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TOPIC 18
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MODELLING AND EVALUATING OF THE PEAK HOUR TRAFFIC CONGESTION UNDER FLEXTIME AND ROAD PRICING SYSTEMS

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Abstract

We develop a traffic equilibrium model and an algorithm for the simultaneous prediction of hourly departure time distribution and traffic assignment. That is, the departure time distribution in this model considered drivers' departure time decisions which change in congested road situation. This model can also be applied to the evaluation of policies of flextime system and road pricing system.

INTRODUCTION

It seems that every commuter by car will estimate the variation of his/her travel time day by day according to his/her departure time from his/her home based on his/her past experiences. He/she chooses his/her departure time by considering the duration time of congestion along the route he/she uses, opening hour of his/her office and so on, having relation to his/her personal attribute. In this paper we will aim to construct a prediction system of peak hour traffic demand which can take influences of travel congestion time on the departure time choice into consideration.

First we develop a traffic equilibrium model for predicting the hourly departure time distribution and traffic assignment simultaneously and it's effective calculation algorithm. The departure time distribution here can be given as a result of commuters' departure time choices in a congested road network. The model proposed here can not only improve the accuracy of hourly traffic demand prediction but also be applied to the evaluation of traffic demand management devices such as a flextime system or a road pricing system.

The procedure of our study is as follows: first we develop a simultaneous model of departure time distribution and traffic assignment and its calculation algorithm. Next we develop a departure time choice model. Finally we apply these models jointly to the real large-scale road network in Toyota City and evaluate the effects of a flextime system and a road pricing system.

A REVIEW AND OUTLINE OF THIS STUDY

Several existing studies (Hendrickson 1981, Ben-Akiva 1986, Alfa 1989 and Arnott 1990), focus on the departure time choice problem and route choice by commuters. However, these can not be applied to traffic demand prediction problems on a real large-scale network because they are very dynamic models operated in short time intervals. The study by Hendrickson and Plank (1984) is interesting since it analyzed departure time choice and mode choice simultaneously by using a disaggregate model. However this model is not applicable to a traffic assignment problem.

The basic concept of the model proposed in this paper is shown in Figure 1. We consider the rush hours in the morning (6:30-9:30), which is divided into three hourly time periods as shown in Figure 1, and OD traffic flow as input data is divided into two flows with different characteristics. We consider that OD traffic flow during rush hours consists of two kinds, variable OD flows and fixed OD flows. The former can choose their departure time under a flextime system considering the time-varying travel time. The latter is the flows given exogenously. The model of hourly departure time distribution/assignment can predict variations of the travel time and the OD flow simultaneously.

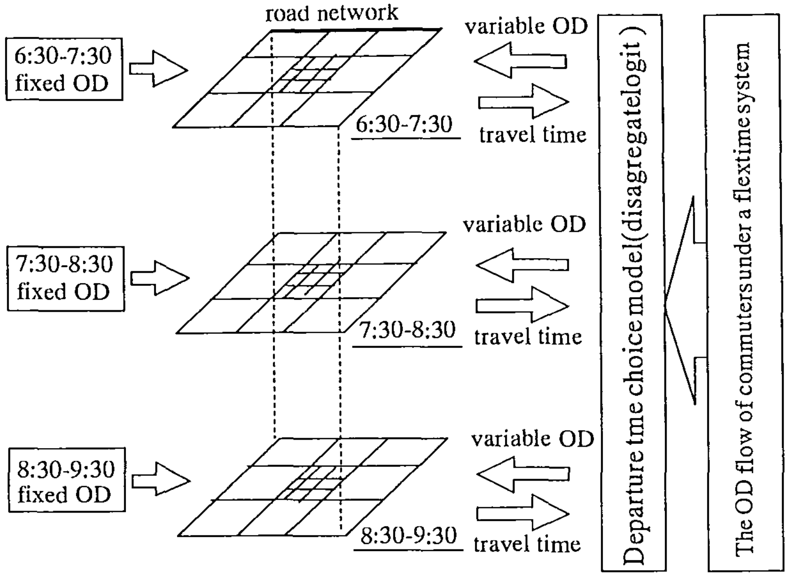

Figure 1 **The model of hourly departure time distribution/assignment**

A SIMULTANEOUS MODEL OF DEPARTURE TIME DISTRIBUTION/ASSIGNMENT

Formulation of the model

We now formulate a simultaneous model of departure time distribution/assignment below.

We will first show that the model can be formulated as the following Bechmann-type user equilibrium (UE) assignment problem:

Min:
$$F1=\sum_n \sum_a \int_0^{x_a^n} Ca(y)dy+\sum_n \sum_i \int_0^{g_i^n} \frac{1}{b}(\ln\frac{z}{G_i}-a_i^n)dz \qquad (1)$$

s.t.
$$\sum_k f_{ik}^n-g_i^n=0 \qquad (2)$$

$$\sum_k h_{ik}^n-H_i^n=0 \qquad (3)$$

$$x_a^n=\sum_i \sum_k \delta_{ika}^n(f_{ik}^n+h_{ik}^n) \qquad (4)$$

$$\sum_n g_i^n-G_i=0 \qquad (5)$$

$$f_{ik}^n\geq 0,\ h_{ik}^n\geq 0,\ g_i^n\geq 0,\ x_a^n\geq 0 \qquad (6)$$

where

x_a^n = flow on link a during time period n

Ca ( ) = travel time on link a

g_i^n = variable OD flow under a flextime system between OD pair i during time period n

H_i^n = fixed OD flow between OD pair i during time period n

f_{ik}^n = flow on path k connecting OD pair i during time period n corresponding to g_i^n

h_{ik}^n = flow on path k connecting OD pair i during time period n corresponding to H_i^n

G_i = sum of g_i^n concerning to time period n

a_i^n = sum of {coefficient × variable value} concerning to variables except travel time given by Table 1 during time period n

b = absolute value of the coefficient of travel time in Table 1

δ_{ika}^n = indicator variable (1 if link a is on path k connecting OD pair i during time period n ,0 if otherwise)

It is supposed that $g_i^n \log g_i^n = 0$ when $g_i^n = 0$ in Equation (1). Equation (1) is very similar with the traffic equilibrium problem with variable demand proposed by Beckmann using an exponential demand function.

Our model is fairly different in formulation and its nature from the previous models (Florian 1977, Florian and Spiess 1983, and Safwat and Magnanti 1981). That is, the previous models use two conditions relating to generated and attracted traffic, however our model uses the only one condition relating to the total OD flow for the entire time period as shown in Equation (5). And Jacobian Matrix of Equation (1) becomes symmetric since the link performance function Ca(x) is equal relating to f_{ik}^n and h_{ik}^n for the entire time period. In addition, we incorporate a multinominal logit model as a demand function into the model.

Next we will prove that the simultaneous model of departure time distribution/assignment can be given by minimizing the following lagrangian function.

$$F2\,(f,g,\lambda,\nu)=F1\,(f,g)-\sum_n\sum_i\lambda_i^n\left\{\sum_k f_{ik}^n-g_i^n\right\}+\sum_n\sum_i\lambda_i^{n\prime}\left\{\sum_k h_{ik}^n-H_i^n\right\}+\sum_i\nu_i\Big(\sum_n g_i^n-G_i\Big) \qquad (7)$$

where

λ_i^n, $\lambda_i^{n\prime}$, ν_i denote the Lagrange multiplier. Optimality conditions for this problem are given by

$$\begin{gathered} f_{ik}^n(C_{ik}^n-\lambda_i^n)=0\\ C_{ik}^n-\lambda_i^n\geq 0 \end{gathered} \qquad (8)$$

$$\begin{gathered} h_{ik}^n(C_{ik}^n-\lambda_i^{n\prime})=0\\ C_{ik}^n-\lambda_i^{n\prime}\geq 0 \end{gathered} \qquad (9)$$

$$\begin{gathered} g_i^n\left[\frac{1}{b}\Big(\ln\frac{g_i^n}{G_i}-a_i^n\Big)+\lambda_i^n+\nu_i\right]=0\\ \frac{1}{b}\Big(\ln\frac{g_i^n}{G_i}-a_i^n\Big)+\lambda_i^n+\nu_i\geq 0 \end{gathered} \qquad (10)$$

where C_{ik}^n is the travel time ($=\Sigma_a\,\delta_{ika}^n\,Ca(x_a^n)$) on path k connecting OD pair i during time period n. From Equation (8) and (9), the Lagrange multiplier, λ_i^n and $\lambda_i^{n\prime}$ of OD pair i are less than or equal to the travel times of any path connecting this OD pair. Thus λ_i^n and $\lambda_i^{n\prime}$ are equal the minimum path travel time between OD pair i. With this interpretation, it is now proved that Equation (8) and (9) is equivalent to the UE principle. When $g_i^n > 0$, Equation (10) becomes

$$\frac{g_i^n}{G_i} = \frac{\exp\left(a_i^n - b\lambda_i^n\right)}{\exp\left(b\, v_i\right)} \tag{11}$$

Furthermore, by Equation (5) the above equation becomes

$$g_i^n = \frac{\exp\left(a_i^n - b\lambda_i^n\right)}{\sum_n \exp\left(a_i^n - b\lambda_i^n\right)} G_i \tag{12}$$

Since λ_i^n represents a minimum path travel time during time period n from Equation (8), Equation (12) is equivalent to a multinominal logit model using the path travel time as a variable. We will incorporate the departure time choice model developed in the next chapter into this equation. If an initial value is set $g_i^n > 0$ in Equation (12), g_i^n can not take 0 through the calculation procedure. Therefore, by minimizing the objective function (1) subject to Equations (2)–(6), we can obtain a set of equilibrium solutions (g_i^n, x_a^n, i=1..s, a=1..r) simultaneously. And this minimization problem has a unique solution since the link performance function in the first term of Equation (1) and the function in the second term are monotonically increasing.

Calculation algorithm

The problem mentioned above is equivalent to the Beckmann-type UE assignment problem which is formulated as the UE assignment with variable demand. The Frank-Wolf (FW) method is well-known as an efficient procedure for solving such a optimization problem. That is, the calculation algorithm for this model is as follows:

Step 1 Determine the study time period, and calculate each OD flow during the time period.

Step 2 Set k=1. Find an initial feasible flow pattern $\{ g_i^{n(k)}, x_a^{n(k)} \}$

Step 3 Find the minimum path and compute the travel time $C_{ik}^{n(k)}$ along the minimum path between each OD pair by making use of $\{Ca(x_a^{n(k)})\}$.

Step 4 Compute $\overline{g_i^n}$ by substituting $C_{ik}^{n(k)}$ for λ_i^n in Equation (12)

Step 5 Compute $\overline{x_a^n}$ by using All-or-Nothing method by which the total OD flow ($H_i^n + \overline{g_i^n}$) between each OD pair is assigned to the minimum path.

Step 6 If either of the following inequalities holds, then finish. If not, go to step 7.

$$\text{a)} \sum_n \sum_a \left(\overline{x_a^n} - x_a^{n(k)}\right) Ca\left(x_a^{n(k)}\right) \leq \varepsilon\, 1$$

$$\text{b)} \max_{n,a} \left|\left(\overline{x_a^n} - x_a^{n(k)}\right) / x_a^{n(k)}\right| \leq \varepsilon\, 2$$

(c) $k > K$ (K: constant value)

Step 7 By setting

$$g_i^n = \alpha^{(k)} g_i^{n(k)} + (1-\alpha^{(k)})\overline{g_i^n}$$

$$x_a^n = \alpha^{(k)} x_a^{n(k)} + (1-\alpha^{(k)})\overline{x_a^n}$$

find the optimal move size $\alpha^{(k)}$ that can minimize the objective function(1) by one-dimensional minimization.

Step 8 The link and OD flow variables are updated by setting

$$g_i^{n(k+1)}=\alpha^{(k)}g_i^{n(k)} + (1-\alpha^{(k)})\overline{g_i^n}$$

$$x_a^{n(k+1)}=\alpha^{(k)}x_a^{n(k)} + (1-\alpha^{(k)})\overline{x_a^n}$$

Step 9 Set k=k+1 and go to step 3.

MODELLING OF DEPARTURE TIME CHOICE

Departure time choice is made by a disaggregate multinominal logit model. When the dispersion parameter in the probability term of utility is set to 1, a disaggregate multinominal logit model is represented as follows.

$$P_n=\exp(V_n)/\sum_{s=1}^{m}\exp(V_s) \tag{13}$$

where

P_n = probability that a commuter chooses time period n as his/her departure time

V_n = utility when a commuter chooses time period n as his/her departure time

In this paper, the study time period was divided into three time periods such as from 6:30 to 7:30, from 7:30 to 8:30 and from 8:30 to 9:30. Table 1 shows the best model with high accuracy, which was made by using 288 data. Parking dummy in the table is set to 1 if no limitation of parking and set to 0 if otherwise. Since the value ρ^2 of the model is 0.21, the model has good accuracy.

The utility of early time period becomes larger since the sign of the working time variable is negative. It is shown that the commuter with longer working time tends to depart earlier in the morning than the commuter with shorter working time. Similarly, we can understand that the commuter with longer travel distance to his/her office tends to depart earlier in the morning than the commuter with shorter travel distance. And the time period which requires the longer travel time for a commuter has a smaller utility because the sign of travel time as a common variable is negative. We can also understand that this model is effective enough as a departure time choice model because the variables used by the model and the resultant coefficients have proper characteristics.

Table 1 Departure time choice model

	Time period	Coefficient (t-statistics)
Constant value	6.30-7.30	-1.91 (-5.30)
Travel time (minutes)	common	-0.0226 (-1.53)
Working time (minutes)	6.30-7.30	—
	7.30-8.30	—
	8.30-9.30	-0.116 (-0.91)
Age	8.30-9.30	-0.0438 (-1.03)
Travel distance (km)	7.30-8.30	-0.0666 (-2.06)
	8.30-9.30	—
Parking dummy	8.30-9.30	0.616 (1.39)
Fitness rate (%)		74.76
ρ^2		0.212

EFFECT ANALYSIS OF A FLEXTIME SYSTEM

We will apply the model proposed here to the evaluation of the traffic demand management policies on a real road network. We will analyze a flextime system in this chapter and a road pricing system in the next section. A road network in Toyota city is used for application, which is a network with 28 centroids, 88 nodes, 278 links (as shown in Figure 2). A revised BPR function is used as a link performance function. Hourly OD flows are given from the master tape of the Person-Trip survey made in 1981. In this paper, only the OD flow of commuters in the manufacturing industry sector is assumed to vary by introducing a flextime system, and the other OD flow is treated exogenously as a fixed flow. For reference the total employees in Toyota city amounts to 164,726 and the employees in the manufacturing industries amounts to 91,782 which is 61% of the total employees.

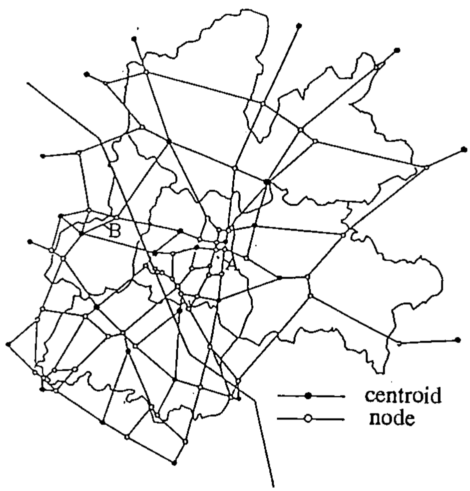

Figure 2 Study network

Situation of convergence

Figure 3 shows the situation of convergence in iterations. Note that the objective function converges enough with about 30 times iterations relatively smoothly.

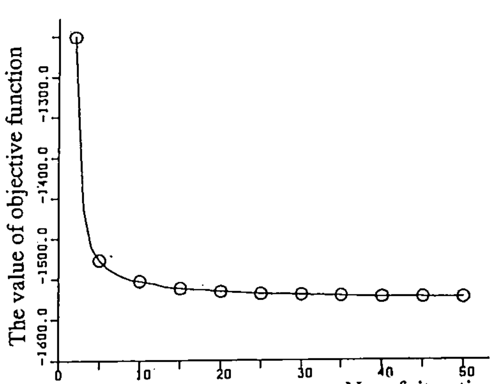

Figure 3 The convergence of objective function

Change of total travel time

Figure 4 shows the change of the total travel time when the rate of popularization of flextime in the manufacturing industries of Toyota city varies from 0% to 100%. It also shows that when the rate of popularization of flextime amounts to 100%, then the total travel time can reduce by 25%. In particular the rate of decrease in total travel time is large when the rate of popularization is in the domain from 0% to 50%. Accordingly, it can be understood that the traffic congestion reduction effects by introducing the flextime system is very large even if the rate of popularization of flextime amounts to about 50%.

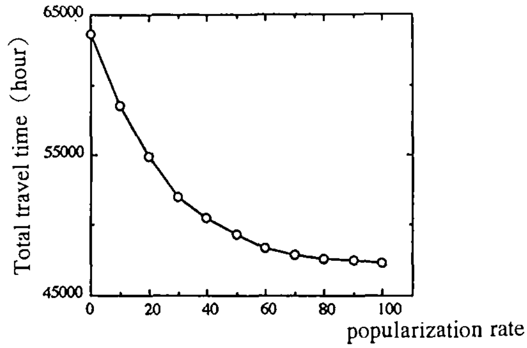

Figure 4 Relationship between the popularisation of flextime and the total travel time

Variations in hourly distribution of OD flow

Figures 5 and 6 show that the variations in hourly distribution of OD flow before and after introducing the flextime system. It is noted that the peak traffic flow disperses to the adjacent time period after introducing the flextime system.

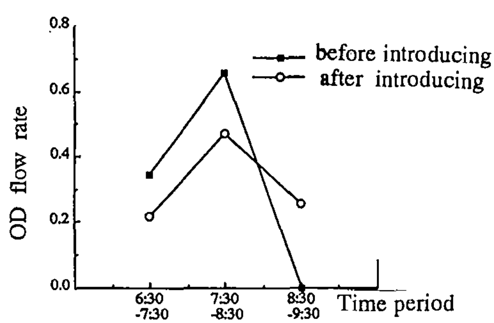

Figure 5 The hourly distribution of OD flow before and after introducing the flextime system (OD pair 2-8)

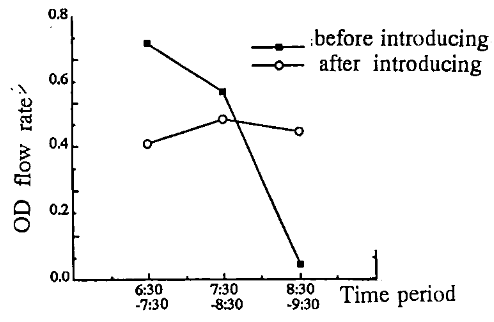

Figure 6 **The hourly distribution of OD flow before and after introducing the flextime system (OD pair 2-8)**

Effects by shortening overtime work

In the departure time choice model as shown in Table 1, working time is assumed as a variable. Figure 7 shows the effects of traffic congestion reduction by shortening overtime work, which are estimated by using working time as a variable and under the assumption that the index of overtime work in 1981 is set to 100% and the rate of popularization of flextime is set to 50%. The figure also shows that the effect increases according to the decrease of overtime work.

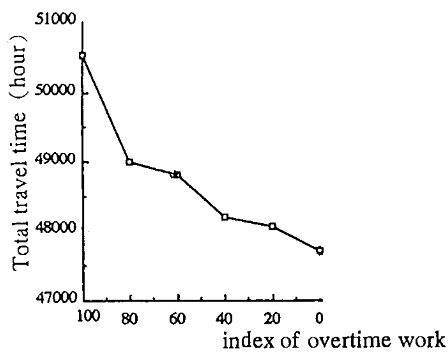

Figure 7 **Traffic congestion reduction by shortening overtime work**

EFFECT ANALYSIS OF A ROAD PRICING POLICY

We will analyze here the traffic congestion reduction effects when a road pricing policy is introduced after the flextime system popularizes to 100%. Application of the model to the effect analysis of a road pricing policy is as follows: some congestion charge during the peak time period is assumed to be imposed on the whole road network. This charge is added to the travel time, which is given by the following equation.

$$Vn' = Vn - b \times Xn \tag{14}$$

where

Vn′ = utility of time period n which includes a congestion charge

Vn = utility used in Equation (13)

b = absolute value of the coefficient corresponding to the travel time in Table 1

Xn = (congestion charge) /(time value)

The time value is set to 2020yen/hour which is the average wage an hour in Toyota city. And, we suppose that the congestion charge is imposed only for a peak hour (7:30-8:30). Figure 8 shows the results of total travel time variation by the effect of this traffic policy.

It is noted that the more the congestion charge rises, the more the traffic congestion are improved. We should note that our model has some possibility to describe the congestion reduction effects by a road pricing policy in a large-scale road network.

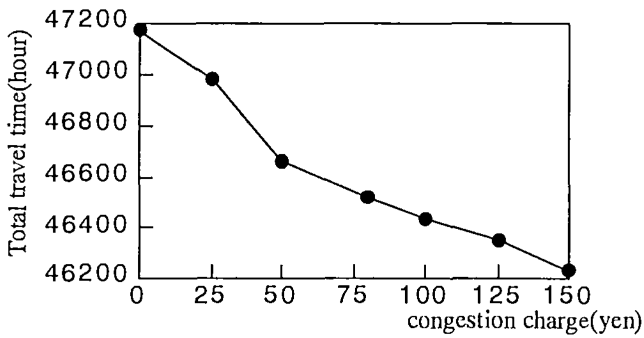

Figure 8 Relationship between the congestion charge and the total travel time

CONCLUSION

We developed the simultaneous model of departure time distribution/assignment in this paper, and analyzed the congestion reduction effects by the application of the model. The conclusions are as follows:

1. The departure time choice model of commuters was formulated as a disaggregate multinominal logit model and showed good accuracy.
2. In addition, we developed the simultaneous model of departure time distribution/assignment, in which the departure time choice model was integrated to the hourly traffic assignment model.
3. The uniqueness of solutions and the calculation algorithm were also given.
4. Furthermore, it is shown that the introduction of flextime and road pricing systems has large effects of traffic congestion reduction through the application of this model to a real road network.

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