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# MODELLING RESPONSIVE SIGNAL CONTROL AND ROUTE CHOICE

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## Abstract

The interaction between responsive traffic signals and route choice is studied in a simple artificial network, using a stochastic process model of day-to-day decisions. The long-term ("stationary") behaviour is seen to be potentially quite different from pseudo-stable conditions that prevail over a shorter time horizon, raising questions about conventional approaches to traffic assignment.

# INTRODUCTION

This paper considers the problem of modelling a traffic network in which a number of the intersections are signalised. More specifically, it considers the interrelationship between the *traffic assignment* problem of modelling flows in the network for a given set of traffic signal timings, and the *traffic control* problem of selecting the signal timings (which must take some account of the flows).

A traffic control *policy* is a systematic method for selecting signal timings, given information on the flows in the network. Two broad types of policy may be distinguished. In *fixed time policies*, the flow information is purely historical. Typically, the timings will switch between different signal timing "plans" at pre-defined times of the day. In *responsive policies*, the timings are adjusted on and during each day, based at least partially on prevailing conditions, estimated from detectors in the road. These two broad categories include policies which control junctions in isolation, as well as those that are able to link information from neighbouring junctions. One of the best known fixed time policies is TRANSYT (Robertson 1969). A wide range of micro-processor controlled responsive policies are in operation or under development—see Shepherd (1994) for a review.

Turning attention to the traffic assignment problem, the established methodology for modelling drivers' long-term choice of route through an urban road network is equilibrium assignment. In the simplest, *steady state* or *static* case (Sheffi 1985), we supply as input an origin-destination matrix (representing the average demand for travel during, say, a peak period) and a set of link travel time functions (representing the relationship between each link's average travel time and the vector of average link flows on the network). The users of the network, as represented in the OD matrix, are assumed to choose their routes through the network according to the generalised travel cost, typically a linear function of travel time and other flow-independent attributes. Equilibrium occurs when the demand for road usage, given by the routes drivers choose to follow, matches the supply of road space, represented by the link travel time functions. Such a condition prevails when no user can reduce their own travel cost by unilaterally changing routes.

Traffic assignment modelling is not traditionally associated with concessions to behavioural considerations. In applying one of the class of equilibrium models (including extensions to the static case, such as dynamic equilibrium, or "taste variation" represented by stochastic or multiple user class equilibrium), the implicit assumption is made that drivers possess perfect knowledge of the traffic conditions they would encounter in the network. When applied to a network where a fixed time signal policy is in operation, then—provided the timings are not adjusted for a significant period of days—such an assumption is probably justifiable. In the case of responsive traffic signals, however, there is a clear contradiction: If traffic conditions really are sufficiently steady for the equilibrium assumption to apply, then why is a responsive policy needed? Clearly, the answer is that conditions *do* vary both between and within days, and it is the equilibrium approach to traffic assignment that is brought into question. (Interestingly, similar issues have arisen in the study of a different type of "responsive" system, namely driver information systems—see Watling and Van Vuren 1993).

The purpose of this paper is to study the use of a relatively new, stochastic process approach to the problem of modelling traffic networks. It is demonstrated how this approach allows a more natural representation of responsive signal control and its interaction with route choice behaviour. A highly simplified example -studied previously in the equilibrium literature—is used to study the *evolution* of the route choice / control process, and comparisons are drawn with the equilibrium approach.

## REVIEW: EQUILIBRIUM AND FIXED TIME SIGNAL CONTROL

To the author's knowledge, the effect of responsive signals on route choice has not previously been studied. Ghali and Smith (1994) studied the effect of allowing control policies to respond to the iterations of an equilibrium algorithm; since real day-to-day adjustments of traffic and driver behaviour are unlikely to be like these iterations, it is not clear that they are modelling responsive control as defined here, nor do they address the objections to equilibrium discussed in the previous section. The interaction between static equilibrium assignment and *fixed time* signal control has, however, been considered by a number of authors. Much of this work has relevance to the problems studied later in the paper, and so a brief review is provided.

Following Allsop and Charlesworth (1977), an equilibrium in the presence of fixed time signals is defined as a point solution (ie an assignment of flows to the links of the network, and a particular choice of signal timings at intersections) at which

a) the flows are in equilibrium, given the signal timings in (b); *and*

b) the signal timings satisfy the control policy at the equilibrium flows in (a).

We shall refer to this as the *(static) control/equilibrium problem.* There are four aspects of this problem that are of particular interest: existence, uniqueness, stability, and network design.

### Existence

Smith (1981a,b) established technical conditions on the link travel time functions and control policy that ensure existence of at least one solution to the control/equilibrium problem. In order to consider traffic signals in such a setting, Smith made a number of simplifying assumptions:

- at any given junction, traffic streams that could otherwise conflict are not given green simultaneously;
- delays at a junction are affected only by flows at that junction, and not additionally by flows at other junctions;
- for given flows, average delays (over, say, a peak hour) are affected predominantly by the aggregate *proportion of green* given to each turning movement, rather than (say) the cycle time, stages/phases, or the absolute durations of green/red.

Smith went on to consider the implications of his results in a simple artificial network, for three control policies, all of which are applied locally (ie independently to each intersection):

a) *Delay minimisation*, where the green proportions are chosen to minimise total expected delay at the intersection, given the flows;

b) *Webster's equi-saturation*, where the green proportions are chosen to equalise the `degree of saturation' (flow divided by saturation flow) on each approach;

c) *Smith's P0 policy*, an artificial policy chosen specifically to satisfy the sufficient conditions of his existence theorem, whereby the green times are chosen such that the relative magnitude of the delays on each approach are inversely proportional to the saturation flows.

These policies are specified more precisely in later.

Smith showed that a) and b)—approximations to policies used in practice—can fail to satisfy the sufficient conditions of his existence theorem. This is not the same as saying that no solution exists (the conditions are not necessary), although Smith (1979a) was able to construct an example in which, for a sufficiently high demand, no feasible control/equilibrium solution existed under Webster's equi-saturation policy.

### Uniqueness

Assuming that existence of at least one equilibrium solution is established, it would be desirable to prove that there is a unique such solution. This uniqueness issue, whether or not related to signal control, is one that has occupied researchers for many years. The seminal papers of Smith (1979b) and Dafermos (1980) gave rise to what are still today the most general known sufficient conditions. The main requirement is that the Jacobian matrix of first partial derivatives of the vector of link travel time functions with respect to the vector of link flows be positive definite. This is satisfied if:

- the travel time on a link is an increasing function of the flow on that link, when other link flows are held constant; and
- the dominant explanatory factor in a link's travel time is the flow on that particular link, rather than any other link flows.

At a signalised intersection operating on pre-specified signal timings, the second condition above may be violated if conflicting flows are given green simultaneously, since the major cause of delay to a vehicle ceding priority is the flow of traffic with the priority. If, however, we allow a fixed time control policy to determine the signal settings based on the (as yet undetermined) equilibrium flows, then even more serious problems may arise in which either condition above may be violated (Heydecker 1983; see also Morlok 1979, for an analogous problem in the context of mode choice and a demand-responsive bus service). For example, at an intersection with two approaches competing for green time, an increase in the flow on one approach will tend to increase its share of the green. Depending on the control policy, it is possible that this combination of an increased flow compensated by an increased green time could lead to a decrease in delay for that approach. The examples constructed by Heydecker (1980) and Cascetta (1989) show that delay functions of such a non-monotone shape may indeed lead to multiple equilibrium solutions.

More specifically, Smith (1979a) showed that at demand levels sufficiently moderate to ensure existence, Webster's equi-saturation policy may lead to multiple control/equilibrium solutions, whereas Smith's P0 policy in the same example gave rise to a unique solution. These examples are studied at greater length later.

## Stability

We consider the issue of whether a control/equilibrium solution is 'stable', for which a number of definitions are possible. In loose terms, stability is concerned with the mathematical behaviour of the route choice/control process in approaching (rather than at) a precise state of equilibrium. The most common definition, and the one adopted in the present paper, concerns properties of this process in a *local* neighbourhood of a control/equilibrium solution (following Netter 1972; Braess and Koch 1979; and Heydecker 1983). Namely, an equilibrium is stable unless arbitrarily small deviations from it may cause the flows to diverge from the original equilibrium state. The behaviour of the route choice/control process at non-equilibrium states is not explicitly defined by control/equilibrium models, except in as much as it is assumed that drivers will tend to divert to lower cost alternatives when they are available. It is assumed in this definition of stability that this diversion will happen in sufficiently conservative amounts, that the process will indeed evolve to the original control/equilibrium solution. Unstable equilibria are considered to be uninteresting in terms of network analysis, since by definition any real system would never persist in such a state, but could at best periodically occupy it. Note that alternative, *global* stability conditions were considered by Smith (1979b) and Horowitz (1984).

## Network design

The classical *network design problem* (NDP) is that of designing a network that—for a given demand matrix—minimises total travel time/cost, subject to flows being in equilibrium for the design (Abdulaal and Leblanc 1979). A special case of the NDP may be considered to be the case where the design variables are traffic signal settings. For example, cycle times and stages may assumed to be fixed, and the green times are then the continuous design variables, giving rise to a continuous NDP. In the context of network design, there is an implicit recognition that multiple control/equilibrium solutions may exist, the NDP aiming to select the best of these from a system controller's point of view.

In particular, we consider the original solution technique proposed by Allsop (1974), and later termed the iterative optimization-equilibrium algorithm (Friesz and Harker 1985). This is a simple technique which, as the name suggests, consists of alternating steps of signal optimization for given flows, and equilibrium assignment for given signal settings. In this context, Allsop and Charlesworth (1977) and Dickson (1981) have constructed examples in which there are, indeed, multiple control/equilibrium solutions which this algorithm may approach. Dickson, and Friesz and Harker, have shown that although the iterative optimization-equilibrium algorithm may converge to such a control/equilibrium solution (and conditions on such convergence have been established by Smith and Van Vuren 1993), this solution may not be optimal in terms of solving the NDP.

From a purely algorithmic standpoint of solving the NDP, the above criticism may be justified; indeed, a variety of alternative solution methods now exist. However, the (starting-condition dependent) solution of the iterative optimization-equilibrium algorithm probably has greater credibility from a behavioural point of view. The alternation between drivers finding a long-term optimal choice of route given fixed time signal settings, and the system controller optimizing the signals for given long-term average flows, is similar to the evolution of real-life systems. Such an approach recognises that the initial conditions (eg existing signals set by a traffic engineer from local knowledge and/or the current observed flows on the network) will affect the long-term evolution of the control/route choice process. In contrast, the solution of the NDP may require a routing pattern which is far-removed from current behaviour, and which may be unlikely to evolve from current conditions or which may only evolve over a very long period of time.

## ROUTE CHOICE AND RESPONSIVE SIGNAL CONTROL: MODELLING FRAMEWORK

The discussion in the previous section, of the evolution of route choice/control processes, is an appropriate introduction to the modelling approach to be pursued in this paper, which explicitly models an analogous kind of evolution. Here, the dynamic *day-to-day evolution* of the traffic system will be considered, from some given starting condition. Models of this kind have grown in popularity in recent years, due to the modelling flexibility they offer (eg Alfa and Minh 1979; Horowitz 1984; Ben-Akiva et al. 1986; Cascetta 1989; Vythoulkas 1990; Chang-Jou and Mahmassani 1994; Emmerink et al. 1995).

Such day-to-day models fall into two distinct categories, deterministic and stochastic process models. From given starting conditions, deterministic process models seek convergence to a single state of deterministic or stochastic equilibrium. In the present paper, however, we favour the *stochastic process* approach championed by Cascetta (1989). Under fairly mild conditions, satisfied by the particular form of model described below, Cascetta proved the existence of a unique long-term *equilibrium* or *stationary probability distribution* for any given system, regardless of the starting conditions. This may be considered the analogue of a deterministic/stochastic equilibrium *state*.

The particular model considered here will have the following general structure (one time period may typically represent one day):

1. *[Initialisation]* Set time period counter k=0. Initialise mean perceived costs (or flows).
2. *[Demand-side]* Increment k. The current mean perceived costs, the assumed probability distribution of perceived costs, the assumed route choice rules and the given origin-destination matrix together define a joint probability distribution for route, and thence link, flows. The link flows in period k are random variables, following this distribution.
3. *[Control]* The green splits for time period k are computed, based on the flows from step 2, according to the given control policy.
4. *[Supply-side]* Calculate the experienced link travel times and hence costs for period k arising from the flows in step 2 and the green splits in step 3, according to specified traffic performance functions.

5. *[Learning]* Update the mean perceived costs based on the new experiences in step 4, using a moving average of experienced costs in at most the last m periods, for some given m. Return to step 2.

In the above model framework, the evolution of the system from a given starting condition can only be determined according to a probability law, ie the process is indeed stochastic. Note that because the time period k link flows are random variables, then implicitly so are the time period k green splits, the period k experienced link travel times/costs, the mean perceived costs at the end of period k, and the initial mean perceived costs. Therefore, for example, in step 3 we could more correctly say 'The green split probability distribution for time period k is computed ...'. Many contrasts may be drawn between this approach and conventional equilibrium, eg that link, route and OD flows are all discrete variables in the above framework, but continuous in equilibrium models. See Cascetta (1989) and Watling (1995) for a further discussion of such contrasts.

Within a framework of the kind above, many types of control system may be analysed, including periodically updated fixed-time policies (responding to past conditions in the last n days, say), and responsive policies that combine, in various ways, historical information with that on prevailing conditions. These are interesting topics for future research. In the current paper, we shall analyse only a highly-responsive form of policy which takes account only of prevailing traffic conditions, not those on previous days. Furthermore, it is assumed that these conditions are perfectly monitored, although introducing some kind of "model" of the passage of information, and the errors and communications delays incurred, would be possible.

More importantly, there are no within-day dynamics of any kind in the model. Instead, the "state" of a road on any one day ('s peak hour) is characterised by a single aggregate flow. This can be contrasted with most studies of traffic signals, where issues such as platooning, signal staging, queue dissipation, and co-ordination are crucial. In keeping with this, we assume (following Smith 1981a,b; see earlier section) that for given flows, average delays are affected predominantly by the aggregate *proportion of green* given to each turning movement. This is not because we believe these dynamic factors to be unimportant. Indeed, as will be discussed in the conclusion, work is underway to develop a modelling framework that can take account of such factors in combination with the day-to-day traffic assignment problem. As we shall see, however, the interpretation of the output of stochastic process models is not a trivial issue, and this is the main reason for starting by considering a simple, within-day static case. These simplifications allow us to gain a first approximation to the long-term effects of signal control policies, allowing some analytical (as opposed to simulation-based) results to be obtained.

## EXAMPLE NETWORK AND CONTROL POLICIES

The example network considered will be the highly simplistic one considered by Smith (1979a) in the context of equilibrium and fixed time policies, where the network consists of a single origin-destination pair connected by two routes. The two routes/approaches meet along their length at a signalised intersection working on a fixed cycle time c. The green time proportions for the two approaches are chosen in response to the prevailing flows. Smith considered two control policies, and a third is added here. Define (for i=1,2):

$\lambda_i$ = green time proportion given to route/approach/link i;

$v_i$ = flow on link i;

$s_i$ = saturation flow of link i; and

$d_i(v_i,\lambda_i)$ = delay on link i.

The green time proportions are constrained by $\lambda_1 + \lambda_2 = 1$—ie there is no "lost-time". For given flows $v_1$ and $v_2$, the policies are:

*(a) Webster's equi-saturation policy*

$$\lambda_i = \frac{y_i}{y_1 + y_2} \text{ where } y_i = \frac{v_i}{s_i} \ (i=1,2)$$

*(b) Smith's P0 policy*

Choose $(\lambda_1, \lambda_2)$ such that

$$s_1 d_1(v_1, \lambda_1) = s_2 d_2(v_2, \lambda_2)$$

*(c) Delay minimisation*

Choose $(\lambda_1, \lambda_2)$ so as to minimise

$$\sum_{i=1}^{2} v_i d_i(v_i, \lambda_i)$$

It is supposed that delays are given by Webster's two-term formula (for i=1,2):

$$d_i(v_i, \lambda_i) = \frac{9}{20}\left[\frac{c(1-\lambda_i)^2}{1-y_i} + \frac{y_i^2}{v_i \lambda_i(\lambda_i - y_i)}\right]$$

Note that a number of other simplifying assumptions need first to be made, such as: the two links cannot have green simultaneously; the travel time along each route consists of a free-flow component and a junction delay component—the free flow component is assumed the same for both routes; and drivers aim to minimise cost = expected travel time = expected junction delay.

Smith considered two cases, with an OD demand of $\rho = {}^1/_2$ and ${}^7/_6$. In order to compare the approach with the stochastic process method (which works in discrete units of demand), we shall scale the demand up to a value q, but in order to obtain the same effect on the delay functions, when substituting in the delay functions $v_i$ is replaced by

$$v_i' = \frac{\rho v_i}{q}$$

where $\rho$ is the demand level assumed by Smith. He assumed further that $s_1 = 1$ and $s_2 = 2$, but did not specify the value used for the cycle time c in his tests—which is clearly important, since it has an effect on the relative influence of the deterministic queuing (first) term and random arrivals (second) term. With a demand factor of $\rho = 0.5$, the link flows, when scaled as above, will be in the range 0 to 0.5. This is not an unreasonable range for flows measured in vehicles per second, and so it seems reasonable to assume c to be measured in seconds; a value c = 60 will be used as a basis for the tests.

Before proceeding, we note two potential problems with the direct application of Smith's approach. Firstly, he imposes no minimum green time constraints, and so the green time proportions $\lambda_1$ and $\lambda_2$ are allowed to be zero. Unfortunately, the limit, as $v_i \to 0$ and $\lambda_i \to 0$, of Webster's delay function $d_i(v_i, \lambda_i)$ does not exist. (Allowing $v_i \to 0$ first gives rise to a finite limit, whereas letting $\lambda_i \to 0$ first means that $d_i \to \infty$). This is not an entirely trivial issue, since Smith shows that under Webster's control policy, the only stable equilibria are precisely at these all-or-nothing solutions. This problem is overcome by imposing a minimum value g on the $\lambda_i$. Therefore, if, for example, a policy gives $\lambda_1 < g$, then we reset the values to:

$\lambda_1 = g \qquad \lambda_2 = 1 - g$ .

In the tests reported, g=0.01 is assumed. This is not particularly realistic, but is chosen so as to make the comparison with Smith's results as close as possible. Tests have indicated that the sensitivity to this value is not great over the range $0 < g \leq 0.05$, the findings being qualitatively the same.

A second problem is the interpretation of Webster's delay formula when flows are greater than capacity ($v_i > \lambda_i s_i$), as may occur in Smith's second example with a demand factor of $\rho = {}^7/_6$. Smith argues that such points are infeasible in 'supply' terms (in any case, Webster's formula is inapplicable in such circumstances). This is really a limitation of 'static' assignment models, which are unable to deal adequately with the non-transient queuing suggested by $v_i > \lambda_i s_i$, which will prevent some drivers from reaching their destination in the modelled period. Since this problem is not of central interest to the paper, it will be avoided by only considering Smith's first example, with a demand factor $\rho = {}^1/_2$. Then:

a) Under Webster's policy, we need do nothing further, since all demand-feasible flow allocations imply $y_1 + y_2 < 1$, and hence the green times given by Webster's control method will always satisfy $v_i < \lambda_i s_i$. The minimum green time value (g=0.01) used in the tests below, together with the assumed values for $s_1$, $s_2$ and $\rho$ will not cause any problems either, since all flow allocations then satisfy $v_i < (1-g)s_i$.

b) For Smith's P0 policy, we explicitly introduce the constraint $v_i < \lambda_i s_i$ during the computation of the $\lambda_i$ (in any case, if this constraint were not included, then for any given set of flows, P0 may not give a unique set of green times).

c) Under delay minimisation, we proceed as for (b), the same comments applying.

It is assumed, finally, that drivers choose between the two routes according to a logit route choice rule with parameter ß (>0):

$$p_1(c_1,c_2) = \frac{1}{1 + \exp \beta(c_1-c_2))}$$

where $c_i$ is the cost of travelling along link i. The limiting case, as ß tends to infinity, corresponds to a deterministic, cost minimising rule. In the equilibrium approach, the cost above is assumed equal to delay, and for time period k+1 of the stochastic process approach, it is equal to the average delay over the last min(k,m) time periods.

## TEST RESULTS

Summarising the discussion of the previous section, the parameter values assumed are:

- Smith's scenario $\rho = {}^1/_2$
- OD demand of q = 100
- Minimum green time proportion g = 0.01
- Saturation flows $s_1$=1 and $s_2$=2, cycle time c=60

The learning parameter m and the dispersion parameter ß are varied (including the limiting case where the choice is a deterministic one). The tests consider and contrast equilibrium + fixed time signals with stochastic process + responsive signals.

Due to space limitations, the techniques used to derive the results are not described (see Watling 1995 for the details). In brief, the equilibrium results are obtained by an exhaustive search over the feasible region. In the stochastic process approach, the stationary distribution is found using Gaussian elimination as the solution to a linear fixed point problem. In addition, to produce Figure 3, the evolution of the transient flow probability distribution was explicitly calculated. In practice, with larger networks and demand levels, neither of these two methods will be feasible, and instead we must resort to Monte Carlo simulation to generate a realisation of the process—this technique is studied in Figure 4.

### Webster's control policy

Applying an equilibrium model and fixed time control according to Webster's method, three equilibria arise under a deterministic choice rule. Two of these equilibria are stable, falling at the all-or-nothing flow solutions with maximum/minimum green. The third is an intermediate, but unstable, equilibrium around $v_1 = 33.67$. As noted earlier, unstable equilibria are not considered to be significant in the analysis of transport networks. The stochastic equilibria, for various values of the choice dispersion ß, are as follows (to two decimal places):

| ß | Stochastic equilibria (u/s = unstable/stable) | | | | |
|---|---|---|---|---|---|
| 0.02 | | | | | 56.46 (s) |
| 0.05 | | | | | 73.63 (s) |
| 0.10 | | | | | 98.40 (s) |
| 0.1174 | | | | | 98.70 (s) |
| 0.1175 | | | 9.38 (s) | 10.46 (u) | 98.70 (s) |
| 0.12 | | | 6.46 (s) | 13.62 (u) | 98.74 (s) |
| 0.13 | 0.49 (s) | 0.57 (u) | 2.95 (s) | 17.95 (u) | 98.90 (s) |
| 0.15 | 0.38 (s) | | | 21.98 (u) | 99.17 (s) |
| 0.40 | 0.00 (s) | | | 30.67 (u) | 100.00 (s) |

For high values of ß, three stochastic equilibria arise, approaching (as expected) the deterministic equilibria as the perception error tends to zero. For low values of ß, perception errors tend to completely dominate the congestion costs; in the limit, drivers chose a route purely at random, and we obtain a 50:50 split. At intermediate values of ß, up to three *stable* stochastic equilibria may exist.

In contrast, we can apply a stochastic process model with responsive signals according to the same policy. The rather interesting case of ß = 0.13 is first examined, where five stochastic equilibria were seen to exist above, and apply the stochastic process approach with m=1 in the driver learning model. The *one-step transition probability matrix* gives the implied probability that the system will occupy any of the 101 possible states $v_1$=0,1,2,...,100 on day k (the columns), given the state occupied on day k-1 (represented by the rows). In this case, the matrix has a distinctly diagonal form (Figure 1).

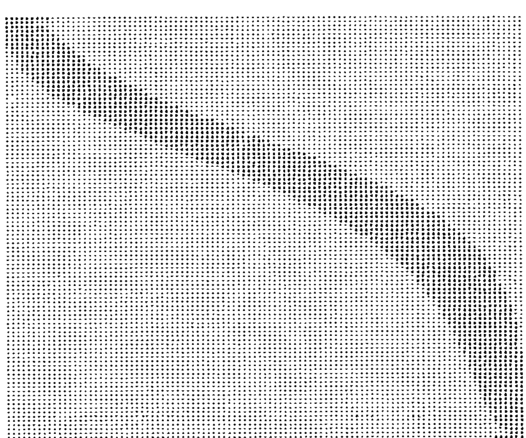

**Figure 1 Schematic representation of one-step transition probability matrix (Webster, ß=0.13, m=1)**

The light part of Figure 1 represents cells with transition probabilities less than 0.005, and the dark part probabilities greater than 0.005. This illustrates that *any* given state occupied will tend to transform into a state *in the same vicinity* on the following day, other states having an extremely small probability of being reached. This is the characteristic pattern of a process which may, depending on the initial conditions, approach the stationary distribution at a very slow rate (see Watling 1995, for other examples, unrelated to signals). Note that all probabilities are in fact non-zero (a feature of using the logit model) but may be extremely small—even outside the range of computer accuracy. For Cascetta's (1989) theory to guarantee convergence of the process to a unique stationary distribution, all such probabilities must be non-zero; as we see later, in practice this "good behaviour" may not be apparent as some transition probabilities tend to zero. It is noted also that increasing ß and/or m reduces the underlying day-to-day variance in flows, thereby causing the transition probability matrix to be more tightly focused on the diagonal—transforming to "far-off" states is now even more difficult.

This matrix may be used to compute the unique stationary distribution of the process, illustrated in Figure 2. For the case of ß=0.13, the distribution is heavily concentrated in the close vicinity of the upper stable stochastic equilibrium state of the fixed-time control/equilibrium problem (the signal timings are also similar). The mean and standard deviation of the stationary distribution are approximately 98.34 and 1.76 respectively. As ß is reduced, the distribution becomes further spread and more symmetric, though still with a mean close to the upper equilibrium. The conclusion is that in the fixed time control/equilibrium problem, one of two equally plausible (locally) stable states of the network will prevail; whereas in the responsive control problem, there is ultimately only one attractive region for the system, the mean of which is closely approximated by one of the stable fixed-time control/equilibrium solutions.

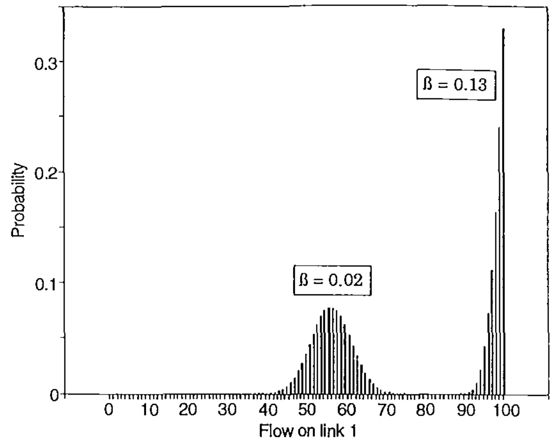


**Figure 2 Stationary distribution (Webster, m=1) for each of the cases ß=0.02 and ß=0.13**

In the long-term, then, the nature of the responsive control / route choice process in this example is clear. However, when attention is directed at the *evolution* of the process through its transient stage (rather than just the long-term stationary behaviour), we begin to pose the more fundamental question: is the stationary distribution a reasonable characterisation of the performance of such a network? Returning to the case ß=0.13 and m=1—then with an initial (day 0) flow on link 1 of $v_1 = 100$ or $v_1 = 50$ (or any intermediate value), less than a hundred days are required for the stationary distribution to be reached, to four decimal place accuracy in estimating the mean and standard deviation. However, with a lower starting flow on link 1, convergence may be much slower—eg see Figure 3 for a starting condition $v_1$=0. In this case, nearer to 50,000 days are required to obtain two decimal place accuracy in the stationary mean and standard deviation. Note that the evolution is quite smooth—the small, local oscillations in the graph are due merely to interpolation between data points at 100 day intervals. Over a planning horizon of, say, 10,000 days—around 30 years—the system is still clearly in its transient stage. Although the results are not given here, increasing ß and/or m only serves to emphasise this effect.

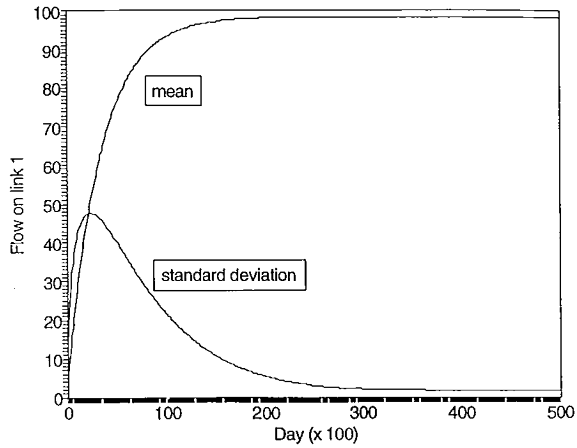


**Figure 3** **Evolution of transient probability distribution, as measured by mean and standard deviation (Webster, ß=0.13, m=1, initial $v_1$=0)**

To illustrate better what might be observed on the street from such a system, a Monte Carlo simulation of the process over a 1000 day period is examined (Figure 4). The trends marked 'Webster 1' and 'Webster 2' are obtained from simulations started at $v_1$=100 and $v_1$=0 respectively. Each gives rise to a (visually) stable flow mean/ variance. If more days were simulated, we would find—with probability almost 1—the Webster 1 series would continue with the same behaviour, whereas eventually the Webster 2 series would be drawn towards the Webster 1 series, and thereafter persist with similar behaviour to the Webster 1 series. However, each of the two series in Figure 4 could be described as "locally stable" *over the time horizon simulated*, and because of this we will refer to them as 'pseudo-equilibria'. This term is meant to convey the idea that we have *apparently* reached a stationary condition of the system. It is also appropriate because, over the period simulated, the two series give mean flows close to the stable stochastic equilibria of the fixed time control/equilibrium problem. It is clear from Figure 2, though, that only the Webster 1 series characterises the true stationary behaviour. Repeating these tests for fifty random number seeds from an initial condition $v_1$=0, after even $10^6$ days six of the simulations were still apparently stable around a mean link 1 flow of 3.2-3.3, with the remaining simulations at various stages on their path to stationarity. These tests indicate the care needed in applying Monte Carlo based stochastic process models. At the very least, sensitivity analyses to the seed value and starting conditions need to be carried out.

## Smith's P0 policy and delay minimisation

In contrast with Webster's policy, when applied in a fixed time control/equilibrium framework, P0 gives rise to a unique, stable deterministic equilibrium solution ($v_1$=0, $\lambda_1$=0.37), and unique stochastic equilibrium for all values of ß. This latter closely approximates the long-term average behaviour predicted by a responsive control/stochastic process approach. The stationary probability distribution corresponding to ß=0.13 and m=1 is illustrated in Figure 5. As ß is increased, this distribution becomes more concentrated and shifts to the left—notably, the complete opposite of Webster's. In contrast to Webster's policy, with P0 a rapid convergence to the stationary distribution occurs, regardless of the initial conditions (see Figure 4 for an illustrative simulation). The one-step transition probability matrix (Figure 6) has a very different form to that under Webster's policy, with a single attractive region into which any state is likely to transform.

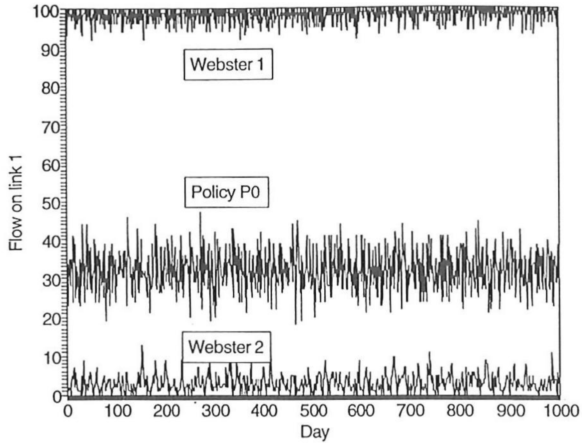


**Figure 4** **Monte Carlo simulations of the process (m=1, ß=0.13)**

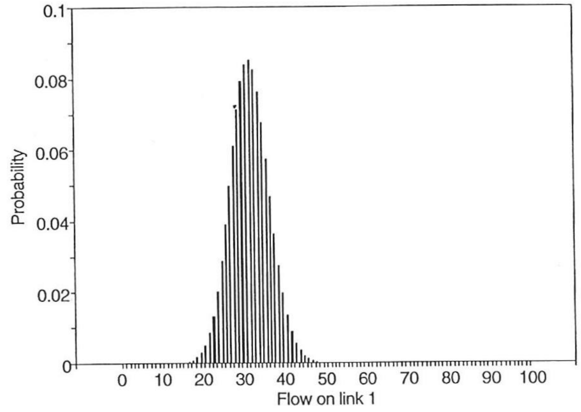


**Figure 5** **Stationary distribution (PO, ß=0.13, m=1)**

The delay minimisation policy gives rise to extremely similar behaviour to Webster's equi-saturation, which is probably not surprising given that the latter is often considered to be an approximation to the former. In particular, the comments earlier regarding the stationary distribution, pseudo-equilibria, and the effect of the initial conditions on convergence rate were found equally to apply. Although it was not the purpose of the study, we note finally that, from a system controller's point of view, both Webster's policy and delay minimisation were considerably superior to P0. Whether we consider either of the pseudo-equilibria or the stationary position, the day-averaged total travel time under the former policies was significantly smaller than that from P0, although with a marginally larger day-to-day variance.

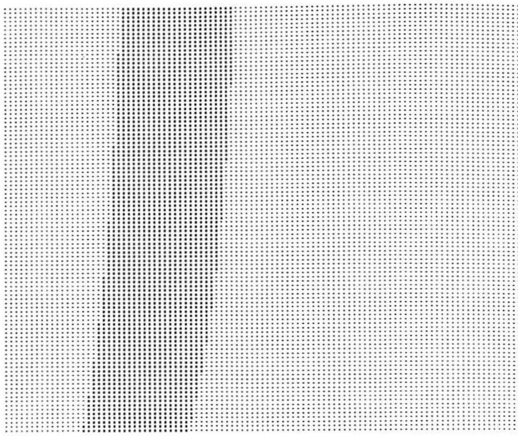

**Figure 6 Schematic representation of one-step transition probability matrix (PO, ß=0.13, m=1)**

## CONCLUSION

It has been argued that the assumptions of equilibrium assignment models are inappropriate in networks where responsive control systems operate. Using a more general, stochastic process model of the day-to-day evolution of the responsive-control/route-choice process, a simple example—previously considered in the literature—was studied. Applying either the Webster equisaturation control policy or delay minimisation, it was seen that:

a) In the *long term*, a single, unimodal stationary probability distribution of flows on the network will ultimately prevail, from arbitrary starting conditions.

b) The related equilibrium/fixed-time control problem was seen to have two stable equilibria, *one* of which approximates the mean of the stationary distribution above. This relationship held under a variety of behavioural specifications (viz. m and ß).

c) Examining Monte Carlo simulations of the responsive-control/choice process, in the *shorter term*—which could mean as long as thirty years!—pseudo-stable but non-stationary behaviour may persist for long periods of time. This was seen to be due to implied transition probabilities between system states being extremely small.

d) Different pseudo-stable conditions may be reached over such a time horizon, *dependent on the initial conditions* and possibly the random number seed.

e) Smith's P0 control policy was seen not to suffer from these problems, though it demonstrated an inferior performance from a system controller's point of view.

The simplistic nature of the test model and network make these findings suggestive of future research, rather than conclusive, but the simplification has allowed analysis which avoids Monte Carlo simulation. Although simulation is likely to be the only feasible approach for realistic networks, an analytic approach provides a better basis for understanding the complex behaviour of stochastic process models, and indeed day-to-day evolutions in real-life. Thus, it is suggested that further such analysis of simple networks should be carried out, studying examples where bimodal stationary distributions prevail and transitions between attracting regions occur less rarely, the effect of different behavioural models, or the impact of using part-historical information in the control response (see Watling 1995, for a coverage of some of these, though not in relation to signal control). It is noted that studying larger networks with overlapping routes, or allowing drivers or the control system to use more distant historical data, will tend to dampen the level of variability, leading to a number of small transition probabilities, and hence more problems of the types (c) and (d); ie intuition suggests that these issues are not artificially created by the example.

In parallel, it is planned to study larger, realistic networks, and more detailed traffic/control models, using a simulation framework currently under development (Liu et al. 1994). This approach combines a detailed representation of driver behaviour, as this evolves within and between days, with a detailed traffic simulation of individual vehicles and complex signal policies. It is therefore possible to study the performance of signals responding dynamically to serve a given demand, in the context of the longer-term evolution of driver choices, as well as studying novel policies such as P0. Strategies for dealing with the problems identified in (d) may be investigated, such as: starting the simulation from current conditions/signal-timings, when testing hypothesised "do-something" schemes/ policies; and using "common random numbers" to reduce the impact of the seed value (Rathi 1992). More generally, the whole philosophy for evaluating policies/networks may be re-evaluated—in particular, should our results be explicitly time-horizon specific?

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