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FORMATION OF TRAFFIC JAMS CAUSED BY FLUCTUATIONS AND RANDOM PROCESSES IN TRAFFIC FLOW

BORIS S. KERNER
Research Institute
Daimler-Benz AG, F1V/VT, HPC: G253
70546 Stuttgart, GERMANY

PETER KONHÄUSER
Research Institute
Daimler-Benz AG, F1V/VT, HPC: G253
70546 Stuttgart, GERMANY

MARTIN SCHILKE
Research Institute
Daimler-Benz AG, F1V/VT, HPC: G253
70546 Stuttgart, GERMANY

Abstract

A kinetic traffic flow model, where fluctuations as well as the effects of on-ramps and off-ramps are taken into account, is proposed. Based on this model, random processes of a spontaneous formation of traffic jams both in an initially homogeneous and in an inhomogeneous traffic flow are investigated.

INTRODUCTION

Numerous experimental investigations of traffic flow have shown that traffic jams can spontaneously appear (Treiterer, 1975; Leutzbach, 1988), when the density of vehicles in traffic flow becomes relatively high. The experimental features and characteristics of traffic jams have recently been found in Kerner and Rehborn (1996).

The phenomenon of "phantom traffic jam" in a homogeneous traffic flow may be explained by the local cluster effect (Kerner and Konhäuser, 1994). The local cluster of vehicles is a localized structure in traffic flow which can be formed in the initially homogeneous traffic flow, if both the density of vehicles in this flow exceeds a boundary (threshold) density and a localized fluctuation appears which amplitude exceeds some critical value. The results of the theory of traffic jams obtained in Kerner and Konhäuser (1994) based on a macroscopic traffic flow model have been confirmed by the investigations of microscopic traffic flow models (Schreckenberg et al. 1995; Nagel and Paczuski, 1995; Bando et al. 1995).

An ideal homogeneous traffic flow is obviously a hypothetical state of traffic flow: Different kind of random processes which for example are linked to a lane changing or to entering or exiting of vehicles to on-ramps or off-ramps, etc., may have a considerable influence on both a spontaneous appearance and properties of traffic jams. In this article a kinetic traffic flow model, where fluctuations as well as the effects of on-ramps and off-ramps are taken into account, will be proposed. Based on this model, processes of a spontaneous formation of traffic jams and their properties both in an initially homogeneous and in an inhomogeneous traffic flow will be investigated and compared.

KINETIC MODEL OF TRAFFIC FLOW

Basic equations

In a kinetic approach (Prigogine and Herman, 1971; Payne, 1971; Whitham, 1974; Leutzbach, 1988; Kühne, 1991), the kinetic model of traffic flow under consideration is described by the continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0, \tag{1}$$

the equation of motion (Kerner and Konhäuser, 1993):

$$\rho\left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right] = \rho[V(\rho) - v] - c_0^2\frac{\partial \rho}{\partial x} + \frac{\partial^2 v}{\partial x^2} \tag{2}$$

and the boundary conditions:

$$q(0,t) = q(L,t),\ v(0,t) = v(L,t) \text{ and } w(0,t) = w(L,t). \tag{3}$$

In (1)—(3) $\rho(x,t)$ is the density $(0 < \rho \le 1)$ and $v(x,t)$ is the average speed $(v \ge 0)$,

$$w = \frac{\partial v}{\partial x}; \tag{4}$$

$$q(x,t) = \rho(x,t) \cdot v(x,t) \tag{5}$$

is the flux of vehicles in traffic flow; V is a safe ("maximal and out of danger") speed which is achieved in a time-independent and homogeneous traffic flow. In (1)—(5) v, V and c_0 are measured in units of ℓ/τ, the length in units of ℓ, the density ρ in units of $\hat{\rho}$ ($\hat{\rho}$ is the maximal possible density of vehicles on the road; for n-lanes road $\hat{\rho} = n/\hat{a}$, where $\hat{a}$ is an average length of vehicles), the time in units of τ, the value w in units of $1/\tau$, the flux q is measured in units of $\hat{\rho}\ell/\tau$, $\ell = \sqrt{\mu\hat{\rho}^{-1}\tau}$; τ is the characteristic relaxation time of the speed v to the "maximal and out of danger" speed V; $V(\rho)$ is a monotonous decreasing function of ρ, ie., its derivative is (Prigogine and Herman, 1971; Whitham, 1974; Leutzbach, 1988):

$$\xi(\rho) = \frac{dV(\rho)}{d\rho} < 0\,; \tag{6}$$

$\tau = \text{const}$; $c_0 = \text{const}$; (Payne, 1971; Whitham, 1974); $\mu = \text{const}$ (Kühne, 1991); L is the length of a road.

The equation of motion (2) formally follows from the Navier-Stokes equations, or to be more precisely, from the "Navier-Stokes-like" equation for traffic flow (Kerner and Konhäuser, 1993):

$$\rho\left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right] = X - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right), \tag{7}$$

where $X = \rho \cdot (V(\rho) - v)/\tau$ (Payne, 1971; Whitham, 1974; Leutzbach, 1988; Kühne, 1991) represents the sum of all inner "forces", which appear due to interactions between individual vehicles. The terms on the right-hand side of Equation (7) look like the forces which enter the Navier-Stokes equations. However, in traffic flow the nature and the meaning of these terms are completely different compared with classical physical systems. In the latter systems for example the pressure p as well as the viscosity μ appear due to the variance of the velocity distribution, ie., due to the temperature of a gas. In traffic flow, contrarily, the terms $\partial p/\partial x$ and $\partial/\partial x \cdot (\mu \cdot \partial v/\partial x)$ are even present when the variance of the speed distribution of vehicles were zero. These terms are present due to the perception, decision making and action of drivers in the presence of nonhomogeneities in the density and the average speed of vehicles. Therefore, both terms $\partial p/\partial x$ and $\partial/\partial x \cdot (\mu \cdot \partial v/\partial x)$ can be considered as some kind of anticipation factors. For example the term $-\partial p/\partial x$ causes acceleration of drivers if the density ρ decreases and slowing down when the density increases (eg Kühne, 1991). This means that the gradient of the pressure $\partial p/\partial x$ should have the same sign as the gradient of the density: $\partial p/\partial x = c_0^2 \cdot \partial\rho/\partial x$, where $c_0^2 > 0$. The value c_0^2 may be, however, in two limit ranges of the density nearly equal zero (Kerner et al. 1995): (i) when the density everywhere on a road is so low that all drivers move practically with their "free speed" v_f and, therefore, do not react on a change in density; (ii) when the average speed due to very high density is nearly zero and drivers cannot move even if the gradient of the density is not equal zero. A variety of functions $c_0^2(\rho)$ which fulfils the conditions (i) and (ii) mentioned above have been implemented in Equation (2) and tested during the numerical investigations of traffic flow. It has been found that in some cases there can be some important differences compared to the case $c_0^2 = \text{const}$. However, for the investigations of traffic jams considered in (Kerner and Konhäuser, 1993, 1994) and in this article, it has been found that one can use in Equation (2) $c_0^2 = \text{const}$ for the following reasons:

The range of density (i) is not relevant for $\rho \geq \rho_b$ (Kerner and Konhäuser, 1994), where traffic jams can be formed. The range of density (ii) is usually realized inside traffic jams, but there we have $\partial\rho/\partial x = 0$ and therefore the product $c_0^2 \cdot \partial\rho/\partial x$ is equal to zero even when $c_0^2 > 0$. In an intermediate range of density, where $c_0^2 > 0$, a possible dependence of the value c_0^2 of the density, as it follows from investigations made, is not important for the qualitative results presented. Therefore, for investigation of traffic jam formations it is possible to express the local pressure p in Equation (7) by $p = c_0^2 \cdot \rho$.

The value c_0 may be considered as the velocity of the propagation of a possible or of a necessary reaction (ie., a change in acceleration or in slowing down) of drivers which is caused by some local change in traffic flow down-stream. In other words, the delay time τ_r of this reaction of the drivers on the local change in traffic flow which is situated on a distance Δx down-stream from the drivers is approximately equal to $\tau_r \approx \Delta x / c_0$. This interpretation of the value c_0 may be confirmed by two facts: (i) near the critical density ρ_{ci} (i=1, 2) of the instability of an initially homogeneous traffic flow, as it follows form the formula for the velocity of a propagation of small amplitude nonhomogeneous perturbations (Kerner and Konhäuser, 1993) $v_p = v_h - c_0$, the value c_0 is equal to this velocity v_p in a system of co-ordinates moving with the average speed of vehicles v_h ; (ii) correspondingly to the formula for the phase velocity v_p (see also Equation (9) below) and to the results of the investigations of instability of traffic flow (Kerner and Konhäuser, 1994), this propagation of local perturbations in a system of co-ordinates moving with the velocity v_h without sharp attenuation is only possible in the direction up-stream form a source of perturbations.

The value τ, strictly speaking, depends on the density: At low density interactions between vehicles are seldom. Therefore, the relaxation time τ is longer than at high density. This circumstance has additionally been taken into account in the kinetic model of traffic flow. However, as it follows from numerical investigations made in the range of the density $\rho \geq \rho_b$, which will be considered below it is found that the dependence $\tau(\rho)$ is not important for the qualitative results. For this reason and for simplifications $\tau = \text{const}$ has been used in all illustrations presented below.

Fundamental diagram

The function $V(\rho)$ in (2) is a phenomenological monotonous decreasing function of ρ. The related dependence on ρ of the traffic volume $Q(\rho) = \rho \cdot V(\rho)$ is obviously a function with only one maximum $Q = Q_{max}$ (Figure 1(b)). Traffic engineers call this relationship *fundamental diagram* (eg., Leutzbach, 1988). This fundamental diagram (Figure 1(b)) makes sense only for a homogeneous and time-independent traffic flow.

For given values of the total number of vehicles on the whole road N and the length of the road L, there is only one homogeneous and time-independent state ρ_h, v_h for the traffic flow under consideration: $\rho_h = N/L$, $v_h = V(\rho_h)$.

The corresponding flux (traffic volume) is $q_h = v_h \cdot \rho_h$. Therefore the given values N and L determine *one point* on the fundamental diagram $Q(\rho)$: $q_h = Q(\rho_h)$ and one point on the speed density relationship $V(\rho)$: $v_h = V(\rho_h)$. In (Kerner and Konhäuser, 1993, 1994) it has been shown that the given number N can represent not only the homogeneous state but also the non-homogeneous solutions in the form of local clusters of vehicles .

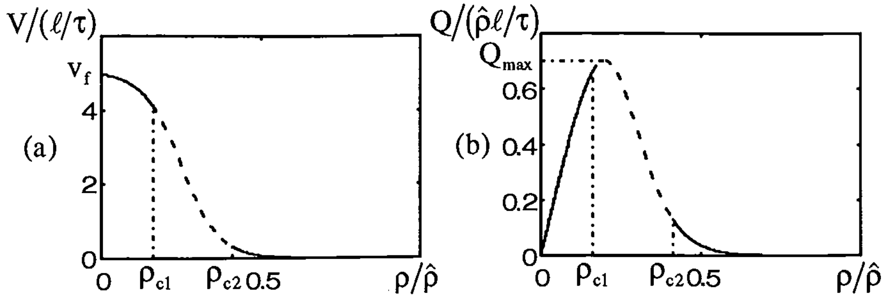


Figure 1 The example of the function $V(\rho)$ a) and of the corresponding fundamental diagram $Q(\rho)$ b). Results of the numerical computations for $V(\rho) = 5.0461\left\{\left[1+\exp\left(\left((\rho/\hat{\rho}) - 0.25\right)/0.06\right)\right]^{-1} - 3.72 \cdot 10^{-6}\right\}\ell/\tau$, $c_0 = 2.0497\,\ell/\tau$. The found critical values of density for $L = 800\ell$ are $\rho_{c1} \cong 0.1613\hat{\rho}$ and $\rho_{c2} \cong 0.4118\hat{\rho}$ (for the meaning of the densities ρ_{c1} and ρ_{c2} see Kerner and Konhäuser (1993)).

Stochastic macroscopic traffic flow model

To investigate the development of critical fluctuations in a vicinity of critical points of the density discussed above, it is possible to use a stochastic macroscopic traffic model which includes the continuity equation (1), the stochastic equation of motion

$$\rho\left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right] = \rho \cdot \left[V(\rho) - v\right] - c_0^2\frac{\partial \rho}{\partial x} + \frac{\partial^2 v}{\partial x^2} + \rho \cdot \zeta(x,t) \qquad (8)$$

and the conditions (3). In (8) the stochastic function $\zeta(x,t)$ describes the random component of acceleration (or slowing down) of vehicles and is measured in unites of ℓ/τ^2 .

A correlation length χ of the stochastic function $\zeta(x,t)$ can not be less than the value $1/\rho$, corresponding to an average distance of vehicles. On the other hand, fluctuations of an acceleration or a slowing down of vehicles being situated far enough from one another may be not correlated. A correlation time ψ of the stochastic function $\zeta(x,t)$ may depend on both the density and the average speed of vehicles. In particular, for small time-intervals an acceleration or slowing down partially correlates but does not correlate only for large time-intervals. These properties of the function $\zeta(x,t)$ have been taken into account in the algorithm, where a random number-generator has been used. In this case an amplitude of the function $\zeta(x,t)$ has been limited by some maximum value. Additionally, a correlation time-interval T as well as a road section of length χ, where the fluctuations are correlated, was taken into account, and it was assumed that the section of length χ moves with the speed v_h.

Stochastic macroscopic traffic flow model for inhomogeneous traffic flow

Due to entering and exiting traffic to on- and off-ramps traffic flow is, as a rule, inhomogeneous. In the kinetic model of traffic flow on a long road with I on- and J off-ramps, the continuity equation has to be modified

$$\partial\rho/\partial t + \partial q/\partial x = \sum_{i=1}^{I} g_i(x - x_i, t) - \sum_{j=1}^{J} r_j(x - x_j, t). \tag{9}$$

It can be assumed that expressions $g_i(x - x_i, t) = g_i^0(t)\cdot\varphi_i(x - x_i)$ and $r_j(x - x_j, t) = r_j^0(t)\cdot\phi_j(x - x_j)$, where the functions $\varphi_i(x - x_i)$ and $\phi_j(x - x_j)$ are strongly localized near the corresponding on- or off-ramp. The equation of motion (8) and the boundary conditions (3) complete the model. The terms on the right-hand side of Equation (9) include entering traffic g_i to the on-ramp "i" situated at $x = x_i$ and exiting traffic r_j from the highway to the off-ramp "j" being situated at $x = x_j$. Values of integrals $q_a^{(i)} = g_i^0(t)\cdot\int_0^L \varphi_i(x - x_i)dx$ or $q_d^{(j)} = r_j^0(t)\cdot\int_0^L \phi_j(x - x_j)dx$ correspond to fluxes $q_a^{(i)}$ and $q_d^{(j)}$ to on-ramp "i" or to off-ramp "j", respectively.

PHYSICS, STRUCTURES AND PROPERTIES OF TRAFFIC JAMS

In this section, based on the results of the original work by Kerner and Konhäuser (1994), a brief review of the physics, the structures and the properties of traffic jams which are formed in an initially homogeneous traffic flow will be given in this section.

Kinetics of traffic jam formation in initially homogeneous stable traffic flow

If a local perturbation appears in an initially homogeneous traffic flow, a local cluster of vehicles which is surrounded by the initial homogeneous flow can spontaneously be formed on a road long enough. As long as the local cluster does not reach one of the boundaries of the road, the latter can be considered as an "open" system.

In the numerical analysis of the kinetic model (1)—(3), a stable initially homogeneous traffic flow has been disturbed at $t = 0$ by the local perturbation

$$\Delta\rho(x) = \Delta\rho_m\left\{\cosh^{-2}\left(0.2(x - x_0)\right) - 0.25\cosh^{-2}\left(0.05\cdot(x - 25\ell - x_0)\right)\right\}. \tag{10}$$

If the amplitude $\Delta\rho_m$ of the local perturbation (10) exceeds some critical value $\Delta\rho_c$, the amplitude of the initial local perturbation (10) grows in time and a local cluster appears on the road (Figure 2(a)). The shape and the properties of the local cluster formed do not depend on the amplitude of this local perturbation. Contrarily, if the amplitude of the initial perturbation $\Delta\rho_m$ (10) is lower than $\Delta\rho_c$, the amplitude of the initial local perturbation fades in time.

The local initial perturbation first moves down-stream with only slightly increasing amplitude (Figure 2(c)). After some time $(\approx 36\tau)$, this local perturbation comes to a stop and its amplitude begins to grow rapidly (Figure 2(d)), forming a cluster of vehicles of large amplitude, ie the traffic jam (Figure 2(e)). Then, the developing cluster moves up-stream.

The width of the cluster L_s, ie the distance between the cluster's fronts, where the density and the average speed of vehicles sharply changes in space, monotonously increases in time. It occurs because the up-stream front of the cluster moves with a higher negative velocity v_{g1}

$\left(v_{gl} \approx -0.8448\,\ell/\tau\right)$ compared to the velocity v_{gr} of the down-stream front $\left(v_{gr} \approx -0.7174\,\ell/\tau\right)$. Therefore, the derivative

$$dL_s/dt = v_{gr} - v_{gl} \tag{11}$$

is positive. The increase in L_s is linked to the fact that the flux of vehicles through the up-stream front of the cluster, ie the flux into the cluster q_{in}, is stronger than the flux through the down-stream front, ie., the flux out of the cluster q_{out} $\left(q_{out} \approx 0.5609\hat{\rho}\,\ell/\tau\right)$ (Figure 2(f)):

$$q_{in} > q_{out}, \tag{12}$$

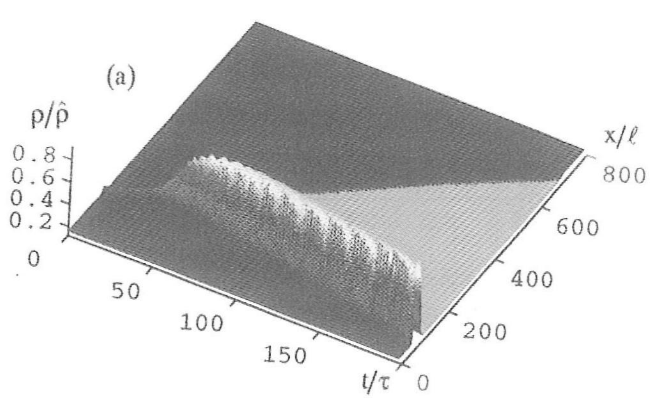
where q_{in} equals to q_h. The condition (12) means that the cluster of vehicles—a traffic jam—acts as some new local source of vehicles on the road moving up-stream which "stores" more and more vehicles in the course of time. This local source of vehicles forms down-stream it a new almost homogeneous traffic flow with smaller density ρ_{min} (Figure 2(a)). For this reason, far down-stream from the traffic jam, there moves a transition layer with positive velocity $v_w \cong \left(q_h - q_{out}\right)\cdot\left(\rho_h - \rho_{min}\right)^{-1}$ between the initially homogeneous traffic flow and the new homogeneous traffic flow formed by the traffic jam. Thus, the local cluster of vehicles is a non-stationary localised structure which consists of:

a) the proper cluster of vehicles, ie., the traffic jam;

b) the new homogeneous traffic flow formed by the traffic jam down-stream;

c) the transition layer between this new and the initially homogeneous flow moving down-steam (Figure 2(a)).

Furthermore, it is instructive to consider the vehicle trajectories on the t-x plane (Figure 3), corresponding to the local cluster shown in Figure 2(a). One can see in Figure 3 that the driver which has caused the local perturbation at $t = 0$ and at the distance $\approx 150\ell$ from the beginning of the road can always travel with high speed. Therefore, this driver is far away from the traffic jam he is responsible for. Consequently, this driver does not know anything about the traffic jam which he has "prepared" for the other drivers moving at some distance behind him.



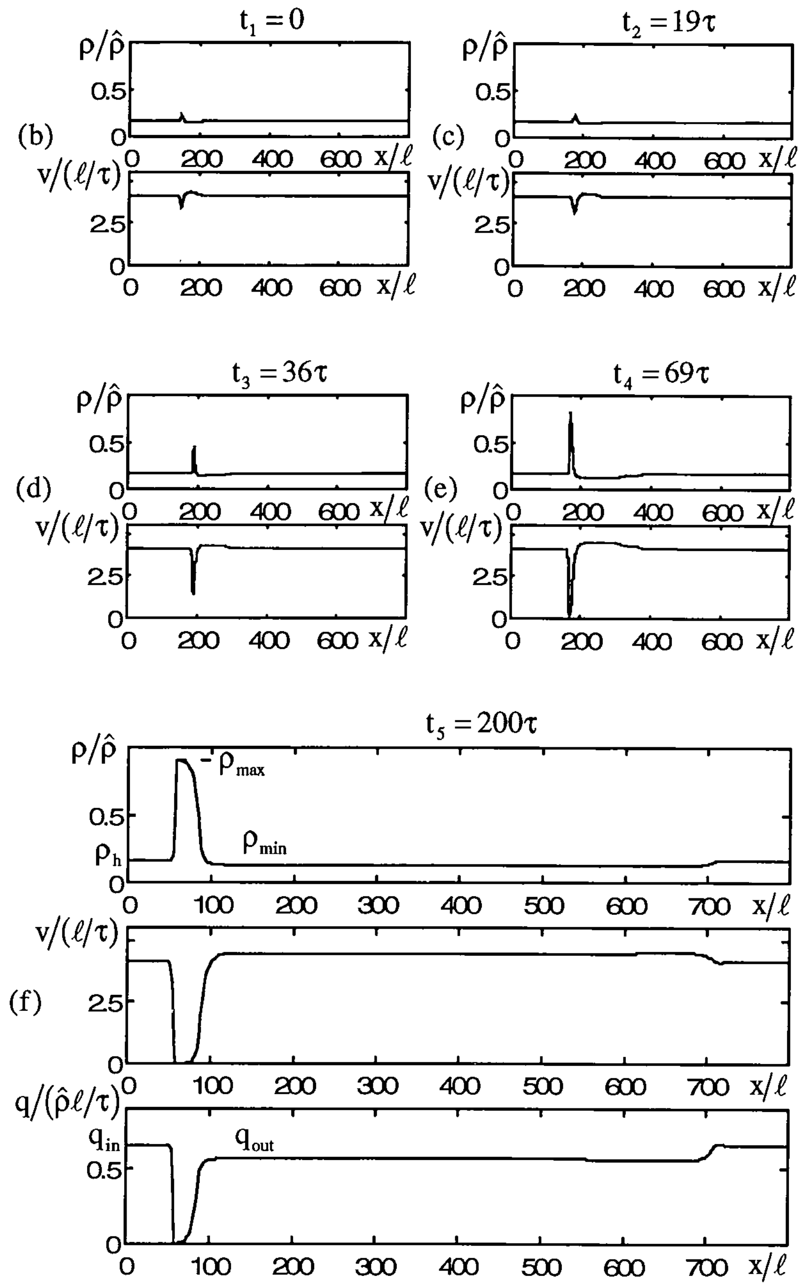


Figure 2 The kinetics of the local cluster formation: (a)—the dependence $\rho(x,t)$, (b, c, d, e, f)—the distributions $\rho(x)$, v(x) and q(x) (f) in the intermediate moments of time ((b)—$t_1 = 0$, (c)—$t_2 = 19\tau$, (d)—$t_3 = 36\tau$, (e)—$t_4 = 69\tau$); (f)—$t_5 = 200\tau$. The initial distribution $\rho(x,0) = \rho_h + \Delta\rho(x)$ with $\Delta\rho$ (12), $\Delta\rho_m = 0.06\hat{\rho}$, $\rho_h = 0.16\hat{\rho}$, $x_0 = 150\ell$. The other parameters are the same as in Figure 1.

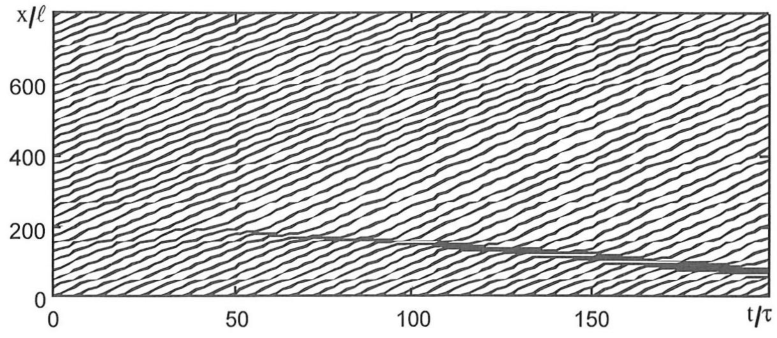


Figure 3 **The vehicle trajectories, corresponding to the local cluster shown in Figure 2(a)**

Traffic jam and fundamental diagram

In the local cluster of vehicles (Figure 2(a)), the values of density ρ, average speed v, and, consequently, the flux of vehicles $q = v \cdot \rho$ depend on position x and time t. It means that in the ρ -q phase plane the dependence $q(\rho)$, which corresponds to the distributions $v(x)$ and $\rho(x)$ at some fixed moment of time t, represent some closed curves. The local cluster formation, which corresponds to the kinetics shown in Figure 2, is shown on the ρ -q phase plane in Figure 4.

One can see that when the amplitude of the local perturbation comes to a stop and begins to grow rapidly (Figure 2 a, c, d)), the local perturbation on the ρ -q phase plane corresponds to the curve which is sharply deviated from the fundamental diagram (Figure 4(c, d)). The developed traffic jam (Figure 2(f)) on the ρ -q phase plane corresponds to the nearly stationary curve which has roughly the shape of a "triangle" (Figure 4(e)).

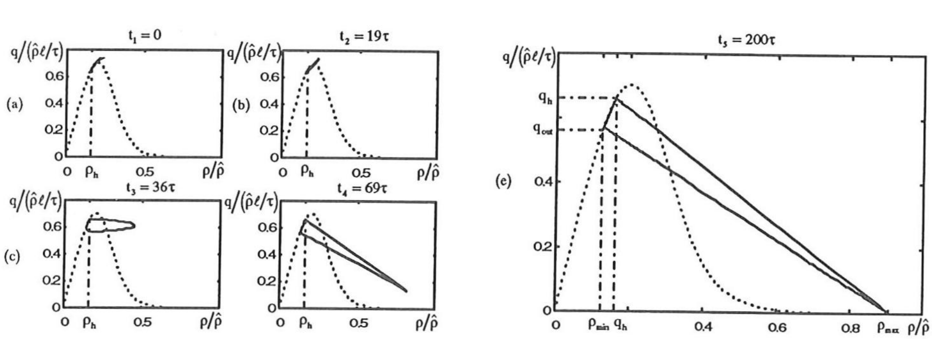


Figure 4 **The kinetic of the cluster formation in the ρ -q phase plane: (a, b, c, d, e)—the curves $q(\rho)$ at the same moment of time as shown in Figure 2(b, c, d, e, f), correspondingly. Dotted lines represent the fundamental diagram from Figure 1(b).**

Velocity and other parameters of traffic jams

When the width L_s of the traffic jam is large enough ($t > 200\tau$, Figures 2 and 3(e)), the average speed of vehicles in the traffic jam approaches $v_{min} \cong 0$ (and consequently the flux $q_{min} = v_{min} \cdot \rho_{max} \cong 0$), ie., the developed traffic jam corresponds practically to a standstill. Taking this into account and also that the developed traffic jam on the ρ -q phase plane represents the triangle (Figure 4(e)), where the upper line with negative slope corresponds to the up-stream front and the lower line with negative slope corresponds to the down-stream front of the traffic jam, one can get the approximate formulas:

$$v_{gr} = -q_{out} \cdot (\rho_{max} - \rho_{min})^{-1}, \; v_{gl} = -q_h \cdot (\rho_{max} - \rho_h)^{-1}, \tag{13}$$

where $q_{out} = v_{max} \cdot \rho_{min}$; ρ_{min} and $v_{max} \cong V(\rho_{min})$ are the density and the average speed of vehicles directly down-stream of the traffic jam, correspondingly. The flux q_{out}, or else the density ρ_{min} and ρ_{max} do not depend upon ρ_h. Therefore, the velocity v_{gr} also does not depend on ρ_h. On the contrary, the velocity v_{gl} becomes more and more negative the higher the density ρ_h is. For this reason, as it follows from (11), the width of the traffic jam

$$L_s(t) = L_s(t_0) + \left(\frac{q_h}{(\rho_{max} - \rho_h)} - \frac{q_{out}}{(\rho_{max} - \rho_{min})} \right) \cdot (t - t_0) \tag{14}$$

is the stronger increasing in time, the higher the density ρ_h is. In (14) $t_0 = \text{const}$. Emphasize that v_{gr}, ρ_{min}, q_{out} and v_{max} are the given characteristics, ie., intrinsic parameters of traffic. They do not depend on the density in the initial flow, on the length of the road and on the initial perturbation which growth leads to the formation of jams. These prediction of the theory (Kerner and Konhäuser, 1994) are in agreement with the results of experimental investigations (Kerner and Rehborn, 1996).

Boundary (threshold) density and boundary flux of traffic jam's existence

There is a boundary (threshold) value of the density ρ_b for an excitation of a traffic jam, ie., in a homogeneous flow there is a boundary flux $q_b = V(\rho_b) \cdot \rho_b$. At $\rho_h < \rho_b$ (ie at $q_h < q_b$), a local perturbation of any amplitude fades in time, ie a traffic jam cannot develop. On the other hand, from this qualitative consideration it follows that if a traffic jam would exist in the flow but up-stream of the traffic jam the density $\rho < \rho_{min}$, this traffic jam should disappear in the course of time. Indeed, in this case $q_h < q_{out}$ and correspondingly to (14) the width of the traffic jam L_s should monotonously decrease. Therefore, the density ρ_b should be close to the density ρ_{min} ; consequently, the boundary flux q_b for an excitation of the traffic jam is close to the flux q_{out} :

$$\rho_b \cong \rho_{min}, \; q_b \cong q_{out} \,. \tag{15}$$

Emphasize that the flux q_{out} is considerably lower than the flux Q_{max} (Figure 1(b)) which corresponds to the maximum point on the fundamental diagram.

It is necessary here to notice that if the density ρ_h equals ρ_b or only slightly exceeds it, narrow traffic jams, as a rule, are formed. Inside such traffic jams the average speed of vehicles v_{min} is considerably higher than zero. As a result the values ρ_{min} and q_{out} which should be used in (15) are a little bit lower for narrow traffic jams than the corresponding values for the wide traffic jam (Figure 2(a), $t > 200\tau$) discussed above. For narrow traffic jams, where $v_{min} > 0$ and $q_{min} = v_{min} \cdot q_{max} > 0$, the formulas (13) should be replaced by

$$v_{gr} = (q_{min} - q_{out}) \cdot (\rho_{max} - \rho_{min})^{-1}, \; v_{gl} = (q_{min} - q_h) \cdot (\rho_{max} - \rho_h)^{-1}. \tag{16}$$

It follows from (16) that the velocities v_{gr}, v_{gl} for narrow traffic jams are higher than for the wide traffic jam. They can even become positive as the density ρ_h decreases.

Amplitude of critical perturbation

The amplitude of a local perturbation $\Delta\rho_m$ (10) should exceed some critical value $\Delta\rho_c$ for the traffic jam to be excited (Figure 5). The critical amplitude $\Delta\rho_c$ is maximal at the density ρ_b (Figure 5).

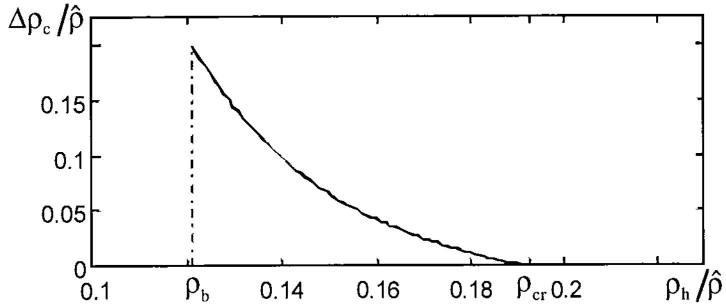


Figure 5 Critical amplitude of a local perturbation (10) as the function of ρ_h

There is also some critical value of the density ρ_{cr} which is higher than ρ_b. At $\rho_h > \rho_{cr}$ any local perturbation grows on the long road in the course of time. This means that the value $\Delta\rho_c$ is a monotonously falling function of ρ_h in the interval $\rho_b \le \rho_h \le \rho_{cr}$ and it tends to zero at $\rho_h \to \rho_{cr}$ (Figure 5). Therefore, in the range $\rho_b < \rho_h < \rho_{cr}$ the homogeneous state of traffic flow is a "metastable" state: A random appearance of a localized critical fluctuation, whose amplitude exceeds a critical value $\Delta\rho_c$ (Figure 4), causes an excitation of a traffic jam (see also Kerner, 1996). This property of traffic jams (Kerner and Konhäuser, 1994) is qualitative similar to the behaviour of localized dissipative structures—autosolitons (Kerner and Osipov, 1994; Kerner, 1996)—which can spontaneously occur in a lot of nonlinear distributed systems.

Notice that the more the amplitude of localized perturbation $\Delta\rho_m$ passes over the critical value $\Delta\rho_c$ (Figure 5), the less the delay time τ_d for the appearance of a traffic jam is (Figure 6). This delay time can be calculated (Figure 6) as the time between the occurrence of the local perturbation $(t = 0)$ and the moment of time, when this local perturbation comes to a stop and begins to grow rapidly.

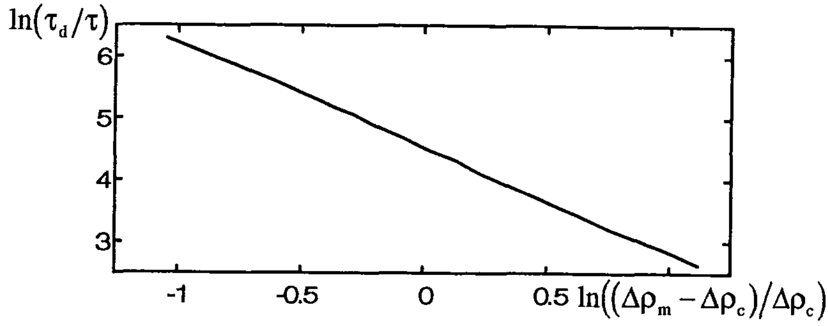


Figure 6 The delay time τ_d of traffic jam appearance as the function of $(\Delta\rho_m - \Delta\rho_c)/\Delta\rho_c$ for $\rho_h = 0.16\,\hat{\rho}$. The other parameters are the same as in Figure 1.

MULTIPLE TRAFFIC JAMS AND OTHER COMPLEX STRUCTURES IN TRAFFIC FLOW

Up till now a simplification of the consideration only the localized structure which consists of one traffic jam has been discussed. Besides complex sequences of different jams can be formed. The traffic jams in these sequences have, as a rule, different amplitudes, different widths, different velocities and are not situated periodically in space (Kerner and Konhäuser, 1993, 1994).

Formation of multiple traffic jam from small amplitude fluctuations

Multiple traffic jam states can spontaneously appear due to a development of fluctuations with small amplitudes in traffic flow, if the density exceeds the critical value ρ_{c1} (Figure 1). The corresponding investigations of the model (3), (8) are shown in Figure 7. It can be seen there that fluctuations with small amplitudes after some time delay lead to the formation of a lot of traffic jams which cover the whole road. These traffic jams build a complex sequence: The traffic jams in this sequence have different amplitudes, different widths, different velocities and are not situated periodically in space. Traffic jams which have different velocities can catch up one another and than merge (Figure 7(b)). Therefore, parameters and the quantity of traffic jams in the discussed complex sequence changes permanently in time (Figure 7(a)).

"Dipole layer" effect in dense traffic flow

Up to now we have considered the range of density $\rho_b \leq \rho_h < \rho_{cr}$, where different kind of traffic jams can be self-formed. In dense traffic flow, exactly, in a range of the density

$$\rho_{cr'} < \rho_h < \rho_{b'}, \tag{17}$$

a new non-linear effect can be realized in an initially homogeneous traffic flow: If a localized fluctuation, whose amplitude exceeds some critical value $\Delta\rho_c$, occurs, a localized structure in traffic flow in the form of a "dipole-layer" (Kerner et al. 1996) can spontaneously be self-formed (Figure 8). The developed "dipole layer" in the ρ -q phase plane corresponds to a nearly stationary curve which again has roughly the shape of a "triangle" (Figure 8(c)).

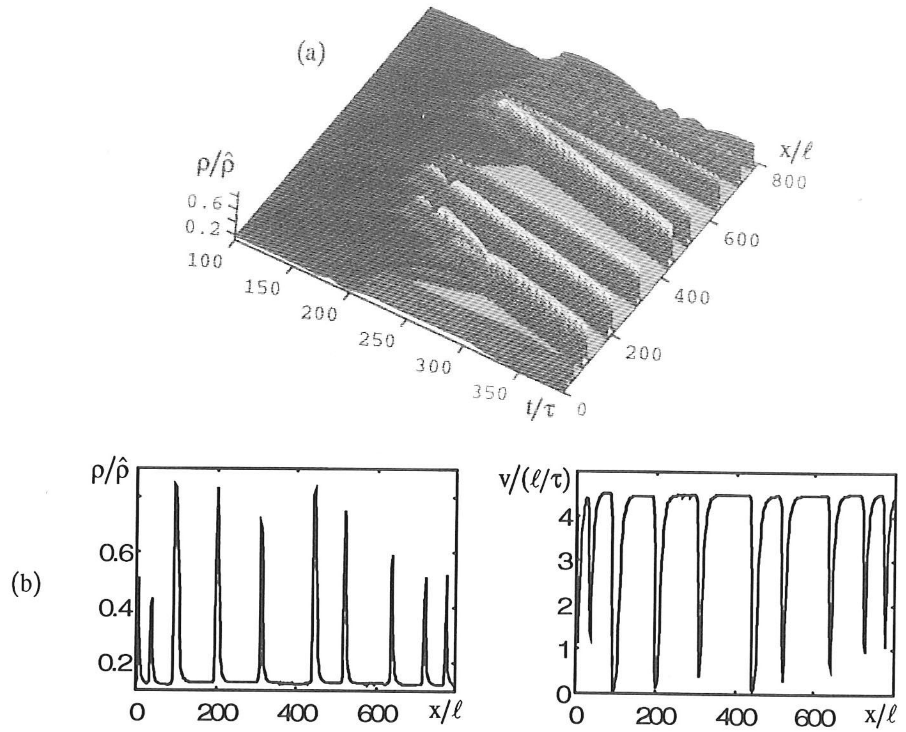
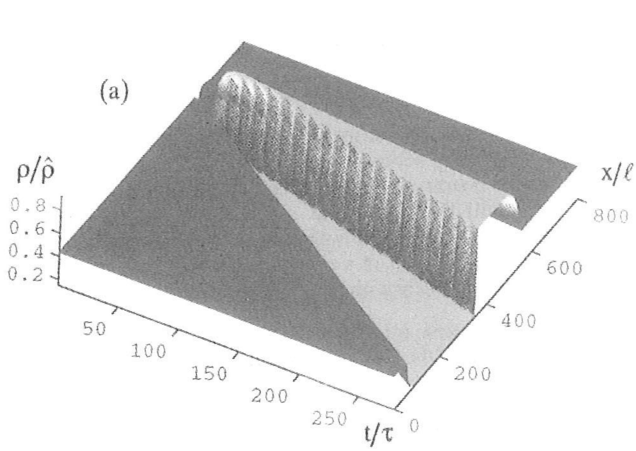


Figure 7 **Kinetics of the formation of a stochastic sequences of traffic jams due to the development of fluctuations in traffic flow: (a) the dependence $\rho(x,t)$, (b) the distributions of ρ and v at moment of time $t = 372\tau$. $\rho_h = 0.18\hat{\rho}$, $\sigma = 0.05\ \ell/\tau^2$. The other parameters are the same as in Figure 1.**



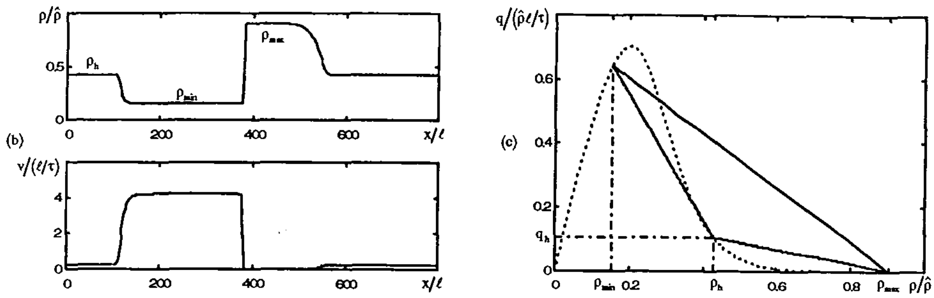


Figure 8 **Kinetics of the formation of the "dipole layer" in traffic flow: (a)—the dependence $\rho(x,t)$, (b)—the distributions $\rho(x)$ and v(x) and (c)—the "dipole layer" in the ρ -q phase plane at $t = 250\tau$. Dotted line in (c) represents the fundamental diagram (Figure 1(b)). The initial distribution $\rho(x,0) = \rho_h - \Delta\rho(x)$ with $\Delta\rho(x)$ (10), $\Delta\rho_m = 0.12\hat{\rho}$, $\rho_h = 0.43\hat{\rho}$, $x_0 = 600\ell$. The other parameters are the same as in Figure 1.**

The maximal density in the "dipole layer" ρ_{max} (Figure 8(c)) nearly coincides with the corresponding density in a wide traffic jam (Figure 2(f)). The characteristic density ρ_{min} (Figure 8) is a decreasing function of ρ_h : At $\rho_h \to \rho_{b'}$, the value ρ_{min} tends to the boundary density ρ_b and the "dipole layer" gradually transform into a wide anti-cluster considered in Kerner and Konhäuser (1994). In other words, when $\rho_h \to \rho_{b'}$ the "triangle" shown in Figure 8(c) degenerates in the ρ -q phase plane into a line, corresponding to the wide anti-cluster.

Notice that in (17) the boundary density $\rho_{b'} \cong \rho_{max}$, where ρ_{max} coincides with the maximal density in a traffic jam (Figure 2(f)). The amplitude of the critical perturbation $\Delta\rho_c$ is an increasing function of the density ρ_h. Exactly, the value $\Delta\rho_c$ tends to zero at $\rho_h \to \rho_{cr'}$ and it is maximal at $\rho_h = \rho_{b'}$.

DETERMINISTIC SPONTANEOUS APPEARANCE OF TRAFFIC JAMS IN SLIGHTLY INHOMOGENEOUS TRAFFIC FLOW

A homogeneous traffic flow is obviously a hypothetical state of traffic flow: A real traffic flow on a highway is always inhomogeneous due to entering and exiting traffic to on- and off-ramps. In this section based on the results of the work (Kerner et al. 1995), where the model (2), (9), (3) has been investigated, it will be shown that a process of a formation of a traffic jam shows qualitatively new peculiarities even in a slightly inhomogeneous traffic flow: A traffic jam on a highway can spontaneously appear in a *deterministic way*, ie., even when fluctuations in traffic flow are so small that their influence on the dynamic processes may be neglected.

We restrict the consideration of usual cases, when a traffic jam in a deterministic way spontaneously appears even if the flux in the traffic flow in any region of the road is considerably lower than the flux Q_{max} which is shown in Figure 1(b). To show this effect, let us assume that at time $t < t_0 = 0$ there is an initially homogeneous traffic flow with some density ρ_h, average speed of vehicles v_h and a flux $q_h = v_h \cdot \rho_h$. Obviously the initially homogeneous traffic flow on a road becomes inhomogeneous when, beginning at the time $t = 0$, some additional flux $q_a = q_a^{(1)}$ appears from an on-ramp: The total flux $q_t = q_h + q_a$ (which is chosen to be lower than Q_{max}) builds on the road directly down-stream from the on-ramp a traffic flow with higher density $\rho_{h1} > \rho_h$. In real traffic flow, a flux q_a is usually not a constant during a long period of time. If, for example, $q_a = \begin{cases} q_a, & 0 \le t \le 95\tau \\ 0, & t > 95\tau \end{cases}$, ie., for $t > 95\tau$ the flux q_a from the on-ramp becomes zero, the transition layer at $t > 95\tau$ begins to move. Although the total flux in the traffic flow is chosen to be considerably lower than Q_{max} (Figure 1(b)), a self-formation of traffic jam in a deterministic way starts and the traffic jam is formed after the transition layer has begun to move (Figure 9). The physics of this effect is linked to the "local breakdown" effect which has been considered in (Kerner et al. 1995).

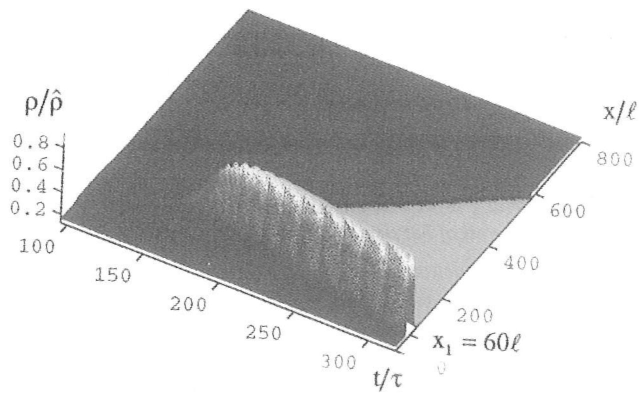


Figure 9 **A deterministic appearance of a traffic jam in inhomogeneous traffic flow at a total flux $q_t < Q_{max}$: the dependence $\rho(x,t)$ for the time $t > 95\tau$, when due to $q_a|_{t>95\tau} = 0$ the transition layer has begun to moves. ($q_t = 0.6960\hat{\rho}\,\ell/\tau$, $\rho_{h1} \cong 0.1833\hat{\rho}$, $Q_{max} = 0.7035\hat{\rho}\,\ell/\tau$,) The other parameters are the same as in Figure 1.**

CONCLUSIONS

Traffic flow of a low enough density of vehicles is a stable state with respect to arbitrary fluctuations. If the density exceeds the boundary (threshold) value, due to fluctuations or due to different kind of inhomogeneities in traffic flow a traffic jam may spontaneously be self-formed in this traffic flow. If the density is further increased, a very complex non-stationary state of many interacting traffic jams with different parameters may appear in the traffic flow. An occurrence of traffic jams without obvious reason is a quite natural general property of relative dense traffic flow and can be considered as some kind of self-organisation in this flow. As the traffic jam moves up-stream, it will reach after some time a source of the road. The spontaneous appearance of the traffic jam somewhere on a long road can cause a sharp change in the whole road network which includes the road under consideration. These very complex non-linear processes in the road network, which are caused by an appearance of traffic jams on one of the roads of this net could be interesting subjects of future investigations.

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