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TOPIC 25
URBAN AND LOCAL
TRAFFIC MANAGEMENT

ON A FUNCTIONAL FORM FOR TRAFFIC-FLOW RELATIONSHIPS

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Abstract

A functional form for the speed-density relationship is presented in this work. The functional form is characterised by a dimensionless function, the generating function, whose argument is a dimensionless headway, the equivalente headway. The functional form is derived by means of two arguments. The first one arises from the set of properties that the speed-density relationship should satisfy and the second from the driver's behaviour.

INTRODUCTION

It has been sufficiently contrasted experimentally that vehicle speed and traffic density are correlated under time-stationary and space-homogeneous traffic flow. Since the work of Greenshields (1935) numerous speed-headway models have been suggested. One of the most popular relationships has been derived from car-following models:

$$V = V_f\left(1-\left(\frac{K}{K_j}\right)^m\right)^n \tag{1}$$

where V and K are the speed and density, V_f is the free flow or maximum speed and K_j is the jam density. Another speed-density models include exponential and logarithmic relationships such as that of Greenberg (1959)

$$V = -V_c \ln\left(\frac{K}{K_j}\right) \tag{2}$$

Drake et al. (1967)

$$V = V_f \exp\left(-\frac{1}{2}\left(\frac{K}{K_j}\right)^2\right) \tag{3}$$

and Underwood (1961)

$$V = V_f \exp\left(-\frac{K}{K_c}\right) \tag{4}$$

where V_c and K_c are the values of speed and density at which maximum flow or capacity is reached.

Any realistic speed-density relationship should satisfy the following properties:

- the values of the speed go from zero to a maximum called *free flow speed,* V_f;
- the values of traffic density lie between zero and a maximum referred to as *jam density*, K_j;
- when the headway tends to infinity, or the density to zero, the speed approaches the free flow speed, thus $V(0) = V_f$;
- vehicles stop at jam density, $V(K_j) = 0$;
- the speed is a decreasing function of the density, $V'(K) < 0$, where (') stands for the derivative with respect to density;
- as traffic flow becomes lighter, the interactions between drivers vanish and so does the dependence of speed on density, thus $V'(0) = 0$.

These properties where already proposed by Greenshields. They can be deduced by considering the traffic flow as a stationary phenomenon. There is still an important property of the speed-density relationship concerning the traffic flow dynamics. The flow-density curve should be concave so that the traffic flow continuity equation may only yield braking shock waves. The mathematical condition is:

$$Q''(K) < 0 \tag{5}$$

where $Q = KV$ is the traffic flow. The explanation of this property may be found in Del Castillo et al. (1994a). We will call this property 'concavity condition'. Ansorge (1990) has given a physical interpretation of the concavity condition in terms of an entropy condition for the continuity equation. The entropy condition allows to select out of all the possible solutions of the continuity equation, the physically relevant one.

It is rather striking that none of the speed-density models mentioned before entirely fulfils the above properties. Therefore, the search of a continuous and differentiable speed-density relationship is an open question that will be addressed in the following pages.

THE FUNCTIONAL FORM

The existence of three fundamental parameters of traffic flow was first established by Franklin (1965). He postulated that the speed-density relationship was determined by three main factors: the jam density, the free flow speed and the kinematic wave speed at jam density, C_j. This last parameter is defined as the slope of the flow-density curve evaluated at the jam density:

$$C_j = Q'(K_j) \tag{6}$$

The kinematic wave speed at jam density is roughly the speed of the braking and starting waves generated by a traffic light.

The values of the jam density and the kinematic wave speed at jam density are weakly influenced by the characteristics of the road. They are mainly determined by the vehicle size, the jam density, and also by the reaction time of drivers, the kinematic wave speed at jam density. Typical mean values are K_j=150 veh/km/lane and C_j=-20 km/h, see Del Castillo et al. (1994b). On the contrary, the free flow speed is strongly conditioned by the road type and its value can have a wide range. Hence, it seems reasonable to choose the parameters K_j and C_j for nondimensionalizing the traffic flow variables. We then define:

$$u = \frac{V}{|C_j|}, \quad \rho = \frac{K}{K_j}, \quad q = \rho u = \frac{Q}{K_j |C_j|}, \quad u_f = \frac{V_f}{|C_j|}. \tag{7}$$

In the above variables the properties of the traffic flow relationships become:

$$u(0) = u_f \tag{8}$$

$$u(1) = 0, \tag{9}$$

$$u'(\rho) < 0, \quad 0 < \rho < 1, \tag{10}$$

$$u'(0) = 0 \tag{11}$$

$$q''(\rho) < 0, \quad 0 < \rho < 1, \tag{12}$$

where the derivatives are with respect to the nondimensional density, ρ. Moreover, the following property should be added due to the nondimensionalization:

$$u'(1) = q'(1) = -1 \tag{13}$$

The speed-density and flow-density curves in dimensionless form are plotted in Figure 1.

The problem is then to find a continuous an differentiable function $u(\rho)$ satisfying the properties (8-13). The naive solution to this problem would be to try a polynomial function and conveniently adjust the coefficients of such a polynomial. In practice, it turns out impossible to find a speed-density relationship by this method because the conditions for the polynomial coefficients are too cumbersome.

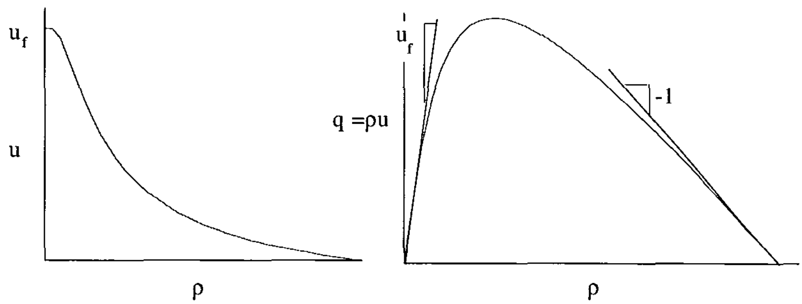

Figure 1 Speed-density and flow-density curves

If we restrict the search of a speed-density relationship to those having the following functional form

$$u = u_f \left[1 - f\left(\frac{1}{u_f}\left(\frac{1}{\rho} - 1 \right) \right) \right] \tag{14}$$

the problem becomes solvable. Writing

$$\lambda = \frac{1}{u_f}\left(\frac{1}{\rho} - 1 \right) \tag{15}$$

the function $f(\lambda)$ will be called *generating function* and its argument, λ, *equivalent headway*, since it is a nondimensional headway.

It is easy to see that the generating function should satisfy the following properties:

$$\lim_{\lambda \to \infty} f(\lambda) = 0, \tag{16}$$

$$f(0) = 1, \tag{17}$$

$$\dot{f}(\lambda) < 0, \quad \lambda > 0, \tag{18}$$

$$\lim_{\lambda \to \infty} \lambda^2 \dot{f}(\lambda) = 0 \tag{19}$$

$$\ddot{f}(\lambda) > 0, \quad \lambda > 0, \tag{20}$$

$$\dot{f}(0) = -1, \tag{21}$$

$$0 < f(\lambda) < 1, \quad \lambda > 0, \tag{22}$$

where $(\cdot)$ means derivative with respect to lambda.

EXAMPLES OF GENERATING FUNCTIONS

In this section, several examples of generating functions are presented. It is important to remark that the transformation given by the functional form (14) does not provide a constructive method to find generating functions. Instead, an analytical expression for the generating function must be a priori suggested, and it should be doublecheked if it satisfies the properties (16-22). In such a manner, we have found up to four types of generating functions that we have called: exponential, double-exponential, rational and reciprocal-exponential.

The exponential generating function is given by:

$$f(\lambda)=\exp\left[1-\left(1+\frac{\lambda}{n}\right)^{n}\right] \tag{23}$$

with $n>0$. Two special cases are, for $n=1$:

$$f(\lambda)=\exp(-\lambda) \tag{24}$$

and for $n\rightarrow\infty$:

$$f(\lambda)=\exp[1-\exp(\lambda)] \tag{25}$$

The double exponential family may be obtained by introducing a parameter in the above generating function:

$$f(\lambda)=\exp\left[n\left(1-\exp\left(\frac{\lambda}{n}\right)\right)\right] \tag{26}$$

with $n\geq 1$. The rational generating functions have the following form:

$$f(\lambda)=\left(\frac{\lambda}{n}+1\right)^{-n} \tag{27}$$

with $n>1$. Finally, the reciprocal-exponential family has the expression:

$$f(\lambda)=\frac{n}{\exp(n\lambda)+n-1} \tag{28}$$

where $0<n\leq 2$.

The speed-headway curves given by expressions (23-28) are shown in Figure 2. The y-axis represents the nondimensional speed v defined as:

$$v=\frac{V}{V_f}=1-f(\lambda) \tag{29}$$

It is quite remarkable that all the speed-headway curves are bounded by the curve given by (25), which approaches the idealized two-linear regime: $v=\lambda$, if $\lambda<1$ and $v=1$ otherwise.

Further, of all the proposed speed-density curves, only one may be found in the literature. The curve generated by the model (24) is:

$$V=V_f\left[1-\exp\left(\frac{|C_j|}{V_f}\left(1-\frac{K}{K_j}\right)\right)\right] \tag{30}$$

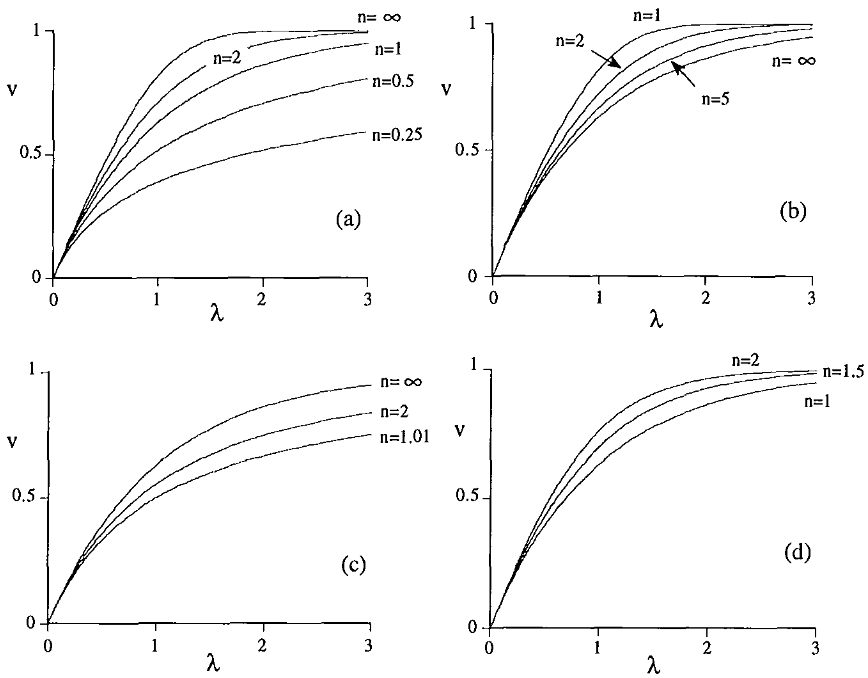

Figure 2 **Nondimensional speed vs. equivalent headway for the exponential (a), double exponential (b), rational (c) and reciprocal-exponential (d) generating functions**

This curve has an interesting history: it was first formulated independently by Newell (1961) and Franklin (1961) as the steady state solution of a car-following model. The curve is later mentioned in the work of Leutzbach and Bexelius (1966) and in the book of Valdés (1971). Since then, it vanishes from the traffic flow literature. That makes one wonder why something so simple and appealing as the exponential function has gone unnoticed for more than 25 years, or if it was known, why it was not exploited.

THE DERIVATION OF THE FUNCTIONAL FORM

In this section, the functional form given by (14) and (15) will be derived by using two different arguments: the first one is of mathematical type and the second is based on the drivers' behavior.

Mathematical argument

The mathematical argument justifies why the proposed functional form simplifies the problem of finding an analytical expression for the speed-density relationship. One could try a generic transformation $\lambda = \lambda(\rho)$ to try to simplify such a problem. Then, differentiating with respect to the density in the expression of the speed

$$u = u_f\left[1 - f(\lambda(\rho))\right] \tag{31}$$

yields the following expressions

$$u'(\rho) = -u_f \dot{f}(\lambda)\lambda', \tag{32}$$

$$q''(\rho) = -u_f[\rho\ddot{f}(\lambda)\lambda'^2 + (\rho\lambda'' + 2\lambda')\dot{f}(\lambda)] \tag{33}$$

where

$$\lambda' = \frac{d\lambda}{d\rho} \tag{34}$$

The choice of a function $\lambda(\rho)$ that satisfies

$$\rho\lambda'' + 2\lambda' = 0 \tag{35}$$

eliminates the dependence of $q''(\rho)$ on $\dot{f}(\lambda)$ and simplifies the final expressions of properties (18) and (20). The solution of equation (35) with the conditions $\lambda(1) = 0$ and $\lambda'(1) = -1/u_f$ leads to the proposed expression of the equivalent headway (15). The choice of the initial conditions for (35) may seem arbitrary but had we chosen another conditions, we could have obtained the same solution for λ by making an appropriate change of variables. Therefore the proposed functional form is the only one that simplifies the properties of the generating function.

Driver's behavior based argument

Next, it will be demonstrated that the functional form given by (14) and (15) may be obtained as the steady state solution of a car-following model. A dimensionless generic car-following model may be written as:

$$\frac{du}{d\tau} = s(u, h, \frac{dh}{d\tau}; u_f) \tag{36}$$

where u is the speed of the follower vehicle, $h = K_j / K$ is the headway between the follower and the lead vehicle and $\tau = t|C_j|K_j$ is a nondimensional time. The function s is the sensitivity of the drivers with respect to the speed, the headway and the relative speed. When traffic flow approaches equilibrium, the headway becomes a function of the speed and the fluctuations of the headway are small in comparison with the mean speed. Thus, the above model may be approximated by

$$\frac{du}{d\tau} = s(u; u_f)\frac{dh}{d\tau} \tag{37}$$

The equilibrium solution of this model is therefore given by :

$$\frac{du}{dh} = s(u; u_f) \tag{38}$$

However, the sensitivity cannot depend separately on u and u_f, since it would imply to depend on the kinematic wave speed at jam density, C_j. This parameter is a macroscopic feature of traffic flow and it would be absurd to admit that the drivers are aware of the waves they generate when accelerating or decelerating. Therefore, the dependence of the sensitivity on C_j should be eliminated. The only way to do that is by supposing a particular functional dependence of the sensitivity on the speed, namely:

$$s(u; u_f) = s\left(\frac{u}{u_f}\right) = s\left(\frac{V}{V_f}\right) \tag{39}$$

Then, the sensitivity would be a function of the ratio between the mean speed and the maximum speed. It is completely reasonable to think that the drivers consider such a ratio as a driving parameter.

Substituting equation (39) in (38) and integrating with the condition $u(h=1)=0$, one obtains:

$$R\left(\frac{u}{u_f}\right) - R(0) = \frac{h-1}{u_f} \tag{40}$$

Solving for u and taking into account that $h = 1/\rho$, the functional form of (14) and (15) is finally obtained, since the constant $R(0)$ is irrelevant.

APPLICATION OF THE THEORY

In order to analyze the performances of the speed-density models previously proposed, a regression analysis was carried out. Prior to this analysis, a procedure for selecting equilibrium traffic data was applied to the available data. The aim of the selection procedure was to guarantee that the data correspond to equilibrium traffic. The original traffic data utilized were collected from a three-lane stretch of the freeway A2 Amsterdam-Utrecht in the Netherlands. The data were taken during the morning rush hour (6.30 to 9.30) of five weekadys. They consist of measurements of number of vehicles, mean speed and mean square speed taken on each lane over 30-seconds intervals. These quantities will be denoted as N_i, $\overline{V}_i$ and $\overline{V}_i^2$ respectively, for the i-th period.

We looked for potentially stationary periods lasting at least 4 or 5 minutes. Thus, periods over which the speed variance was significative in comparison to the mean speed were directly discarded. The imposed duration,' about five minutes, was considered as the necessary minimum to accurately estimate the traffic mean variables. These variables would be estimated averaging over the stationary periods. Hence, we needed a great number of such a periods for each detector. Only two groups of three detectors showed a sufficient number of potentially stationary periods, about 15 at least. These detectors have been named according to their location, U for upstream and D for downstream, and to the lane, L for the left lane, C for the center lane and R for the right lane. Then, they are: UL, UC, UR and DL, DC, DR.

Once we have selected the potentially stationary periods by visual inspection of the graphs, a stationarity test based on the Kendall's τ test was applied to the series of the mean square speed in each period. The hypothesis tested was

H_0: no trend exists in the sample $\overline{V}_i^2, i = 1 \ldots n$

against the alternative of the existence of a trend. A detailed description of the application of the test is given in Del Castillo et al. (1994c). The test is applied sequentially by decreasing the sample size n until a sample is accepted as stationary. This procedure ensures that the longest stationary period is selected. Next, the same test is applied to the series of the number of vehicles.

The mean vehicle number in each stationary period is estimated as the arithmetic sample mean:

$$\hat{N} = \frac{1}{n}\sum_{i=1}^{n} N_i \tag{41}$$

The mean speed that we should consider is the harmonic mean, since the traffic flow and the traffic density are related through it. Although we do not have the reciprocal of the individual speeds, we may approximate the harmonic sample mean by means of the arithmetic sample mean and the sample variance as follows:

$$\hat{V}_e = \frac{\hat{V}^3}{\hat{V}^2 + \hat{\sigma}^2} \tag{42}$$

where $\hat{V}$ is the arithmetic sample mean and $\hat{\sigma}^2$ is the sample variance. Finally, the estimate of the headway in meters is:

$$\hat{H} = \frac{1000\hat{V}_e}{120\hat{N}} \tag{43}$$

for the speed in km/h and the volume in vehicles per 30 seconds.

RESULTS OF THE REGRESSION ANALYSIS

The most common regression model assumes an additive independent and identically distributed normal error. For the sake of simplicity, we chose such a regression model and took the headway as the control variable. This last choice is somewhat arbitrary. The model may be written as:

$$\hat{V}_e = V_f\left[1 - f\left(\frac{|C_j|}{V_f}\left(\frac{\hat{H}}{H_j} - 1\right)\right)\right] + \varepsilon \tag{44}$$

with

$$\varepsilon \approx N(0, \sigma_\varepsilon^2) \tag{45}$$

The choice of a generating function is of crucial importance for the regression model. We have chosen the generating function given by (24) and (25) since these two curves differ sufficiently as to enable us to clearly select the best fit between them. Thus, for each of these curves and for each detector, the estimates of the parameters of the regression model, $V_f, |C_j|$ and $H_j = 1/K_j$, were calculated by minimizing the square error. We have preferred the regression on the headway instead of that on the density because the parameters of the model have a clearer interpretation in terms of speed and headway. Nevertheless, both regression models would have given the same estimates.

We will refer to the chosen speed-headway curves as exponential curve, corresponding to the generating function (24), and maximum sensitivity curve, that corresponding to (25). This name is justified by the fact that the sensitivity $s(u/u_f)$ reaches its maximum for a given value of the speed over the remaining speed-headway curves.

For the right lane detectors, LR and UR, the regression analysis was carried out assuming the exponential and the maximum sensitivity curves. However, the speed-headway observations plots of the center and left lane detectors showed an abrupt change of slope at the transition from the congested regime to the uncongested regime. It was obvious that the exponential curve had not properly fitted the observations. Then, we adopted a two-linear regression as an alternative model to the maximum sensitivity curve.

The goodness of fit of the models was mainly assessed by the values of the estimates of the parameters. For the model to make sense, the estimates should yield realistic values. Then, for instance, a value too large of the estimate of the jam density, say 250 veh/km would have implied the rejection of the model. The classical methodology involves the examination of the residuals. In our case this is not a good method since the sample size is rather limited. Nevertheless, the residuals have been used as a secondary criterion.

For the right lane detectors, the fit achieved by the exponential curve is superior to that of the maximum sensitivity curve. The estimates given by this last model are undoubtedly unrealistic. The values of the estimates are given in Tables 1 and 2, for both regression models and for each detector. Figures 3 and 4 show the resulting speed-headway curves and the observations, that is, the mean variables obtained by averaging over the stationary periods.

Table 1 Results of the regression analysis for detector DR

	exponential curve	maximum sensitivity curve
V_f	86.4 km/h	74.88 km/h
$\lvert C_j \rvert$	11.92 km/h	5.86 km/h
K_j	161.75 veh/km	230.4 veh/km
σ_ε	1.96 km/h	2.65 km/h

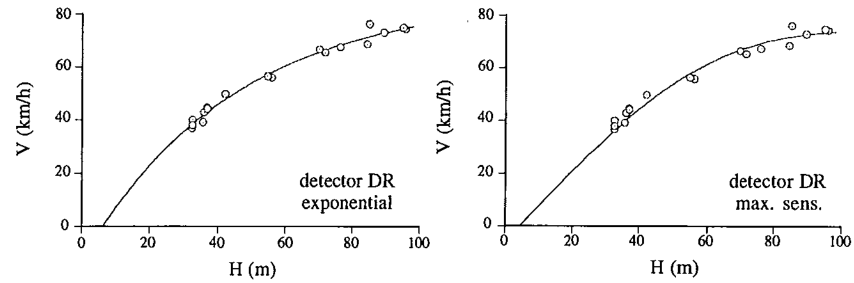

Figure 3 Speed-headway curves and observations for detector DR

Table 2 Results of the regression analysis for detector UR

	exponential curve	maximum sensitivity curve
V_f	106.85 km/h	94.85 km/h
$\lvert C_j \rvert$	21.22 km/h	6.14 km/h
K_j	123.79 veh/km	274.73 veh/km
σ_ε	3.1 km/h	3.47 km/h

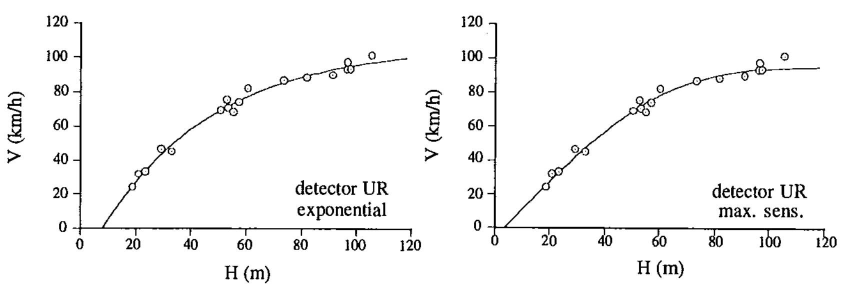

Figure 4 Speed-headway curves and observations for detector UR

As for the center lane detectors, the fit achieved by the maximum sensitivity curve is excellent. The two-linear regression models do not adequately fit the data, but they give an estimate of the transition point between the congested and uncongested regime. The results are given in Tables 3 and 4. The two-linear regression model does not provide an estimate of the free-flow speed, V_f. Instead it provides two different values of the error standard deviation, σ_ε, the first one being that of the congested regime (high densities) and the second one that of the uncongested regime (low densities). The resulting speed-headway curves are plotted in Figures 5 and 6.

Table 3 Results of the regression analysis for detector DC

	maximum sensitivity curve	two-linear model
V_f	113 km/h	-------
$\|C_j\|$	17.98 km/h	11.3 km/h
K_j	147.77 veh/km	202.2 veh/km
σ_ε	2.21 km/h	1.5 and 2.1 km/h

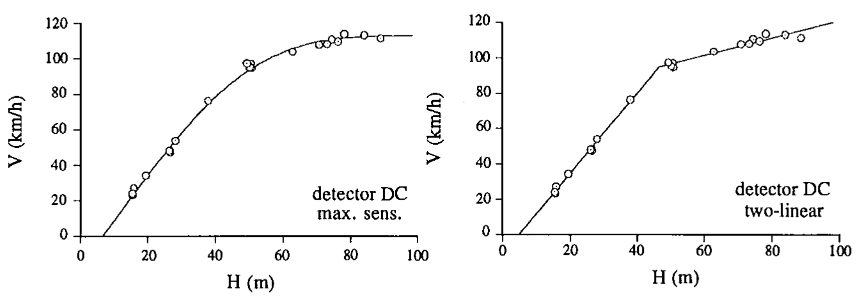

Figure 5 Speed-headway curves and observations for detector DC

Table 4 Results of the regression analysis for detector UC

	maximum sensitivity curve	two-linear model
V_f	110.4 km/h	-------
$\|C_j\|$	19.8 km/h	2.3 km/h
K_j	143.51 veh/km	871.68 veh/km
σ_ε	3.68 km/h	4.74 and 2.29 km/h

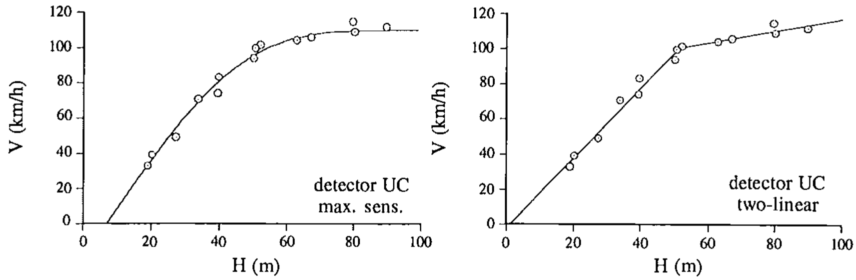

Figure 6 Speed-headway curves and observations for detector UC

Finally, the maximum sensitivity regression model does not fit in a realistic manner the observations of the left lane detectors, DL and UL. The values of $|C_j|$ are clearly overestimated. On the contrary, the two-linear model provide a fairly good description of the observations. The results are presented in Tables 5 and 6, and in Figures 7 and 8.

Table 5 Results of the regression analysis for detector DL

	maximum sensitivity curve	two-linear model
V_f	122 km/h	-------
$\|C_j\|$	34.94 km/h	25.67 km/h
K_j	100.74 veh/km	117.14 veh/km
σ_ε	3.74 km/h	2.63 and 2.31 km/h

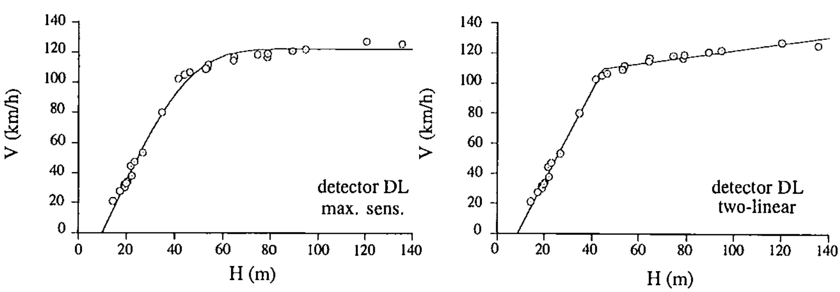

Figure 7 Speed-headway curves and observations for detector DL

Table 6 Results of the regression analysis for detector UL

	maximum sensitivity curve	two-linear model
V_f	126 km/h	-------
$\|C_j\|$	38.85 km/h	18.99 km/h
K_j	100.74 veh/km	138.73 veh/km
σ_ε	5.25 km/h	2.8 and 2.63 km/h

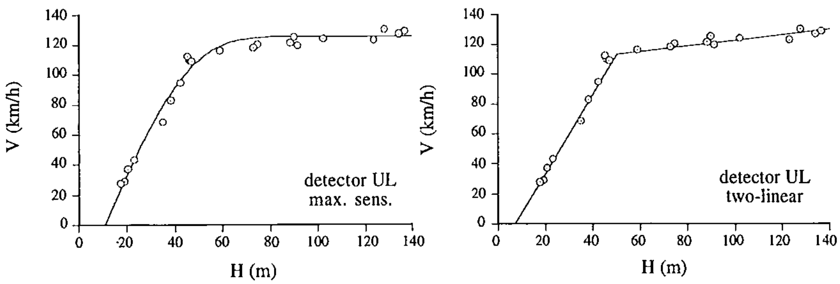

Figure 8 **Speed-headway curves and observations for detector UL**

In conclusion, the regression models that yield the best goodness of fit are the exponential for the right lane, the maximum sensitivity for the center lane and the two-linear model for the left lane detectors. The asymptotic bias of the estimates was calculated according to the expression given in Ratkowsky (1983). The asymptotic bias of the estimates turned out to be extremely low, in fact below 1%, when expressed in percentage of the estimate value. Following Ratkowsky, a percentage above 1% appears to be a good rule of thumb for indicating nonlinear behavior of the estimates. This fact indicates that the parameterization of the regression model in terms of V_f, $|C_j|$ and K_j renders a close to linear regression model. In other words, a reparameterization of the model is not necessary. Another nice feature of the exponential and maximum sensitivity models is the easiness for guessing initial estimates that are very close to the final estimates. This is very important in an iterative procedure such as that required for finding the least squares estimates of a nonlinear regression model.

CONCLUSIONS

In this work, a functional form for the speed-density relationship has been formulated. This functional form is given in a dimensionless form by a function called *generating function* whose argument is a dimensionless headway, named *equivalent headway.* Two different arguments lead to the proposed functional form. The first one consists on a mathematical trick that simplifies the set of properties that the speed-density relationship should satisfy. The second argument is based on the drivers' behavior. The functional form is obtained by applying the dimensional analysis to a generic car-following model. The steady state solution of such a model leads to the functional form.

Several examples of generating functions have been presented. Only one of the corresponding speed-density curves had been previously proposed in the traffic flow literature. Two speed-density curves have special interest: the exponential curve and the maximum sensitivity curve. These two curves have been used as a regression model for fitting data collected at a three lane freeway.

The goodness of fit achieved by the exponential and maximum sensitivity models is excellent for the right and center lane detectors, respectively. However, a two-linear regression model is necessary to explain the observations from the left lane detectors. Hence, the proposed speed-density models seem very adequate provided that the free-flow speed doest not exceed a value about 115 km/h. Above this value, the sudden change of slope from the congested to the uncongested regime, cannot be fitted by the new speed-density models.

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