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MINIMIZATION OF LOGISTIC COSTS WITH GIVEN FREQUENCIES

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Abstract

This paper deals with the problem of minimising the transportation and inventory cost of shipping freight from an origin to several destinations, in the case in which only a given set of shipping frequencies is available. Heuristic solution procedures are presented together with computational results.

INTRODUCTION

One of the most common problems facing a logistic manager is to devise appropriate shipping strategies for some products from where they are available to where they are demanded. Two basic cost factors affect such kinds of decisions: the transportation cost and the inventory cost. Since the former rises and the latter drops as much as more frequent shipments are performed, a reasonable compromise must be found between such two contrasting aims.

Since the pioneering paper by Harris (1913), several researchers have proposed optimization models facing these issues for a wide spectrum of practical situations, with a constant effort towards producing more efficient solution methods for more realistic models. In particular, considerable attention has been devoted to study periodic shipping strategies, in which shipments are repeated at a constant rate when the demand for the products is constant. For this kind of strategy, Blumenfeld et al. (1985) have proposed models for the shipping of a single commodity, which may represent the combination of different products, in several situation, when any real value for the shipment frequency can be used. In particular, they studied the case in which the commodity has to be sent from a single origin to a single destination. For the same situation, Speranza and Ukovich have investigated the more realistic case where only a given finite set of frequencies is available. Different shipment strategies for this case have been analyzed in Speranza and Ukovich (1994), and some optimization models have been proposed. An efficient solution method for the most interesting of such models was presented in Speranza and Ukovich (1991). It uses a branch-and-bound approach for a particular location-like formulation of the problem. Using this method it is experimentally shown in Speranza and Ukovich (1992) that approximate solutions obtained by rounding off the continuous solution provided by the model of Blumenfeld et al. as proposed by Hall (1985), often fail to give the true optimum, and in some cases produce costs that are significantly larger than the minimum.

The scope of this paper is to investigate the problem of selecting shipment frequencies, among a set of given available values, when several products have to be shipped from one single origin to several destinations. In other words, we want to generalize the methods of Speranza and Ukovich (1991 and 1994) to the central depot, multiple destination case. A similar problem, but with no restrictions on the values the shipment frequencies may assume, has been studied, among others, by Burns et al. (1984). They first partition the destinations in sets of equal size (with the possible exception of a single set), then for each set they determine the best continuous shipping frequency, by shrinking the region of the destinations to a single node, and finally they use a routing procedure to determine, given the shipping frequency, the transportation route. A similar problem is also considered by Anily and Federgruen (1990).

Due to the complexity of the multiple destination case, models looking for the exact solution would be impractical. Thus, we propose several heuristic procedures for the solution of the problem and then assess experimentally the solutions they provide.

In particular, experimental evidence is sought to give some answer to questions such as: To which extent is it convenient to consolidate loads? Is it appropriate to use different shipment frequencies? Is it convenient to split a product between different shipments, at different frequencies? Which are the factors according to which different destinations should be included in a same route? As it was discussed in Speranza and Ukovich (1992), the fact of having only a finite given set of available shipment frequencies may produce different answers to these questions with respect to the case of continuous frequencies.

The plan of the paper is as follows. In the next section the single origin, multiple destination problem with given frequencies is formally described. In the second section a basic heuristic approach for it is formulated, based on the model of Speranza and Ukovich (1994) for the single destination case. In the section that follows some variants of the basic procedure are proposed. Computational experience with these heuristics is presented in the final section, giving elements for a practical assessment of the proposed solution methods.

DESCRIPTION OF THE PROBLEM

In this section we formally state the problem we are interested in. Basically, we aim at minimizing the sum of the transportation cost and the inventory cost of shipping products from one origin to several destinations or, equivalently, from several origins to one destination. For reasons of symmetry, we refer to the former situation. A set of products, indexed by i, $i \in I = \{1,2,...|I|\}$, is made available at a common origin A at given constant rates q_i. Product i has unit inventory value h_i and volume w_i. A subset $I_m \subseteq I$ of products is required at destination B_m, $m \in M = \{1,2,...,|M|\}$ at given constant rates q_{mi} such that $\sum_{m \in M} q_{mi} = q_i$. Shipments from A to each destination B_m may only occur at given frequencies f_j, $j \in J = \{1,2,..,|J|\}$. We denote by $t_j = 1/f_j$ the period associated to frequency f_j, $\forall j$, and by H the time horizon obtained as the minimum common multiplier of the periods t_j.

The quantity of a product required by a destination may be split between different shipments, possibly at different frequencies. We assume that the distance d_m between the origin A and the destination B_m, $\forall m$, and the distance $d_{mm'}$ between destinations B_m and $B_{m'}$, $\forall m,m'$, are known. Trucks traveling at frequency f_j have a known capacity r_j. The cost per unit distance of a truck travelling at frequency f_j is denoted by c_j and includes both the fixed component and the operational component of the transportation cost. Once a transportation route is defined for a specific truck, the corresponding transportation cost is obtained by multiplying the cost c_j by the length of the route. The contribution of the truck to the transportation cost per unit time is obtained by multiplying the latter quantity by f_j. Likewise, the inventory cost per unit time of a product i which is partly shipped at frequency f_j is obtained by multiplying its unit inventory cost h_i by its mean waiting time (that is, $1/f_j$) and by the quantity (per unit time) which is shipped at that frequency.

For that situation a shipping strategy is sought, which satisfies the demands, complies with the truck capacities and minimizes the overall transportation and inventory costs. More specifically, it must be decided which shipments to make, where a shipment is characterized by:

- how many trucks are used;
- which products are loaded, and in which quantities;
- at which frequency it is repeated;
- which are the destinations it visits;
- which is the route followed by the trucks.

The constraints of the problem are:

- the cumulative volume of the products attributed to a shipment cannot be larger than the global volume available by the used trucks;
- the demand of each product at each destination must be satisfied.

THE BASIC HEURISTIC PROCEDURE

In this section we present the basic heuristic procedure for the solution of the problem described in the previous section. We refer to this procedure, based upon the decomposition of the network, as *Dec*.

The main idea of the procedure is to start from a link-by-link solution of the problem and then to perform a local search looking for improvement through consolidation. In this phase destinations are aggregated into routes; in general, we admit that each destination may belong to different routes.

In the first phase each destination is considered independently and a one origin-one destination problem is solved using the algorithm of Speranza and Ukovich (1991). This phase produces a set of shipments whose routes are $A - B_m - A$, for all m.

On the basis of the solution produced by the first phase, we identify subsets of destinations which are served by shipments having the same frequency and consider the products shipped from these destinations at that frequency. If a product is only partly shipped at that frequency, only this part is considered. The second phase of the procedure considers each of such subsets and tries to reduce the associated cost through possibly modifying the frequency and aggregating destinations into routes.

More precisely, the possibility of shipping all the considered products at a different, but unique, frequency is evaluated for each available frequency and the most convenient one is chosen. While the inventory cost is influenced by the transportation frequency, the transportation cost depends on the number of needed trucks and therefore on the route of each truck. The transportation cost depends also on the frequency, since, when the frequency is modified, the volume of the products shipped every time is modified as well. If the frequency is increased, then the quantity is reduced, while the opposite happens if the frequency is decreased. As a consequence, we need an estimate of the minimum transportation cost for each possible frequency. This is obtained by means of a heuristic procedure which aggregates destinations into routes. Each destination may be visited by several trucks.

The second phase produces, for each frequency, a set of destinations to which some products are partly shipped at that frequency. In the third phase each set is considered separately and again a heuristic procedure is used to determine an estimate of the minimum transportation cost to ship the products to all the destinations of the given set.

Now we give a more formal description of the solution procedure.

Phase 1: Single link problems

In the first phase, each destination B_m is considered independently. Thus, we have $|M|$ independent problems, each concerning one single link. This problem can be formulated as a mixed integer linear programming problem. It has been studied in Speranza and Ukovich (1994), and a solution method for it has been proposed and assessed in [7].

For each given $m \in M$, we have to decide:

- how many trucks to use for each admissible frequency f_j, $j \in J$; the corresponding decision variable is denoted by y_{mj};
- the fraction of product $i \in I$ required by B_m that has to be shipped at frequency f_j, $j \in J$; the corresponding variable is denoted by x_{mij}. In this way $x_{mij}q_{mi}$ is the quantity of product i shipped to B_m at frequency f_j in unit time.

These are the decisions to be taken. Obviously, a transportation plan is fully defined when the routings of the trucks are given. As we are dealing with a one-to-one connection, the shortest path from A to B_m can be found independently for each m. We assume here it is known and we denote its length by d_m, as already mentioned. We denote by $c_{mj} = c_j d_m$ the transportation cost from A to B_m.

For each destination B_m, the following mixed integer linear programming problem must be solved.

Problem P_m

$$\min \sum_{i \in I_m} \sum_{i \in J} h_i q_{mi} x_{mij} H t_j + \sum_{j \in J} c_{mj} y_{mj} H f_j \quad (1)$$

$$\sum_{j \in J} x_{mij} = 1 \quad i \in I_m \quad (2)$$

$$t_j \sum_{j \in I_m} w_i q_{mi} x_{mij} \leq r_j y_{mj} \quad j \in J \quad (3)$$

$$x_{mij} \geq 0 \quad i \in I_m j \in J \quad (4)$$

$$y_{mj} \text{ integer } j \in J. \quad (5)$$

In this formulation, the first term in (1) expresses the inventory cost and the second term the transportation cost over H. Then (2) imposes that the quantity q_{mi} required in each time instant for product i is actually shipped, and (3) represents the volume capacity constraint. Phase 1 consists in solving Problem P_m for each m. Therefore, at the end of the first phase, the optimal values of the variables x_{mij} and y_{mj}, $\forall m$, are known. The total inventory and transportation cost per unit time is obtained by summing up, on all the destinations, the optimal values of the objective function [1].

Phase 2: Aggregation of destinations

In the second phase the subsets of destinations which are served at the same frequency, according to the results obtained by Phase 1, are considered separately. For any given $j \in J$, we consider the set M_j of destinations which are served at frequency f_j:

$$M_j = \{B_m | y_{mj} > 0\}. \quad (6)$$

The quantity shipped to destination B_m at frequency f_j is $x_{mij} q_{mi} t_j$ for each product i. The total quantity which must be shipped at frequency f_j is $Q_j = \sum_{m \in M_j} \sum_{i \in I_m} x_{mij} q_{mi} t_j$. In this phase we evaluate if it is convenient to ship the whole quantity Q_j at a different frequency and, if so, we select the most convenient frequency. The procedure of the second phase can be summarized as follows.

For each frequency f_j

1. Build the set M_j.

   For each frequency f_k do the following steps.

   (a) For each destination $B_m \in M_j$

      i. Calculate the volume W_{mk} of the products to be shipped every time to destination B_m in the case frequency f_k is adopted; note that the volume W_{mk} must be proportional to $t_k = 1 / f_k$ in order to ship the whole quantity $q_{mi} x_{mij} H$ in the time horizon H. The volume W_{mk} is:

$$W_{mk} = t_k \sum_{i \in I_m} w_i q_{mi} x_{mij}. \quad (7)$$

      ii. Split the volume W_{mk} in two components.

The component $W'_{mk} = \lfloor W_{mk} / r_k \rfloor r_k$ represents the volume of the products which completely fill a number $V_{mk} = \lfloor W_{mk} / r_k \rfloor$ of trucks. Such trucks serve the destination B_m directly from the origin.

The second component $W''_{mk} = W_{mk} - W'_{mk}$, which is smaller than the capacity r_k, represents the volume of a less-than-full load. This volume, together with partial loads of other destinations, will be loaded on a truck which will serve several destinations.

(b) Apply the saving algorithm by Clarke and Wright (1964) to the nodes of M_j, where each node B_m requires the volume W''_{mk}. The algorithm produces a set of routings, indexed by s, $s \in S_k$, with

$$|S_k| = \left\lceil \frac{\sum_{B_m \in M_j} W''_{mk}}{r_k} \right\rceil$$

We denote by d_{ks} the length of routing $s \in S_k$. The corresponding transportation cost is $c_k d_{ks}$.

(c) Calculate the total cost per unit time needed to ship the products of the destinations of M_j at frequency f_k

$$C_{jk} = \sum_{B_m \in M_j} \sum_{i \in I_m} h_i q_{mi} x_{mij} / f_k + \sum_{B_m \in M_j} c_{mk} V_{mk} + c_k \sum_{s \in S_k} d_{ks}.$$

The first component of C_{jk} represents the inventory cost; the second component represents the transportation cost of the full load trucks which serve destinations with a direct shipment; the third component represents the transportation cost of the remaining trucks.

2. Select the value of k which minimizes the total cost C_{jk}. $C_j = \min_k C_{jk}$ is the cost of shipping the quantity Q_j at the most convenient frequency. If the frequency f_j is obtained, this means that no frequency is less expensive than f_j. In this case the related inventory cost does not change with respect to the first phase, while the transportation cost may drop due to the application of the routing algorithm.

The total cost obtained by Phase 2 is $\Sigma_j C_j$ and is always not larger than the cost obtained in the Phase 1.

Phase 3: Further consolidation

As a result of the second phase, the frequency and the number of the shipments may have changed with respect to the shipments obtained in the first phase. As a consequence, the sets M_j, $\forall j$, of the destinations to which products are shipped at frequency f_j may have changed too. Suppose for example that only two frequencies $f_{k'}$ and $f_{k''}$ are available and that in the second phase the shipments of $M_{k'}$ have been attributed to, say, the frequency $f_{k''}$, and that for the shipments of $M_{k''}$ the frequency did not change. Then, at the end of the second phase, the new $M_{k'}$ is empty, whereas the new $M_{k''}$ includes also the previous $M_{k'}$. It follows that a saving may be obtained in the transportation cost of all the destinations served at the same frequency are considered all together.

In the third phase, we consider the sets of destinations to which fractions of some products are shipped at the same frequency. Then the algorithm of Clarke and Wright (1964) is applied separately to each such a set of destinations to determine an estimate of the minimum transportation cost.

In the third phase the total inventory cost does not change, while a reduction in the transportation cost may be obtained.

VARIANTS OF THE BASIC HEURISTIC PROCEDURE

In this section we describe three variants of the procedure presented in the previous section.

Phasing

The rationale for this variant is that products shipped at different frequencies might, in some cases, share the same truck. For example, suppose that one truck travelling every day is half empty and that one truck traveling every two days is half empty too. In this case, it is obviously convenient to load every two days all the products of the two trucks on a single truck. In general, the idea is to "phase" all the frequencies at time 0, that is at time 0 the shipments of all frequencies are made. The following shipments are made according to their frequency. It follows that at each time instant t a set of shipments at different frequencies are made and we can determine the total volume V_t of the products which must be shipped at t.

In this variant, which we refer to as *Dec-P*, the first two phases of the basic heuristic *Dec* are applied. Then the frequencies are phased and for each time instant t the number of trucks needed to ship the volume V_t is calculated. While, in the solution produced by the basic heuristic, products shipped at different frequencies never share a truck, this can happen in the solution produced by this variant.

Preliminary zoning

The second variant we present is based upon an idea used in several heuristics which appeared in the literature for similar problems: first cluster the destinations, and then work separately on each cluster. The rationale of this idea is not only to decompose the initial problem in order to reduce the computational time, but also to deal with more homogeneous sets of destinations, from the geographical point of view. We refer to this variant as *Dec-Z*.

This variant is partially inspired by the heuristic proposed in Burns et al (1984), where a formula was given for determining the "optimal" size of the clusters. We used their formula to determine the number of destinations per cluster. Then, given this number, say n^*, the assignment of destinations to clusters is made by means of a procedure based upon the saving algorithm of Clarke and Wright (1964). The saving algorithm creates routings starting with "small" routings and successively joining some of them on the basis of the saving. When the number of destinations of a routing created by the saving algorithm reaches n^*, the routing is "freezed", and the saving algorithm is applied to the remaining destinations. This procedure is repeated until the number of remaining destinations is less or equal to n^*. Each routing determines a cluster of destinations to which the procedure *Dec* is applied.

Phasing and preliminary zoning

The third variant, referred to as *Dec-PZ*, is obtained by applying the procedure *Dec-P* to each of the clusters determined as in the *Dec-Z*.

Another set of heuristics

It can be convenient, in some cases, to distinguish, among the destinations M_j, those served by trucks with full loads and those served by trucks with less-than-full loads. The rationale of this variation is that the latter ones provide larger consolidation opportunities.

For each $j \in J$, let R_j denote the subset of the destinations M_j which are served by a full load shipment with frequency f_j, and let S_j denote the subset of the destinations M_j which are served by a less-than-full load shipment with frequency f_j. If a destination is served at frequency f_j both by full load trucks and a partial load truck, it belongs to both R_j with the full loads and S_j with the partial load.

Four new heuristics are obtained by applying the procedures *Dec*, *Dec-P*, *Dec-Z* and *Dec-PZ* separately to subsets R_j and S_j in place of M_j. We refer to the new heuristics as *Dec+*, *Dec-P+*, *Dec-Z+* and *Dec-PZ+*.

COMPUTATIONAL EXPERIENCE

The procedures proposed in the previous sections have been implemented in FORTRAN on a personal computer with an Intel 80386 processor and tested on a large set of randomly generated problem instances.

Since no other solution procedure is known in the literature for the problem with discrete frequencies, we modified the procedure proposed by Burns et al. (1984) and compared the modified version with our procedures. The original method by Burns et al. allows any continuous value for the shipping frequency. In the modified version of the method, we select the smallest feasible frequency not smaller than the frequency provided by the original version, in order to obtain a feasible frequency for the discrete case. If no such frequency exists among the feasible frequencies, the largest feasible frequency is selected. Moreover, after the shipping frequencies have been determined, the routings of the trucks, and thus the transportation cost, are obtained by means of the saving algorithm of Clarke and Wright (1964). We refer to this modified version as *B-mod*.

We also report results on the comparison of the best solution provided by the proposed procedures, the solution provided by the procedure *B-mod* and the original procedure of Bums et al. in which the routings are obtained through the saving algorithm. We refer to the latter as *B-orig*.

Evaluation of the proposed procedures

Two basic different sets of problem instances have been generated, corresponding to different practical situations. In the first situation the destinations are distributed around the origin, while in the second one the destinations are distributed in an area at some distance from the origin. Formally, the destinations are uniformly generated in a square of given edge. In the first situation, the origin is located in the center of the square, while in the second one the origin is located at a distance from the center of the square equal to three times the edge of the square. For each basic situation (internal and external origin) 12 specific situations have been tested, corresponding to different size of the square (length of the edge equal to 10, 100, 300) and different number of destinations (5, 10, 15, 20).

For each situation 5 problem instances have been generated with the following data:

- number of products: $|I| = 5$;
- capacity (volume) of a truck: $r_j = 1$, $\forall j$ (the capacity is normalized to 1);

- transportation cost per km: $c_j = 3, \forall j$;
- unit volume of each product: w_i randomly selected from 10^{-3} and 10^{-2};
- unit inventory value of each product: h_i randomly selected from 1 to 5;
- number of products required at each destination: randomly selected from 3 to 5;
- quantity of each product required at each destination per unit time: q_{mi} randomly selected from 80 to 100;
- available frequencies: 1, 1/2, 1/5, 1/10.

In all cases, random selections have been performed according with a uniform distribution.

Each of the instances has been solved using the procedures described previously and the discretized version of the procedure by Burns et al. Moreover, in the reported results we included the link-by link method, referred to as *L-by-L*, that is the solution obtained through the best direct shipping origin-destination, obtained in the first phase of the basic procedure *Dec*. The reason for including this solution is that it is interesting to see when and how much it is possible to gain through routing strategies with respect to the direct strategy.

The solution of the *L-by-L* represents the first step of each of the proposed procedures. It requires a computational time of few seconds and always below 30 seconds. The additional time required by each procedure and by the procedure *B-mod* is again of the order of seconds, with an average of about 30 seconds.

For each of the tested problem instances the minimum experienced cost, among all the procedures, has been identified and the percent increase provided by the various methods has been calculated. The average percent increase (on the 5 homogeneous instances) produced by each method for each of the 24 situations is shown in Tables 1 and 2, for the situations with internal and external origin, respectively. The figures in parentheses give the number of instances for which each method has produced the minimum cost. In some cases, the same minimum cost has been obtained by more than one procedure.

Table 1 Average percent increases on 5 instances with internal origin

dest.	edge	Dec	Dec-P	Dec-Z	Dec-PZ	Dec+	Dec-P+	Dec-Z+	Dec-PZ+	L-by-L	B-mod
5	30	1.50 (0)	1.66 (0)	2.12 (0)	1.66 (0)	0.30 (2)	0.26 (2)	0.30 (2)	0.26 (2)	2.12 (0)	0.70 (3)
10	30	1.18 (0)	0.84 (0)	3.27 (0)	1.32 (0)	0.44 (0)	0.21 (3)	0.89 (2)	0.98 (1)	3.51 (0)	1.85 (0)
15	30	2.14 (0)	2.33 (0)	2.82 (0)	2.39 (0)	0.72 (0)	0.55 (2)	0.85 (0)	0.87 (2)	2.85 (0)	1.00 (1)
20	30	1.58 (0)	1.81 (0)	2.12 (0)	1.30 (0)	0.65 (0)	0.55 (0)	0.34 (2)	0.25 (1)	2.12 (0)	0.25 (2)
5	100	1.72 (0)	1.84 (0)	1.84 (0)	1.19 (0)	0.86 (1)	1.10 (0)	0.99 (1)	1.09 (0)	1.84 (0)	0.18 (3)
10	100	4.56 (0)	3.06 (0)	8.41 (0)	5.01 (0)	0.98 (1)	0.63 (3)	3.98 (0)	3.59 (2)	8.41 (0)	6.16 (0)
15	100	1.68 (1)	2.11 (0)	3.28 (0)	2.96 (0)	2.11 (0)	2.19 (0)	1.51 (2)	1.39 (0)	3.28 (0)	1.16 (2)
20	100	4.07 (0)	4.07 (0)	4.07 (0)	3.40 (0)	0.55 (2)	0.66 (0)	2.40 (0)	2.65 (0)	4.07 (0)	1.85 (3)
5	300	1.22 (0)	2.04 (1)	2.89 (0)	2.38 (1)	0.68 (2)	1.35 (0)	2.31 (0)	1.88 (0)	2.89 (0)	3.94 (1)
10	300	7.30 (0)	3.70 (0)	12.44 (0)	8.56 (0)	6.32 (0)	2.92 (2)	8.80 (1)	8.00 (0)	12.44 (0)	14.29 (0)
15	300	1.71 (0)	1.71 (0)	1.71 (0)	1.62 (0)	1.01 (2)	1.23 (0)	0.44 (1)	0.92 (0)	1.71 (0)	1.63 (2)
20	300	1.66 (1)	2.13 (0)	2.71 (0)	2.57 (0)	0.73 (3)	1.46 (0)	1.59 (0)	1.55 (1)	2.71 (0)	3.25 (0)

Table 2 Average percent increases on 5 instances with internal origin

dest.	edge	Dec	Dec-P	Dec-Z	Dec-PZ	Dec+	Dec-P+	Dec-Z+	Dec-PZ+	L-by-L	B-mod
5	30	2.14	2.14	2.14	2.14	0.05	0.96	0.08	0.96	2.14	1.66
		(0)	(0)	(0)	(0)	(3)	(0)	(1)	(0)	(0)	(2)
10	30	2.71	2.71	2.71	2.71	0.37	0.91	0.81	0.76	2.71	4.17
		(0)	(0)	(0)	(0)	(3)	(2)	(1)	(0)	(0)	(1)
15	30	3.12	3.12	3.12	3.12	0.47	0.72	1.48	1.46	3.12	1.81
		(0)	(0)	(0)	(0)	(4)	(1)	(0)	(0)	(0)	(1)
20	30	3.53	3.48	3.53	3.46	0.11	0.14	0.70	0.77	3.53	0.96
		(0)	(0)	(0)	(0)	(3)	(1)	(0)	(0)	(0)	(2)
5	100	0.99	0.95	1.16	1.01	0.08	0.09	0.71	0.33	1.36	6.55
		(0)	(1)	(0)	(1)	(2)	(3)	(1)	(2)	(0)	(0)
10	100	2.40	2.40	2.40	2.31	0.15	0.52	1.34	1.46	2.40	3.98
		(0)	(0)	(0)	(0)	(3)	(1)	(1)	(0)	(0)	(1)
15	100	2.34	2.46	2.55	2.42	0.00	0.52	1.27	1.30	2.55	5.92
		(0)	(0)	(0)	(0)	(5)	(0)	(0)	(0)	(0)	(0)
20	100	2.59	2.59	2.59	2.40	0.00	0.42	1.30	1.32	2.59	5.66
		(0)	(0)	(0)	(0)	(5)	(2)	(0)	(0)	(0)	(0)
5	300	0.88	0.62	1.10	1.10	0.27	0.00	0.71	0.70	1.14	9.48
		(1)	(2)	(1)	(1)	(3)	(5)	(1)	(1)	(1)	(0)
10	300	1.75	1.68	1.75	1.75	0.15	0.12	0.91	0.70	1.75	9.60
		(0)	(0)	(0)	(0)	(3)	(3)	(1)	(0)	(0)	(0)
15	300	1.87	1.87	1.87	1.81	0.13	0.16	1.36	1.28	1.87	7.41
		(0)	(0)	(0)	(0)	(3)	(2)	(0)	(0)	(0)	(0)
20	300	1.80	1.80	1.80	1.80	0.00	0.19	0.77	0.67	1.80	7.88
		(0)	(0)	(0)	(0)	(5)	(2)	(0)	(0)	(0)	(0)

The first two columns of Tables 1 and 2 give the number of destinations and the length of the edge of the square. The other columns give the performance of the different procedures with respect to the best one.

The following conclusions can be drawn from these results.

- The procedure *Dec+*, which deals separately with *R* and *s*, with neither zoning nor phasing, shows the best global performance, both in the cases with internal and with external origin. The procedure gives the best results in 15 out of 24 situations; in the remaining 9 situations, the performance is never much worse than the best available. In a single case only a cost increase of over 3% is experienced. More specifically, the procedure *Dec+* is always the best with an external origin, except in just one case; in this case the average cost increase is of less than 0.3% with respect to the best average performance;
- each procedure of the second set, ie. *Dec+*, *Dec-P+*, *Dec-Z+* and *Dec-PZ+*, shows a better behavior than its corresponding procedure in the first set (eg *Dec-P+* shows a better behaviour than *Dec-P*); in other words, the results obtained dealing separately with the subsets *R* and *s* of *m*, are almost always better than those obtained considering the whole set *m*;
- in the case with an external origin, the procedure *Dec* is seldom significantly better than the Link-by-Link method;
- phasing is effective in 6 situations out of 12 with separate *R* and *s* for the cases with internal origin; in general, however, phasing does not affect the performance in a very relevant way;
- introducing the preliminary zoning is seldom useful, and sometimes gives rather poor results;
- as it was expected, each of the proposed procedures gives results that are never worse than those obtained by the Link-by-Link method. The improvement obtainable by the best version of the new procedure is always larger than 1%, and is usually much larger;
- in the case with internal origin, only in one situation the modified procedure by Burns et a.l shows a better performance than the best of our procedures; in the case with external origin it is always worse than the best of our procedures and of most of the specific procedures.

Comparison with the original method by Burns et al. (1984)

We have also compared, on all the above tested problem instances, the best solution provided by the procedures described in earlier sections with the solutions of the method of Burns et al. (1984)

both in the discretized version *B-mod* and in the original version. The rationale of this comparison is that it may be interesting to evaluate the impact on the total cost of limiting the feasible values of the frequencies. If this comparison would be carried out on the optimal solutions, the relaxed problem, with continuous values of the frequencies, would provide a lower bound for the problem with given set of feasible frequencies. Our results show that this dominance relation does not hold when heuristic procedures are used for both problems.

We present the results of the comparison in Tables 3 and 4, for the situations with internal and external origin, respectively. In the case with internal origin, the original method by Burns et al. (1984) is always better than the other two. However, the error decreases when the edge of the square, and thus the transportation cost, increases. Moreover, the best of our procedure is almost always better than the discretized version of Burns et al. (1984). The results for the case with external origin are substantially different from those obtained for the case with internal origin. In this case, the best of our heuristics is always better than the others, with only one exception. Moreover, the error produced by the other heuristics, and in particular, by the original method by Burns et al. (1984) is sometimes very large.

Table 3 Average percent increase on 5 instances with internal origin

dest.	edge	Best	B-mod	B-orig
5	30	192.78	194.08	0
10	30	135.76	139.62	0
15	30	182.18	183.31	0
20	30	199.00	199.02	0
5	100	26.84	25.91	0
10	100	52.68	25.91	0
15	100	30.88	30.55	0
20	100	59.28	64.78	0
5	300	14.19	17.97	0
10	300	3.65	15.04	0
15	300	20.94	22.36	0
20	300	23.89	26.98	0

Table 4 Average percent increase on 5 instances with external origin

dest.	edge	Best	B-mod	B-orig
5	30	0	1.97	53.10
10	30	0	2.33	7.02
15	30	28.25	30.00	0
20	30	0	0.89	12.68
5	100	0	6.58	71.00
10	100	0	3.94	19.62
15	100	0	5.96	29.93
20	100	0	5.62	41.11
5	300	0	1.21	42.94
10	300	0	9.39	7.13
15	300	0	7.32	21.43
20	300	0	7.89	10.07

The results shown by Table 4 are more surprising than those shown by Table 3. The reason for the poor performance of the method of Burns et al. in the case with external origin is in the way the heuristic works. In fact, the heuristic identifies a single shipping frequency, which is in general lower than 1. In all cases, each truck visits all destinations. This is the basic reason why, in the cases where the transportation cost is large, the performance of the original method by Burns et al. is rather poor. In the modified version, each truck visits less frequently only a subset of destinations. Therefore the solution obtained by the modified version shows, with respect to the original one, a larger inventory cost but a smaller transportation cost. When the transportation cost is large, such as in the case with external origin, the modified version shows a better performance than the original one, even if it complies with more restrictive assumptions about frequencies.

CONCLUSIONS

The problem of selecting the minimum cost frequencies for shipping products from an origin to a set of destinations is a very complex problem. The practical relevance of the problem makes very important the design of algorithms able to give good solutions.

It turns out, from the comparison of the different procedures, that disaggregating destinations on a geographical basis is not convenient. Conversely, it is convenient to disaggregate destinations according to the fact that they are served by full load trucks or by less-than-full load trucks. In this way relevant savings may be achieved. Finally, phasing may have some impact especially when the origin is internal, but of limited relevance.

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