ is semi-definite positive.

Hypothesis (I): pre-hypothesis 0 holds and the matrix $\underline{\Psi}$ is also diagonal.

$$(\psi_{lm} = \psi_{ml} = 0 \forall l,m)$$

The matrix $Jac[\underline{t}(\underline{z})].\underline{\Psi}$ is a diagonal matrix with non-negative elements (the elements of $\underline{\Psi}$ are non-negative by definition). It is thus semi-positive definite and condition b) is satisfied.

Hypothesis (II): pre-hypothesis 0 is valid but the matrix $\underline{\Psi}$ is not diagonal

Although the matrix $\underline{\Psi}$, it is nonetheless a diagonal block matrix. As a result, the product matrix $Jac[\underline{t}(\underline{z})].\underline{\Psi}$ is a diagonal block matrix whose positive semidefinite character may be studied for each generic block (Salce 1993). It is not possible in general to prove b´´) for a generic block. Thus, we are unable to demonstrate, in general, uniqueness of the multiuser equilibrium (even if we cannot exclude it, given that b), b´) and b´´) are only sufficient conditions).

Hypothesis (II.i): pre-hypothesis 0 holds, matrix $\underline{\Psi}$ is not diagonal but, being block diagonal, it has all blocks with a maximum size of 2

In this case it may be proved (by applying Gashnigore's theorem, Salce 1993, to each "symmetrised" block of the $Jac[\underline{t}(\underline{z})].\underline{\Psi}$ matrix) that a sufficient condition (not necessary) b) to be hold is that:

$$\begin{cases} \psi_{mm} \geq \psi_{ml} \\ \psi_{ll}\dfrac{\partial t_l}{\partial z_l} \geq (\psi_{ml}\dfrac{\partial t_m}{\partial z_m} + \psi_{lm}\dfrac{\partial t_l}{\partial z_l}) \end{cases}$$

Hypothesis (II.ii): hypothesis (II.i) holds and hence

$$z_m = z_l e \frac{\partial t_l}{\partial z_l} = k\frac{\partial t_m}{\partial z_m} \qquad \forall z_m$$

To satisfy the sufficient condition mentioned in hypothesis II.i, the system of inequalities must occur:

$$\psi_{mm} = \psi_{ml}$$
$$\psi_{ll} = \psi_{lm}$$
$$\psi_{ll} \geq \psi_{mm}/4$$

The above hypotheses may be interpreted and referred to some particular cases of multimodal and multicategory assignment.

I. There is only one transport mode and only one user category (hyp. I holds);

II. There is only one transport mode and several user categories (hyp. I holds);

III. There are several transport modes and one or more user category, but each transport mode has exclusive lanes (hyp. I holds);

IV. There are several transport modes and one or more user category. On every physical network link there are at most two transport modes with non-exclusive lanes. The other modes running on the same link have exclusive lanes. One of the modes exerts a strong influence on overall flow conditions while the other exerts far less influence (hyp. II.ii holds);

Case IV holds when private vehicle and bus transport modes are considered, supposing that, for links on which buses have non-exclusive lanes, the disturbance caused by buses to car traffic is less than 1 ($\psi_{bus,\ auto} < 1$), that the congestion function of buses is equal to that of cars multiplied by a k parameter greater than one, that the standardized flow on the bus link/mode equals the standardized flow on the car link/mode and that it is chiefly due to the contribution of the car link/mode ($\psi_{bus,\,auto} = \psi_{bus,\,bus}$, $\psi_{auto,\,auto} = \psi_{auto,\,bus}$). To give an example, the following parameters could be assumed which respect hypothesis (III.ii) and which thus ensure equilibrium uniqueness: $k = 2.5$, $\psi_{bus,\,auto} = \psi_{bus,\,bus} = 0.2$, $\psi_{auto,\,auto} = \psi_{auto,\,bus} = 1$.

Since the bus service frequency is fixed ("what if" model approach), the contribution of collective transport flows/modes to congestion must thus be seen not as an increase in the number of vehicles which condition the flow, but rather as an increase in the number of users boarding at the stops, thereby causing growing disturbance.

With regard to Florian's multimodal model (1977), we reached the conclusion that, to ensure the existence and uniqueness of the multimodal equilibrium solution, the contribution of buses to congestion cannot be overlooked. Indeed, the reciprocal interactions between the various modes and the dependence among various congestion functions should be appropriately evaluated.

Besides the theoretical properties of equilibrium models for assignment, an appropriate algorithmic approach must clearly be studied which allows solution of the multimodal fixed point problem. In particular, on the theoretical plane, reference has been made several times to the arc/path incidence matrix of the multimodal network (A), used to describe the topology of the transport system. Such an incidence matrix is not easily determined. Also for a small-size network the explicitation of the incidence matrix, that is the explicitation of all the transport alternatives available to the user, may be somewhat time-consuming. In operational practice, it is very often preferred to avoid explicitation of all the alternatives and consequently forgo the time-consuming determination of matrix A. Assignment occurs in these cases by means of network assignment techniques which implicitly allow all possible paths (alternatives) to be considered. A different approach which could theoretically be pursued (it is often used in within-day dynamic assignment models, currently being theoretically developed) is that of using, prior to assignment, some algorithms for the explicitation of all the "relevant" transport alternatives. However, it is above all possible in this way to exercise better and direct control on the congruence of the succession of network links and stretches which comprise the paths, thereby easily avoiding illogical sequences of transport modes.

The number of transport alternatives available to users in a real size system is quite high. The application of exhaustive enumeration algorithms of the alternatives would thus appear to necessitate a very large availability of memory and calculation time.

However, in the following, hypothesy is made of adopt explicit numeration for a small test network. In this case, a heuristic algorithm can be used which solves the equilibrium problem with an MSA-type approach (Method of Successive Averages, Sheffi 1985). Consider the following non-negative function may be adopted to express a necessary equilibrium condition. Indeed:

$$Y(\underline{W}) = (\Sigma_i \alpha^i \underline{\Phi}^i(\underline{W}) - \underline{W})^T . (\Sigma_i \alpha^i \underline{\Phi}^i(\underline{W}) - \underline{W}) \tag{21}$$

where

$$\underline{\Phi}^i(\underline{W}) = \underline{\Phi}^i(\underline{C}_0^i + b^i \underline{\Gamma}(\underline{W})) \tag{22}$$

If $\underline{W}^*$ does correspond to an equilibrium configuration, by definition we obtain:

$$\left.\begin{array}{l} \forall i \quad \underline{\Phi}^i(\underline{W}^*) = \underline{F}^{i^*} \\ \text{with } \underline{W}^* = \Sigma_i \alpha^i \underline{F}^{i^*} \end{array}\right\} \Rightarrow Y(\underline{W}^*) = 0 \tag{23}$$

The fixed point problem may then be solved by searching for all the values for which $Y(\underline{W})=0$ and by verifying which of these corresponds to an equilibrium solution. The theoretical problems of convexity and unimodality of $Y(\underline{W})$ were not investigated, nor were any decreasing directions. Filling in these lacunae, to be achieved in the course of further research, could lead to the formulation of an appropriate problem of mathematical optimization equivalent to the multiuser and multimodal fixed point problem and to a relative solution algorithm.

Two user categories are considered for the test network, both relative to a fixed Home-Work trip purpose, but with parking duration of 3 and 6 hours. Transport demand between the pair (o,d) amounts to 1500 people for each user category.

Figure 2 shows the scheme of the system which we intend to model.

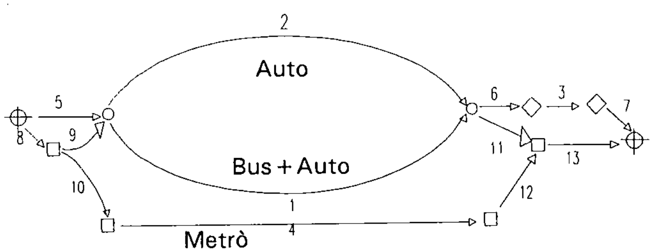

Figure 2 Test network

Link 1 is affected by traffic flows both in car mode and on a public transport line whereas link 2 is affected only by private vehicle flow. Link 1 has the same capacity characteristics than link 2, although it has congestion arising from multimodal flow. Link 3 represents the parking link for private vehicles, on which parking place search costs (for simplification, considered independent of occupancy) and costs of parking fees are loaded. Link 4 represents a direct underground line, on which the transport cost is constant and independent of congestion. Links 5 to 13 are entry or exit links to/from the public or private network, access links to the parking infrastructure, waiting links, pedestrian links, etc. It is assumed that between trip origin and destination, two user categories move and that both may use any transport alternative configured in the network. The set L of links/modes is described below together with the physical network links to which such flows are associated:

1	car mode on link 1	1
2	bus mode on link 1	1
3	car mode on link 2	2
4	car mode on link 3 (parking)	3
5	underground mode on link 4	4
6	car mode on connection link 5	5
7	car mode on access link to car park	6
8	pedestrian mode from car park to destination	7
9	pedestrian mode to collective transport (common) stop	8
10	bus mode (bus line wait)	9
11	underground mode (underground service wait)	10
12	bus mode (alighting from bus line)	11
13	underground mode (exit from underground)	12
14	pedestrian mode (egress from collective transport stop)	13

Let the path set K^i_{od} (n_p=4) be identical for both user categories:

1 Car trip passing along link 1

2 Car trip passing along link 2

3 Bus trip

4 Underground trip

The matrix $\underline{A}^T$, transposed of the link-path incidence matrix, becomes:

$\underline{A}^T$	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	0	0	1	0	1	1	1	0	0	0	0	0	0
2	0	0	1	1	0	1	1	1	0	0	0	0	0	0
3	0	1	0	0	0	0	0	0	1	1	0	1	0	1
4	0	0	0	0	1	0	0	0	1	0	1	0	1	1

The network link costs independent of congestion are determined according to the following indications:

- the unit of measure of the generalized cost is assumed to be the time spent on board of private vehicles;
- the zero-flow travel time on link 1 by car mode is 5 cost units for each user category;
- the zero-flow travel time on link 1 by bus mode is 12 cost units;
- the zero-flow travel time on link 2 by car mode is 5 cost units for each user category;
- the searching time for a free parking space is negligible for each user category;
- the parking cost is 50 cost units for category 1 (3 hour parking duration) and 100 cost units for category 2 (6 hour parking duration);
- the average connection time with the private network (car mode) is 1 cost unit for each user category;
- the access time to parking infrastructures is one cost unit for each user category;
- the egress time from the car park and walking time to the final destination is one cost unit for each user category;
- the average access time to public service stops is 3 cost units;
- the average waiting time for the bus service is 16 cost units for each user category;
- the average waiting time for the underground service is 30 cost units for each user category;
- the monetary cost of the bus service is 13 cost units for each user category;
- the monetary cost of the underground service is 25 cost units for each user category;
- the average egress time from public service stops to destination is 3 cost units for each user category.

Hence:

$\underline{c}_0^{1T}$=[5,12,5,50,5,1,1,1,3,16,30,13,25,3] $\underline{c}_0^{2T}$=[5,12,5,100,5,1,1,1,3,16,30,13,25,3]

Let the delay functions for links/modes 1,2 and 3 be expressed in simplified form as follows:

$$t_1(z_1) = 10(z_1/450)^4 \quad t_2(z_2) = 2.5 \times 10(z_2/450)^4 \quad t_3(z_3) = 10(z_3/450)^4$$

On all the other links/modes the congestion functions have null value (non-congested links). Let us assume that for the determination of standardized flow on each link the following criteria are adopted:

- each user category contributes to the standardized flow of each link/mode through the arithmetic sum of their flows ($\alpha^i=1 \ \ \forall i \in \{1,2\}$)
- $b^i=1 \ \ \forall i \in \{1,2\}$
- the standardized flow on links not subject to congestion may be considered at any rate zero since the cost of such links is independent of the standardized flow
- the matrix $\underline{\Psi}$ assumes the form:

$$\underline{\Psi} = \left(\begin{array}{ccc|c} 1 & 1 & 0 & \\ 2/5 & 2/5 & 0 & \underline{0} \\ 0 & 0 & 1 & \\ \hline & \underline{0} & & \underline{0} \end{array} \right)$$

Lastly, suppose that the vector $\underline{V}_0$ of fixed utilities (modal, socio-economic attributes, etc) is zero. This assumption does not affect the equilibrium properties, vector V being constant. It is adopted merely for the sake of simplicity.

The results obtained by using the MSA-type algorithm are shown in Figure 3. In particular, the flow of the path/mode configuration is presented for each category. The first iteration values are reported, considering costs at null flow. The same values are reported for the last iteration of the algorithm. The equilibrium is verified by comparing path/mode flows of the last iteration ($\underline{F}^i$) with those obtained by the choice model ($\underline{\Phi}^i$).

The performances in terms of time and memory requirements of the application are, of course, of any interest mainly due to the very small size of the test network.

Iteration: 379	Equilibrium	Y(W)=10									
	Category 1 links						Category 2 links				
	f	z	t	c_0	c		f	z	t	c_0	c
1	341	485	13.5	5	18.5	1	5	485	13.5	5	18.5
2	112	485	33.8	12	45.8	2	236	485	33.8	12	45.8
3	452	458	10.8	5	15.8	3	6	458	10.8	5	15.8
4	793	0	0	50	50	4	11	0	0	100	100
5	595	0	0	5	5	5	1253	0	0	5	5
6	793	0	0	1	1	6	11	0	0	1	1
7	793	0	0	1	1	7	11	0	0	1	1
8	793	0	0	1	1	8	11	0	0	1	1
9	707	0	0	3	3	9	1489	0	0	3	3
10	112	0	0	18	18	10	236	0	0	18	18
11	595	0	0	30	30	11	1253	0	0	30	30
12	112	0	0	13	13	12	236	0	0	13	13
13	595	0	0	25	25	13	1253	0	0	25	25
14	707	0	0	3	3	14	1489	0	0	3	3
Paths	$\underline{F}^i$	cost	Probab.		i	Paths	$\underline{F}^i$	cost	Probab.		i
1	341	71.5	0.228332		342	1	5	121.5	0.003245		5
2	452	68.8	0.301199		452	2	6	118.8	0.004281		6
3	112	82.8	0.073901		111	3	236	82.8	0.155897		234
4	595	66	0.396569		595	4	1253	66	0.836577		1255

Figure 3 Results obtained by using the MSA algorithm

Some further considerations on path explicitation

Tests were carried out for path explicitation on the private traffic network of the town of Salerno (southern Italy). The network consists of 60 centroids, 526 nodes and about 1147 links. There was an initial attempt to make all the acyclical paths explicit between every O/D pair on the network. This attempt proved disproportionately time-consuming (related to the medium/small size of the network) because of the large number of possible path alternatives. It was necessary to consider a subset of all acyclical paths. However, this subset must be such as to minimize the probability of not containing any paths which, if an equilibrium algorithm were to be applied with implicit path enumeration, would prove loaded with a non-zero flow at equilibrium. A technique was thus sought which could make explicit a subset of all possible acyclical paths (say APH) on the network so as to contain the "reference set" (say RPH) of all paths with a non-zero probability, at equilibrium, of being chosen. Such a reference set RPH was obtained by a LOGIT-type stochastic equilibrium assignment with implicit path enumeration. Although the RPH set was obtained by a LOGIT path choice model, it would seem reasonable to suppose that the same set would also have been obtained with a PROBIT-type stochastic assignment. For the purposes of the study, it was necessary to define how to search for the path set APH so that it could be successfully compared with the reference set RPH. The criterion adopted for determining the minimum acyclical path set consists in making explicit only those paths with a total-cost (TCOST) not exceeding TR times the minimum path cost (MINCOST), the latter being calculated at zero flow (TCOST≤TR*MINCOST). Consequently, the number of possible acyclical alternatives is re-expressed in more or less acceptable terms depending of the value of the threshold TR. Small values of TR (TR→1) produce small APH but there is almost no chance of "cover" the reference set RPH (RPH⊆APH). However large values of TR produce huge APH which almost surely contains RPH. The following table shows the result of several programs (written in ANSI-C) for the explicitation of acyclical paths with a given threshold value with regard to minimum paths, for the LOGIT stochastic assignment to the same networks and for comparison between the set of paths made explicit and the reference set. A threshold value equal to 2.4 already allows us to make explicit a path set which includes almost the whole comparison set. The 2.6 threshold value ensures the complete "coverage" of the reference set for equilibrium.

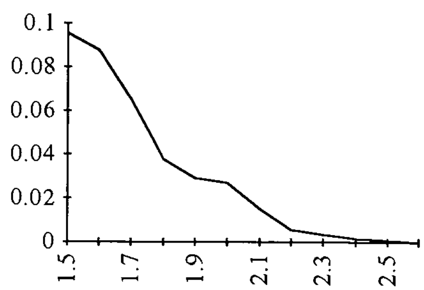

Figure 4 **Coverage of the reference set respect to threshold variation (0 = total coverage)**

Figure 4 shows graphically the degree of coverage of the comparison set depending on the variation of the threshold. By using threshold 2.6 (which ensures total "coverage") a total time of about 4 hours has been employed to explicite all the paths in set APH (PC 486/66Mhz computer with 16 MGbytes of RAM memory). The authors feeling is that these performance could be drastically increased by using more effective explicitation algorithms, which is not the point of this paper. The original contribution consists in trying to proof that path explicitation is possible and that not all acyclic path have to be explicitated.

Similar results have been obtained for other networks of smaller size. The validation of the results on a large network (the city of Napoli, Southern Italy, 200 centroids, 1500 nodes, 3000 links) is currently faced by the authors.

Similar explicitation techniques may be pursued for collective transport networks once the supply system characteristics have been defined along with line operating parameters and some aspects of hyperpath theory relating to user behaviour at stops (adaptive choice by cost comparison or indifferent) (Nguyen and Pallottino 1991).

PERFORMANCE INDICATORS

The performance indicators which may be adopted to make a comparison between the various planning alternatives can be subdivided according to mode and to the relevant objectives. A series of indicators, useful for evaluating planning alternatives, are listed in the following.

Private and pedestrian mode:

- average commercial speed for the whole network weighted on flows;
- average coefficient of saturation on the entire network weighted on flows;
- number of conflict points at intersections weighted on flows;
- road surface area reserved for pedestrians;
- global parking capacity in spaces x hour.

Collective mode:

- number of buses;
- places on transport vehicles supplied per km;
- overall network length;
- number of transfers (average and maximum) weighted on flows;
- speed weighted on flows.

Demand indicators:

- number of motorized trips for each transport mode;
- passenger km per transport mode.

External indicators:

- number of conflict points at intersections weighted on flows and speed;
- levels of equivalent sound pressure in dB(A) (as a function of flow);
- pollutant emissions (as a function of flow and speed);
- fuel consumption of all vehicles (private and public).

EVALUATION OF THE ALTERNATIVES

From the logical procedure described several multimodal networks has been identified. The multimodal assignment and the performance indicators calculation allow to evaluate the "best working" among the network schemes (in the following referred as "planning alternatives"). Given both the large number and the intrinsic diversity of the performance indicators, it is possible to derive conclusions from the indications of preference among the various planning alternatives only by comparative and organic examination. Evaluation of the alternatives is carried out by multiobjective analysis.

The problem of the multiobjective decision may be defined as a problem containing many objectives to satisfy, but which may not in any way be combined. From the mathematical point of view, it may be expressed as a problem of vectorial optimization (Chon 1978):

$$\mathrm{MAX}[\underline{Z}(X_1, ..., X_n)] = \mathrm{MAX}[Z_1(X_1, ..., X_n), ..., Z_p(X_1, ..., X_n)]$$

subject to: $g_i\ (X_1,X_2,...X_n) \le 0\ i= 1,2,...m\ ;\ \ X_j \ge 0\ ;\ \ Z_{j(...,X_i,...)} \in R$

where $\underline{Z}$ is the vector of p objective functions $Z_1,...,Z_p$; g_i are the constraints; X_j are the decisional variables and $[\underline{Z}(X_1),...,\underline{Z}(X_n)]$ is the matrix (p $\times$ n) where each column provides the results for a given alternative on a set of $\underline{Z}$ objectives. However, since there is a non-linear and non-convergent system of functions, it is not possible to find the optimum value analytically. For an alternative search for a sub-optimal value in a set of possible solutions, one of the methods suggested in the literature is the ELECTRE method (Giuliano 1985).

This evaluation methodology allows us to take various objectives into account at the same time and, as a consequence, several judgement criteria, thereby permitting comparison between non-monetizable costs and/or benefits or, more generally, non-quantifiable ones. The ELECTRE methodology has been also used by Di Gangi and Montella (1991) with quite satisfying results; such a procedure consists of the following phases:

- identification of planning alternatives;
- definition of the system of objectives according to social groupings interested in the plan;
- completion of the decision matrix;
- transformation of the matching measurements into utility measurements;
- comparison of alternatives and final choice.

CONCLUSIONS

This paper states a logical procedure for searching an effective configuration of a private and public transport system on wheels in an urban area. Theoretical aspects relative to multimodal assignment were also examined and some solutions were proposed with regard to operative aspects for evaluating the various hypotheses of a multimodal network configuration. The computational aspects were developed on test networks whose theoretical foundations and results are reported. Research is still under way to validate the procedure and the obtained theoretical results by studying its applicability on large networks with various data bases.

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