TOPIC 9
ADVANCED TRAVELLER
INFORMATION SYSTEMS

ON EFFECTS OF FAMILIARITY ON ROUTE CHOICE BEHAVIOR IN THE PRESENCE OF INFORMATION

TSIPPY LOTAN
Free University of Brussels, ULB, CP 210/01
Boulevard du Triomphe, B-1050 Brussels, BELGIUM

Abstract

Route choice behaviour of familiar and unfamiliar drivers is explored and compared. The results obtained indicate larger homogeneity among the unfamiliar drivers in terms of their switching and diverting behaviour, while familiar drivers demonstrate larger taste and preferences variations. Two choice models are implemented and compared: the approximate reasoning model, and a multinomial logit model.

INTRODUCTION

Modeling route choice behavior is not an easy task. It gets even more complicated when traffic information is available to drivers. Not only all the attributes related to the information (such as type, context, spatial and temporal relevance) have to be considered, but other factors, such as information processing capabilities, perceived reliability, and information integration have to be taken into account as well. Furthermore, existence of on-line traffic information forces decisions to be made in real-time, often under time pressure, and while the driver is primarily occupied with the driving task.

Many factors affect route choice behavior. Bovy and Stern (1990) categorized them into four major groups:

- Route attributes which relate to road characteristics, traffic conditions, and environmental considerations.
- Personal characteristics of the driver.
- Trip characteristics such as trip purpose, mode, etc.
- Other circumstances such as weather conditions, time of day, and traffic information.

In this paper we focus on *familiarity* and its effects on route choice behavior. Familiarity is a compound factor, and can not be easily categorized into one of the above four groups. Although naturally belonging to the category of personal characteristics, familiarity is not independent of route and trip characteristics. Drivers can be familiar with certain routes during certain times, for specific purposes, and under given circumstances, and at the same time be unfamiliar with the same routes during different times, for other trip purposes, and different circumstances. Thus familiarity encompasses interactions of personal characteristics with route and trip attributes.

Familiarity has many viewpoint in the context of route choice behavior in the presence of information, and can be roughly divided into the following two groups:

- *familiarity with the network*
- *familiarity with the information system*

Familiarity with the network, as illustrated in Figure 1, pertains on the static level to knowledge of the network structure and infrastructure and includes knowledge of routes in the network, type of roads, and available facilities. More dynamic familiarity with the network includes knowledge of traffic conditions and network performance (eg traffic composition and density, traffic flows, and travel speeds). And, of course, the utmost level of familiarity is achieved by actual experience, which combines static and dynamic knowledge. Other dimensions of network familiarity can be characterized according to spatial, temporal, trip-type, and other external factors (such as weather, time-of-day, etc). Familiarity with the information system pertains to knowledge of its potential features and operating routines, to previous experience and interaction with the system, and more conceptually, to its perceived reliability and to the interpretation of the information received.

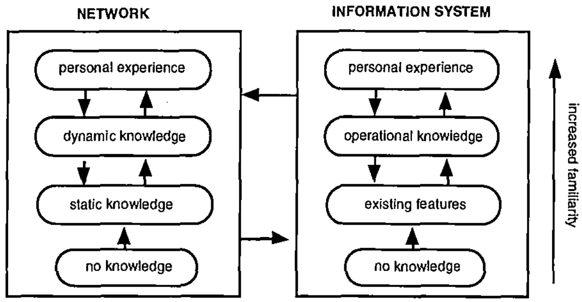

Figure 1 Levels of familiarity

There are important interactions between the two types of familiarities, influenced by the specific type of information provided. Information systems can, for example, change the knowledge of the available routes in the network by informing drivers about route blockages, or alternatively by suggesting alternatives that were not previously considered. Similarly, information systems might change the dynamic knowledge of the network by providing updated information on traffic volumes, current delays, and expected bottlenecks.

The importance of familiarity on modeling route choice behavior in the presence of information has been lately recognized by several researchers. Adler and McNally (1994) investigated effects of familiarity on route choice behavior using the FASTCARS driving simulator. Their results indicate a significant influence of familiarity on drivers behavior and performance. Bonsall (1995) investigated the role of variable message signs on route choice decisions using the VLADIMIR driving simulator, and his results suggest that increased familiarity brings greater rationality and consistency to choices.

In this paper effects of network familiarity on route choice behavior are explored and investigated using a driver simulator. The level of network familiarity is controlled by the design of the experiment, and two extremes participate in the study: very familiar subjects with extensive personal experience with the network, and completely unfamiliar subjects who have never been to the area of the network.

THE EXPERIMENT

A driving simulator was used to compare route choice behavior of familiar and unfamiliar drivers. The familiarity aspect that was controlled for was knowledge of the alternatives in the choice set, and actual experience with the use of the network. The choice scenario that was simulated referred to commuting behavior from south-west Newton Massachusetts to M.I.T. in Cambridge Massachusetts, USA. The simplified network appears in Figure 2 and includes three major alternatives connecting the origin, marked by "HOME", with M.I.T., the destination node: Beacon Street, Commonwealth Avenue, and The Massachusetts TurnPike.

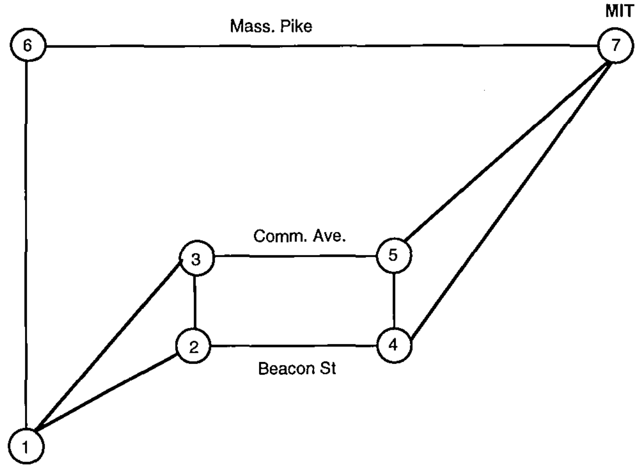

Figure 2 The Newton network

The sample included familiar and unfamiliar drivers. The familiar sample population consisted of 10 subjects who were very familiar with the network, and were actually living in the close neighborhood of the "HOME" node and were commuting regularly to M.I.T. During a preliminary interview, the choice-set of each subject was identified, and the alternatives that the subject actually knew and used were associated with the given alternatives in the simulated network. In addition, subjects were asked to give travel times estimates on the three alternatives in the choice-set. The unfamiliar sample population consisted of 15 subjects taken from the Brussels metropolitan area in Belgium (all University associated), that have never been to Newton and Cambridge, Massachusetts. During the preliminary interview a map of the commuting corridor was shown to the subjects, and the three alternatives were identified on the map. The subjects were asked to provide travel time estimates on the three alternatives. Thus, the unfamiliar drivers were given static information about the network, however they lacked dynamic knowledge and personal experience. Both the familiar and unfamiliar drivers did not have any prior experience with the information system, and were given detailed information about its features and operating instructions. Each subject performed a total of 22 trips from the "HOME" node to M.I.T. The first two trips were considered to be practice trips.

A driver simulator (Koutsopoulos et al. 1994) was used for data collection. The driving environment (as illustrated in Figure 3) is simulated by two main windows: the driving/observation window (on the left of the screen) and the information window (on its right). The driving/observation window corresponds to the driving task itself and is dynamic in nature; at each intersection the driver has to make a choice on the next link to follow. The information window communicates information on traffic conditions, which at the current experiment included link congestion level, and accidents indication. Traffic conditions on links were indicated by colors, where each color corresponded to one of the following labels: bumper-to-bumper, heavy traffic, usual traffic, light traffic, or free flow. Accidents were indicated on the network map according to their occurrence. Traffic scenarios for the 20 trips varied randomly according congestion levels, accidents occurrences, and information availability.

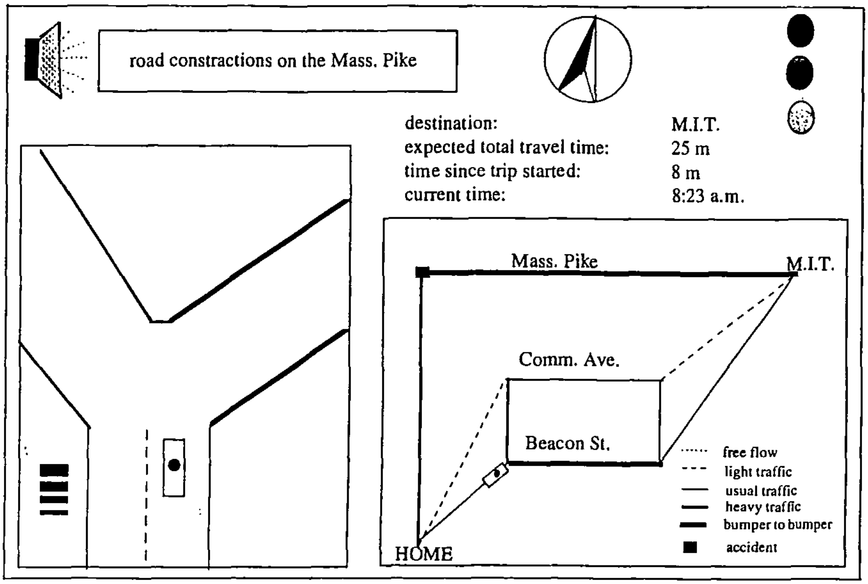

Figure 3 Driving simulator

OBSERVED BEHAVIOR OF FAMILIAR AND UNFAMILIAR DRIVERS

Decisions made at the origin

In each trip drivers had to make a choice decision at the "HOME" node on which alternative out of the available three to choose (Beacon Street, Commonwealth Avenue, or The Massachusetts TurnPike). The distributions of choices made at the origin among the three alternatives for the two populations are presented in Figure 4. It is clear that the unfamiliar sample population has a more uniform distribution among the three alternatives while the familiar sample population shows a clear preference for the Beacon street alternative over the Mass. Pike, supporting the existence of a "favorite" or "usual" path among the experienced users. This phenomenon can be seen even more distinctly in Figure 5, which illustrates a breakdown of choices by subjects. It can be seen that whereas each of the unfamiliar drivers experimented with each of the three alternatives, the familiar drivers show a clear tendency to have a route that is more preferable on the others, and four of them have not chosen the Mass. Pike. alternative even once. This observation holds true even without looking at the specific traffic conditions, which were generally worse than usual, thus having the potential to cause even the experienced drivers to change their usual behavioral pattern.

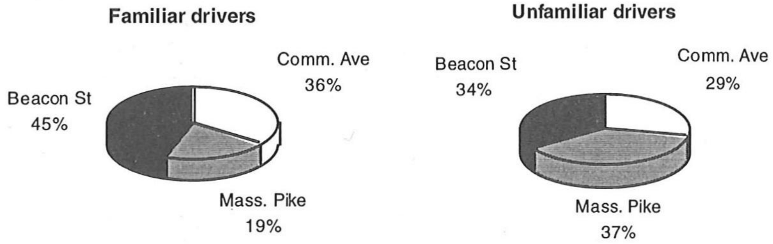

Figure 4 Choice distribution at the origin

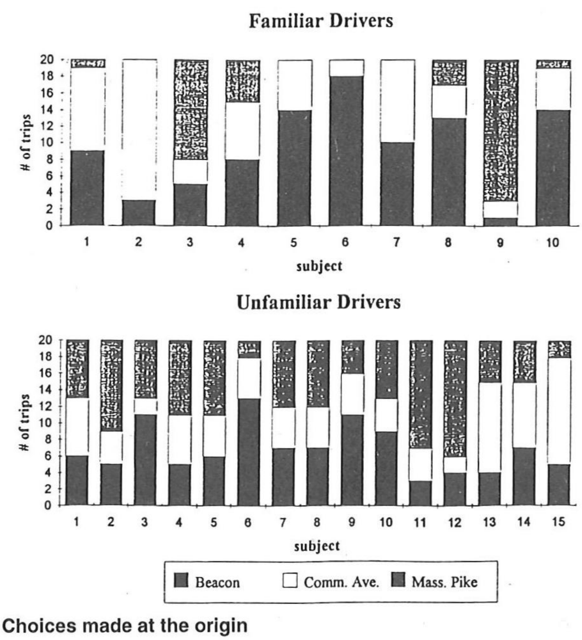

Figure 5 Choices made at the origin

Day-to-day variability

Day-to-day variability corresponds to the temporal aspect of the sequence of choices. A choice is considered to be a "switch" if it differs from the previous choice made at the same decision point. Figure 6 presents the distribution of route switching among familiar and unfamiliar drivers concerning choices made at the origin, (the subjects in the Figure are sorted according to ascending number of switches).

Clearly, the unfamiliar drivers performed more switches than the familiar drivers, and their average number of switches was significantly larger. This phenomenon can be explained by the learning and experimenting nature of the behavior of the unfamiliar drivers; whereas familiar drivers are more likely to stick to their previous choice, unfamiliar drivers keep searching for better choices, since they can not easily evaluate how good their choice was. It is important to recall that switching behavior is also related to the specific traffic conditions encountered which were not accounted for in Figure 6, however, the sequences of traffic scenarios for all subjects (familiar and unfamiliar) were sampled from the same distributions, thus we can conclude that indeed there is a difference in the day-to-day variability of the two sample populations.

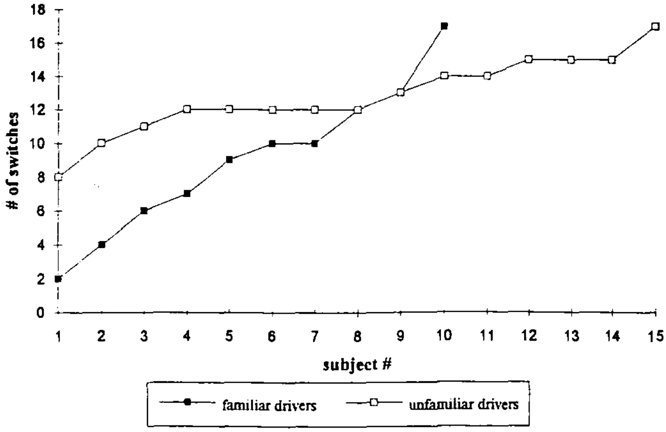

Figure 6 Route switching behavior

Another interesting phenomenon apparent from the switching behavior is the variances of switching behavior within each group; the unfamiliar population is more homogeneous in terms of their switching behavior (they all switch a lot), while the familiar population demonstrates larger variance of number of switches among its members. The above observed phenomenon can be explained by the differences in levels of familiarity: the unfamiliar drivers have limited static knowledge on the network and start with no dynamic knowledge, hence they investigate the dynamic performance of all the alternatives in the network. The experienced drivers, on the other hand, have well-established dynamic knowledge, and hence act according to their personal judgements and preferences and therefore demonstrate larger variability in their behavior.

Diversion behavior

Diversion is defined as a change of planned route, where the planned route is determined by the choice made at the origin. Diversions were possible only if Beacon Street or Commonwealth Avenue were chosen, and there were two possible diversion points on each: between nodes 2 and 3, and nodes 4 and 5. Diversions were generally considered to prolong the duration of the trip (if not motivated by traffic conditions), thus it was expected that diversions occur only if there was a good enough traffic-oriented reason to divert. Figure 7 presents the distributions of diversions among familiar and unfamiliar drivers, the subjects in the Figure are sorted in ascending number of diversions. Obviously familiar drivers diverted much more than unfamiliar drivers, with an average of 4.6 diversions for familiar drivers compared to 1.9 for the unfamiliar ones. A possible explanation is the relative conservatism of the unfamiliar drivers, and their efforts to gain dynamic knowledge of network performance. The unfamiliar drivers were learning the performance of the network on the three major alternatives. When diversions are allowed, then four more alternatives are added to the choice set (not counting routes with two diversions in each), forcing users to be able to process and remember more information. Hence, whereas experienced drivers showed more flexibility to experiment with variations of their usual alternatives, inexperienced drivers demonstrated a more conservative approach in their choices.

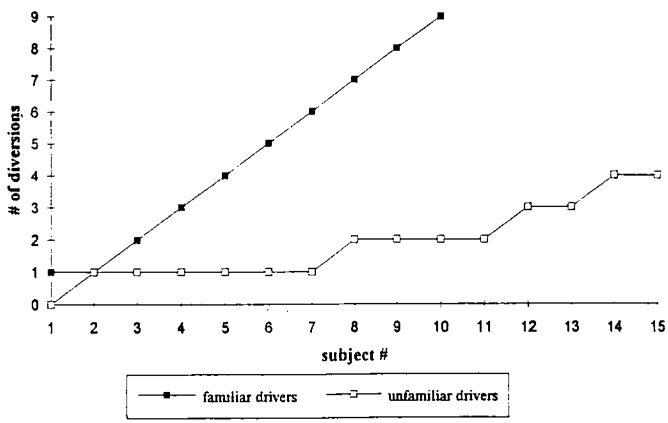

Figure 7 **Diversion behaviour**

Safety and time scores

Safety and time scores were presented to the users of the driving simulator after the completion of each trip. The safety score indicated how well the subject performed the driving task (simulated by the requirement to keep a randomly-moving ball within a car frame without hitting its edges). Both the familiar and unfamiliar drivers performed the driving task very well with an average safety scores of above 99%, exhibiting no significant difference between familiar and unfamiliar drivers. This observation was expected as both populations were unfamiliar with the use of the simulator, and all users managed to learn and master its operation quickly and efficiently.

The time score presented to the users after each trip corresponded to:

$$\text{time score} = \frac{\text{travel time on shortest path}}{\text{travel time on chosen path}} \tag{1}$$

and was computed based on the actual traffic conditions simulated during the trip.

We want to emphasize here that users were not instructed to travel on the shortest path, rather were told to act as they usually do (the familiar population) and as they think they would have done (the unfamiliar population). Moreover, even if the goal of minimizing travel time had been conveyed to users, it would not have been easy to achieve as the simulator did not present exact travel time estimates, rather its corresponding color categories.

Surprisingly at first, the familiar drivers got a lower average time score of 88.7% compared to 90.6% of the unfamiliar drivers, and the difference is statistically significant. However, a second thought reveals that whereas familiar drivers indeed acted according to their usual behavioral pattern (which does not necessarily minimize travel time), unfamiliar drivers, lacking such behavioral pattern, relied more on time scores and were trying harder to minimize travel time, since other preferences and personal tastes among the alternatives were not so obvious to them.

MODELING ROUTE CHOICE BEHAVIOR OF THE TWO POPULATIONS

Data obtained from the driver simulator includes three major groups of factors that influence route choice decisions: traffic conditions on the traveled link (as conveyed by congestion levels translated into the appropriate color category), traffic conditions on all the links of the network (as conveyed by colors through the information window), and accidents indication. Those three groups of factors are used to model route choice behavior of familiar and unfamiliar drivers.

The ART model

The Approximate Reasoning for Transportation (ART) model is based on linguistic rules which are used to describe attitudes towards choosing a specific alternative given (possibly vague) perceptions on system's attributes. The rules are used as anchoring schemes for decisions, while the adjustment of the rules to changing conditions is done by an approximate reasoning mechanism. The use of the fuzzy, approximate reasoning methodology, facilitates a flexible rule interpretation by automatically deriving rules that are close to the original rules. All the adjusted rules are then applied simultaneously (each with the appropriate degree) resulting in a final attractiveness of each alternative. The alternatives are then compared, and the most attractive alternative is chosen. The ART model is briefly described in the Appendix, and in more detail in Lotan and Koutsopoulos (1993).

One of the advantages of the ART model is that it can accommodate fuzzy inputs, allowing for a more natural and intuitive modeling of perceptions relating to observations and to the received information. For modeling existing perceptions and information we use fuzzy sets modeled on the scale of possible travel times. The sets are trapezoidal-shaped fuzzy numbers (TrFN) and correspond to: "travel times experienced along a certain facility". A TrFN, K, is determined by 4 points: k_1, k_2, k_3, and k_4, and is characterized by having a range, $[k_2,k_3]$, of very possible values (with membership degree of 1). A TrFN presentation is argued to be appropriate for modeling travel time perceptions since it is expected to have a range of travel times that are very possible, corresponding to travel times that occur under "usual" conditions. The existence of this range is the result of the fact that even under similar traffic conditions, different travel times realizations may occur, and thus more than one travel time gets a membership degree of one. The extreme points of the TrFN correspond to unusual conditions: k_1 and k_4 correspond to the shortest and the longest travel times respectively that are thought to be possible.

The building blocks of the decision process are rules of the form: $<$*if* A_i *then* $B_i$$>$, which associate the state of the system with choice-related attitudes and preferences. The use of rules resembles other rule-based systems in which decisions are related to specific input conditions (eg expert systems). However, the condition and the consequence part of the rules can include linguistic labels, and thus the rules become more general and intuitive, especially for modeling human thinking. There are three groups of rules for in the initial rule matrix which corresponds to decisions made at the origin. The first group pertains to observed traffic conditions on the three alternatives as observed from the observation window in the simulator, and translated into their travel time linguistic equivalents. The second group pertains to information concerning traffic conditions as received through the information window, and the last group deals with accidents and is of boolean nature: accidents either occur or not since in the simulator there is no indication of their severity. Consequently, the left-hand-side (LHS) of the rules relates to travel time perceptions as categorized into one of the following five fuzzy sets:

- VL - Very Low
- L - Low
- M - Medium
- H - High
- VH - Very High

These underlying design sets do not correspond directly to perceptions, rather they serve as a conceptual basis for comparison. Rule consequences, as appear in the right-hand-side (RHS) of the rules, relate to attitudes and preferences towards choosing among the possible alternatives. The RHS values are measured on a scale ranging from -1 to 1, with -1 corresponding to the case of complete aversion towards choosing alternative j, 1 corresponding to the case of choosing alternative j without reservations, and 0 corresponding to the indifference point. Five fuzzy sets are used to represent driver's attitude towards choosing an alternative:

- N - "I will Definitely Not choose this alternative",
- PN - "I will Probably Not choose this alternative",
- I - "I am Indifferent with respect to choosing that alternative",
- PY - "I will Probably choose this alternative",
- Y - "I will Definitely choose this alternative".

The initial rule matrix, is based on the most intuitive and common-sense rules resulting from the trivial mapping between the 5 LHS and RHS categories:

- VL —> Y
- L —> PY
- M —> I
- H —> PN
- VH —> N

The initial rules concerning accidents are again intuitive: *<if there is an accident on path j then I will definitely not choose path j>*. Thus the initial rule matrix contains 33 rules, 15 pertaining to observation (5 for each of the 3 alternatives), 15 to information, and 3 to accidents.

A rule calibration procedure is implemented to update the initial rule matrix and allow for more complex rules such as rules corresponding to interactions among alternatives (eg *<if travel time on path i is low then path i will probably be chosen and path j will definitely not be chosen>*). Thus, the decision variables are the RHS entries of the rule matrix, which can take one of 5 possible outcomes (Y/PY/I/PN/N), or be left empty. Their initial values are determined by the trivial mapping above. The only constraint imposed on the RHS values is that the preference towards choosing a specific alternative satisfies weak monotonicity with respect to traffic conditions on that alternative. The improvement procedure for rule calibration is based on a heuristic which sorts the rules, picks "bad" rules, and improves them if possible by modifying the existing RHS entry and by trying to add RHS interactions. The sorting of rules is based on "rewarding" good rules (rules that supported correct choices) and "punishing" bad rules (which supported incorrect choices).

Tables 1 and 2 summarize the results of implementing the approximate reasoning model for the familiar and unfamiliar populations. The entries in the tables correspond to the percentage of correctly explained choices where the initial fit is based on the initial rule matrix, and the improved fit on the calibrated rule matrix.

Table 1 Performance of the approximate reasoning model for familiar drivers (% of correctly explained choices)

Subject	1	2	3	4	5	6	7	8	9	10	Average
Initial fit	30	95	75	45	65	25	60	60	85	80	62
Improved	70	95	80	45	80	95	100	90	85	80	82

Table 2 Performance of the approximate reasoning model for unfamiliar drivers (% of correctly explained choices)

Subject	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Avg.
Initial fit	45	60	65	50	50	70	55	50	55	35	60	70	55	30	60	54
Improved	75	80	80	75	75	85	75	85	60	50	85	80	80	65	80	75.33

The fit obtained for both populations for the initial (and trivial) rule matrix is surprisingly high, giving a strong motivation for the appropriateness of the rule-based ART model. The percentages of explained choices obtained for the unfamiliar population are in general smaller in magnitude than those obtained for the familiar population. The unfamiliar group has a significantly lower variance of explained choices among its members (both for the initial and the improved fit), exhibiting again more homogeneity than the familiar group.

While the results of the initial fit of the models are global, in the sense that the same rule matrix was used for all the individuals, the improved results are disaggregate corresponding to individual calibrated rule matrices. Although disaggregate analysis is interesting for understanding individual behavior patterns, it has limited value from practical and predictive points of view. Hence an aggregate calibration was performed, trying to fit the same rule matrix to all individuals. The underlying assumption was that there exists a global rule matrix for all subjects and that individual differences are accounted for through the different perceptions which serve as inputs to the rules.

Table 3 Aggregate performance of the approximate reasoning model

	Initial fit (%)	Improved fit (%)
Familiar drivers	62	75
Unfamiliar drivers	54	60

The results of the aggregate analysis, as appear in Table 3, are of course lower than the results of the disaggregate analysis. However, for the unfamiliar sample population, the difference between the disaggregate and aggregate improved fit is much bigger (about 20%) than the difference for the familiar sample population (less than 10%). This observation indicates more difficulty in trying to fit a single rule matrix to all the unfamiliar drivers, thus displaying less homogeneity among the unfamiliar drivers in terms of their specific decision rules.

A random utility model implementation

A random utility model (RUM) was used for modeling route choices of familiar and unfamiliar drivers. Following the simultaneous modeling approach, the following factors were included in the utility function: observations on traffic conditions (as seen through the observation window), received traffic information (as conveyed by the information window), and existence of accidents. Several observations collected by the simulator had to be omitted as they corresponded to trips in which information was not available. For each individual 4 out of the 20 trips were with no information, and hence for the familiar population a total of 160 observations was considered, and 240 observations for the unfamiliar population.

The following logit model was estimated:

$$U_i=\beta_0+\beta_{tt}X_{tt_i}+\beta_{inf}X_{inf_i}+\beta_{acc_i}X_{acc_i}+\varepsilon_i \quad (2)$$

where: U_i is the utility associated with alternative i, X_{tit} is the observed travel time on alternative i, X_{infi} is the information regarding travel time on alternative i, X_{acci} is a dummy variable which equals to 1 if there is information on an accident on path i, E_i is the random error term, and $ß_i$'s are the coefficients to be estimated. The crisp values for the input variables X_{tit} and X_{infi} were extracted from the relevant fuzzy sets (corresponding to colors representing perceptions on observed and received traffic conditions) using the mid-range of the flat of the TrFN.

For the familiar Boston sample, estimation of a multinomial logit model for the utility function as specified in equation (2), resulted in a non-significant coefficient values for the observed travel time ($ß_{tt}$) and for accidents on Beacon street and on the Mass. Pike. ($ß_{acc1}$ and $ß_{acc3}$). Hence a reduced model with only 4 coefficients was estimated and the following results were obtained:

Variable	*Coefficient estimate*	*t statistics*
Beacon - constant	0.78	3.09
Comm Ave - constant	0.69	2.66
Traffic information	-0.17	-5.82
Accident on Comm Ave	-1.86	-1.71

with $\overline{\rho^2}$ =0.17, and 65% of correctly predicted choices. It is interesting to note that observed traffic condition turned out to be not significant, and more surprisingly neither accidents on Beacon street and the Mass. Pike. The insignificance of the accidents might be related to their rare occurrences (there were 13 indicated accidents on Beacon Street and only 4 on the Mass. Pike). Consequently, the most important factors attributing to the route choice decisions turned out to be the constants corresponding to inherent preferences with respect to the alternatives (that are unrelated to the actual traffic conditions occurred), and the information conveyed through the information window regarding traffic congestion levels. For the unfamiliar Brussels sample population, the *only* significant coefficient turned out to be $ß_{inf}$ which is associated with the received information.

Variable	*Coefficient estimate*	*t statistics*
Traffic information	-1.469	-7.296

with $\overline{\rho^2}$= 0.12, and 50.42% of correctly predicted choices.

The results obtained reveal an intuitive difference between the two populations: whereas the familiar drivers have inherent preferences towards the alternatives (represented by the constants of the utility functions), the unfamiliar drivers have no such a priori preferences and their main and only significant factor is the information received. The sample reconstruction performance of the ART model was better than that of the random utility model.

CONCLUSION

Several aspects of familiarity were discussed in the context of making route choice decisions in the presence of traffic information. A small case study was used to investigate effects of familiarity on route choice behavior. Data was collected using driver simulator from two groups of users: a familiar group that had extensive previous use with the network under consideration, and an unfamiliar group that had absolutely no previous experience. Both groups were unfamiliar with the information system.

The observed behavior of the two groups differed in several aspects; the unfamiliar group exhibited more uniform distribution of choices, while the familiar group showed clear preference among the alternatives. Furthermore, the unfamiliar group switched a lot from day-to-day while the familiar drivers showed a tendency to stick to their previous choice. On the other hand, the familiar group demonstrated larger flexibility in their diversion behavior en-route, while the unfamiliar drivers turned out to be less daring and more conservative. Both groups did very well in terms of performing the act corresponding to the driving task, however the unfamiliar drivers did better in terms of achieving high time-scores although the goal of minimizing travel time was not specified. This phenomenon was explained by the lack of inherent preferences among the alternatives for the unfamiliar drivers, which caused them to try and reach the unspecified goal of traveling on the shortest path.

Two models were implemented: the ART model and a logit model. Both models provided interesting insights into the choice behavior of the two groups. However, for the presented case study, the ART model outperformed the logit model terms of percentages of explained choices. It also had the advantage of being able to handle missing data (corresponding to trips without information).

REFERENCES

Adler, J.L. and McNally M.G. (1994) In-Laboratory experiments to investigate driver behavior under advanced traveler information systems, *Transportation Research C* 2, 149-164.

Bonsall, P. (1995) Analysing and modeling the influence of roadside variable message displays on drivers' route choice, Paper presented at the *7th WCTR*, Sydney, Australia.

Bovy, P.H. and Stern, E. (1990) *Route Choice: Wayfinding in Transport Networks*, Kluwer Academic Publishers.

Koutsopoulos, H.N., Lotan, T. and Yang, Q. (1994) A driving simulator and its application for modeling route choice in the presence of information, *Transportation Research C* 2, 97-107.

Lotan, T. and Koutsopoulos, H.N. (1993) Approximate reasoning models for route choice behavior in the presence of information, In: C.F. Daganzo (ed) *Transportation and Traffic Theory*, pp. 71-88. Elsevier Science Publishers, The Netherlands.

APPENDIX: THE APPROXIMATE REASONING MODEL

Each rule has the form: if A_i then B_i, and associates the state of the system A_i, with choice-related attitudes and preferences B_i. In general A_i and B_i are multi-dimensional vectors defined as:

$A_i = (A_i^1, \ldots, A_i^m)$ $\quad i=1,\ldots,N$

$B_i = (B_i^1, \ldots, B_i^m)$ $\quad i=1,\ldots,N$

where m is the number of alternatives in the choice set. A_i^j is the j'th component of the left-hand-side (LHS) of the i'th rule and corresponds to the state of alternative j. Correspondingly, B_i^j relates to the attractiveness of choosing alternative j in view of the information conveyed by the vector A_i. While the LHS characterizes a given performance measure according to fuzzy labels, the right-hand-side (RHS) of the rules corresponds to aspects of the final decision. It serves as an intermediate step in the decision process and corresponds to the stage at which attractiveness of each alternative is evaluated based on the input. The multi-dimensionality of the RHS representation captures the fact that even if the LHS of a rule relates to a specific alternative j, it could also affect perceptions of the attractiveness of another alternative k.

A single rule i execution is based on a_i, the degree of overlap between the LHS of the i'th rule, A_i, and the relevant input A^*. It serves as the degree with which the i'th rule is being "fired" or executed, and is given by:

$$\alpha_i \max_x \min(\mu_{A^*}(x), \mu_{A_i}(x)) \tag{A1}$$

where $\mu_F()$ is the membership function of the fuzzy set F. Based on the amount of overlap between A^* and A_i, the membership function of the attractiveness of a certain alternative j based on rule i, B_i^{j*}, is derived. Using the correlation-product encoding scheme the membership function of the set B_i^{j*} is given by:

$$\mu_{B_i^j}{*}(y)=\alpha_i{*}\mu_{B_i^j}(y) \tag{A2}$$

It is clear that more than one rule can have $a_i>0$, hence several rules can contribute to the final decision. All rules whose LHS have non-empty overlap with current inputs (ie rules i with $a_i>0$), are being fired simultaneously, each with a different degree a_i. For each alternative j, we combine the individual B_i^{j*}s over all the rules i into a score set B^{j*}which then corresponds to the attractiveness of alternative j and is given by:

$$S_{B_i^?}{}^{?*}(y)=\sum_{i=1}^{N} \mu_{B_i^j}{*}(y) \tag{A3}$$

Finally, the defuzzification phase translates the combined RHS's, B^{j*}, into a choice. As in most fuzzy control applications, a center-of-gravity based defuzzification scheme is used. Using equations A2 and A3, the center of gravity is given by:

$$z=\frac{\sum_{i=1}^{N} \alpha_i V_i S_i}{\sum_{i=1}^{N} \alpha_i S_i} \tag{A4}$$

where α_i is the degree to which the i'th rule was fired, V_i is the centroid of the fuzzy set corresponding to the RHS entry of rule i, and S_i is the area of this set (if $\sum_{i=i}^{N}\alpha_i S_i=0$ then z is equal to 0). Finally the (crisp) centroids are compared using either deterministic or a random utility scheme resulting in a single chosen alternative.