



**TOPIC 2**  
MARITIME  
TRANSPORT (SIG)

## **OPTIMAL FLEET DEPLOYMENT FOR BULKERS AND LINERS: INSIGHTS FROM A DECADE OF RESEARCH**

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### **Abstract**

Due to the very different degree of competition in the bulker and liner markets, and also due to the very dissimilar constraints on their respective operations, optimal fleet deployment is quite different for each one. Over the past ten years, we have provided “exact” and “approximate” algorithms for single or multi-origin and destination problems for bulker fleet deployment.

**INTRODUCTION: A SIMPLE FLEET DEPLOYMENT PROBLEM**

Managers of merchant ship fleets, especially bulk carriers and tankers, frequently find themselves with excess transport capacity, and hence must decide which ships to use (and at what speeds) and which to keep idle (or perhaps make available to another fleet by sale or charter). Moreover, if fuel prices become relatively high, excess transport capacity offers the potentially profitable strategy of slow steaming some or all of their ships. Such a strategy not only substantially reduces the operating costs of the fleet, but furthermore reduces the supply of ton-miles of the existing total bulker fleet, thereby improving the depressed freight rates. On the other hand, a sharp drop in fuel prices could make it advisable to 'fast-steam' ships built during the expensive fuel era, although this would be limited by their design speed and associated operating margin.

The ships of a fleet could be assumed to belong to  $N$  different groups, each consisting of  $n(i)$  sister ships,  $i = 1, \dots, N$ , of equal cargo carrying capacity, speed and fuel consumption (or in general, operating costs). Design speed, cargo capacity and operating costs will in general be different among different ship groups. This is both an efficient and general model, since the case of no two ships in a fleet being identical is obviously covered by setting  $n(i) = 1, i = 1, \dots, N$ . The mission of a fleet is assumed, for our purposes, to be the movement of one commodity between two given ports.

A simple (but realistic) bulker fleet deployment problem was defined in Benford (1981). Most of the assumptions inherent in the solution approach, such as no penalty lay-up of unneeded vessels, a contract to move a given quantity of a given commodity between one origin and one destination port, availability of more than enough ships (tonnage) suited to the trade, etc., were not unrealistic. However, the method proposed for its solution did not give the optimal answer, primarily because of an artificial constraint that all vessels must be operated at a speed resulting in the same unit cost of operation per ton of cargo delivered, imposed for ease of solution, but not a natural constraint of the problem. Table 1 below presents the approach adopted in Benford (1981) and its results.

In Perakis (1985), the problem was correctly solved analytically, without the above equal unit cost constraint, using Lagrange multipliers. The results (see Table 2) showed an improvement of at least 15% over those of Table 1, thus verifying once more that 'constraints impair performance'. More realistic and complicated versions of the problem solved in that paper were subsequently formulated and solved.

**Table 1** The Benford (1981) approach

Ship group	Annual transport capacity ( $\div 10^6$ tons)	Operating cost per ton	Annual operating cost ( $\div 10^6$ )
1. A, B, C	4.902	\$4.562	\$22.36
2. D, E	2.884	\$4.546	\$13.11
3. F, G, H	3.726	\$4.562	\$17.00
4. I, J	—	—	—
TOTAL	11.500		\$52.47

**Table 2** The Perakis (1985) approach

Ship	Annual transport capacity ( $\div 10^6$ tons)	Operating cost per ton	Annual operating cost ( $\div 10^6$ )
1. A, B, C	4.158	\$3.531	\$14.684
2. D, E	2.450	\$3.692	\$ 9.046
3. F, G, H	3.241	\$4.023	\$13.038
4. I, J	1.651	\$4.707	\$ 7.771
TOTAL	11.500		\$44.539

Comparing Tables 1 and 2, we see an annual cost reduction of \$7.93 million, or over 15%. This represents a considerable improvement. The difference in costs could be even greater if lay-up charges are levied against ships I and J in the solution presented in Table 1. On the other hand, this difference could possibly be reduced if ships I and J could be chartered or sold to a third party at a particular price.

Perhaps it is now appropriate to clarify that in the above problem, the annual demanded transport capacity is assumed to be a given output (constant). This is the case for vessels operating under relatively long-term charters, which normally specify, among other things, the freight rate and the amount of cargo to be carried annually. In a normal market environment, long-term charters are the overwhelming majority of fixtures, whereas vessels operating in the spot market constitute less than 10% of the available capacity.

The conclusion from the above is that, in contrast to past practices where significant effort has been directed toward the optimization of the design and operation of individual ships, an owner of a fleet of ships (usually non-uniform in terms of age, size and operating speed) should operate each ship in a manner generally quite different from that dictated by single-ship optimization. Adoption of the results of this and subsequent research should result in significant cost savings in the operations of several shipping companies.

## **MORE REALISTIC (SINGLE ORIGIN, SINGLE-DESTINATION) FLEET DEPLOYMENT MODELS**

Perakis and Papadakis (1987a and b (Parts I and II)) and Papadakis and Perakis (1989) presented more realistic and complicated fleet deployment problems and their "optimal" solutions. The problem of single-origin, single-destination fleet deployment was first studied. A computer program was developed to solve the problem and to aid the fleet operator make slow steaming policy decisions. A detailed discussion of the problem solution and a sensitivity analysis are presented in Perakis and Papadakis (1987a). Sensitivity analysis provides the user with an understanding of the influence on the total fleet operating cost of its various components. For small to moderate changes of one or more cost components, the user can get a very accurate estimate of his new total operating cost without having to re-run the computer program. Some interesting conclusions were made on the basis of the sensitivity results.

The fleet deployment problem with time-varying cost components was also formulated and solved. A computer program was developed to implement the solution of this problem (Perakis et al. 1985). The relevant algorithms are briefly described here as well. The problem of fleet deployment when the cost coefficients are random variables with known probability density functions was formulated in detail (Perakis and Papadakis 1987b), where analytical expressions for the basic probabilistic quantities were presented. A shorter description of the above is included in this presentation.

### **Objective function and constraints**

A fleet, consisting of a given number of ships, is available to move a fixed amount of cargo between two ports, over a given period of time, for a fixed price. Each vessel in the fleet is assumed to have known operating cost characteristics. The problem objective is to determine each vessel's full load and ballast speeds such that the total fleet operating cost is minimized *and* all contracted cargo is transported.

A first constraint imposes upper and lower bounds on the vessel full load and ballast speeds. These speed constraints are necessary to insure a feasible solution to the problem; which is, that each speed is less than or equal to its maximum and greater than or equal to its minimum operating limits. In practice, the minimum speed is non-zero and determined by the lower end of the normal operating region of the vessel's main engine. The minimum speed should also be adequate for

purposes of ship safety in maneuverability and control. The equality constraint must be satisfied to insure all contracted cargo is transported.

This formulation is based on the following assumptions:

- 1) A vessel carries a full load of cargo from load port to unload port.
- 2) When the vessel is operating in restricted waters, it has a known and constant restricted speed which is usually the maximum allowable speed in the region in question, hence requiring a known, fixed power and fuel rate.
- 3) The number of days a vessel spends in the load port and unload port per round trip is known and constant.
- 4) The charges incurred at the load port and unload port per round trip are known and constant.
- 5) The amount of fuel burned per day in the load port and unload port is known and constant.
- 6) The annual costs of manning, stores, supplies, equipment, capital, administration, maintenance and repair, and make ready for sail are known and constant.
- 7) The power of vessel  $i$  may be expressed by

$$P_i = a_i \cdot X_i^{b_i} \text{ for the full load and by}$$

$$P_{bi} = a_{bi} \cdot Y_i^{b_{bi}}$$

for the ballast condition, where  $X_i$  and  $Y_i$  are the full load and ballast speeds of ship  $i$  respectively and the rest are appropriate constants.

- 8) The all-purpose fuel rate for a fully loaded vessel  $i$  may be expressed by

$$(R_p)_i = g_i \cdot p_i^2 + s_i \cdot p_i + d_i \text{ for the full load and by}$$

$$(R_f)_{bi} = g_{bi} \cdot p_{bi}^2 + s_{bi} \cdot p_{bi} + d_{bi}$$

for the ballast condition where  $p_i$  and  $p_{bi}$  are the normalized (percent)  $p_i$  and  $p_{bi}$  respectively, and the rest are appropriate constants.

- 9) The *total* annual cost of laying up vessel  $i$  is known for all  $i = 1, \dots, z$ .
- 10) The number of days per year vessel  $i$  is out of service for maintenance and repair is known and constant.
- 11) This problem formulation and solution is for a single stage, "one-shot" decision.

In the literature, the number of tons carried per year is assumed to be a linear function of a ship's full load and ballast speeds. In our research, we have shown that this assumption can be quite unrealistic. As Figure 1 indicates, this function is quite nonlinear in nature. A derivation of this function may be found in Perakis and Papadakis (1987a).

Operating costs, developed in detail in Perakis and Papadakis (1985), are considered to fit into one of two categories, those that do not vary with ship speed, or *daily running costs*, and those that vary with ship speed, or *voyage costs*. Typical plots for the total (*not* per ton) operating costs per year for a particular ship, for various ballast speeds, are given in Figure 2.

A typical plot of  $F(X_i, Y_i)$  is shown in Figure 3, as a function of the full load and ballast speeds. It is seen that  $F$  is a smooth convex curve or surface with a single minimum. There is also a finite speed range in which  $F$  is not very different from its minimum value, a property which allows approximate solutions to the problem using very different speeds for individual ships to produce total fleet costs very close to one another and to the optimum cost itself. For  $X_i$  and/or  $Y_i$  going towards either 0 or  $\infty$ ,  $F$  approaches infinity. Figures 2 and 3 are for the same ship and for constant route data.

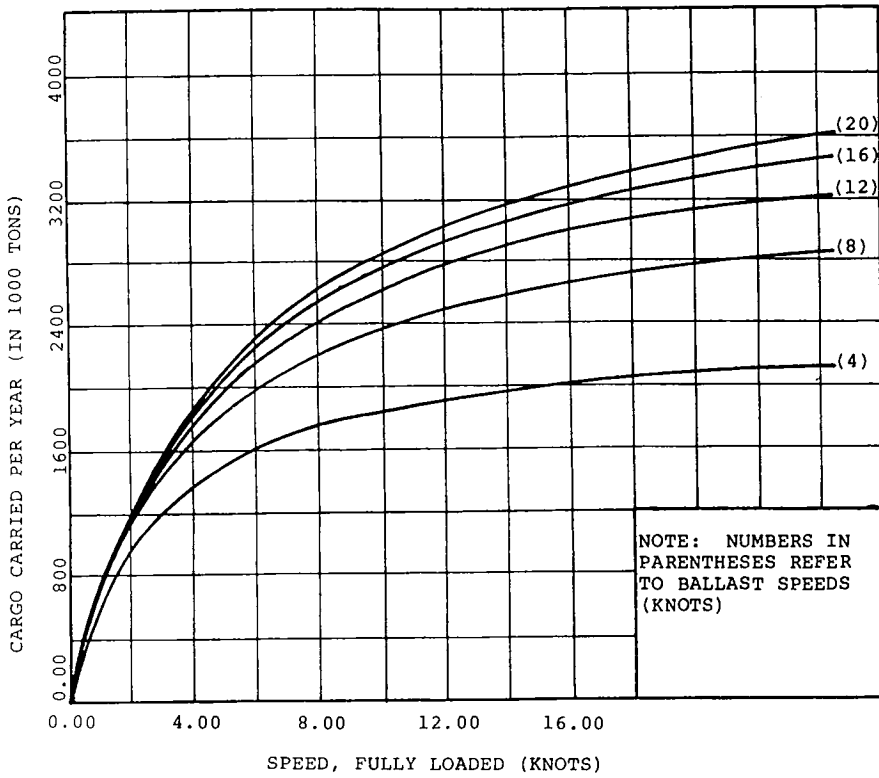


Figure 1 Typical plot of cargo carried per year as a function of ship speeds

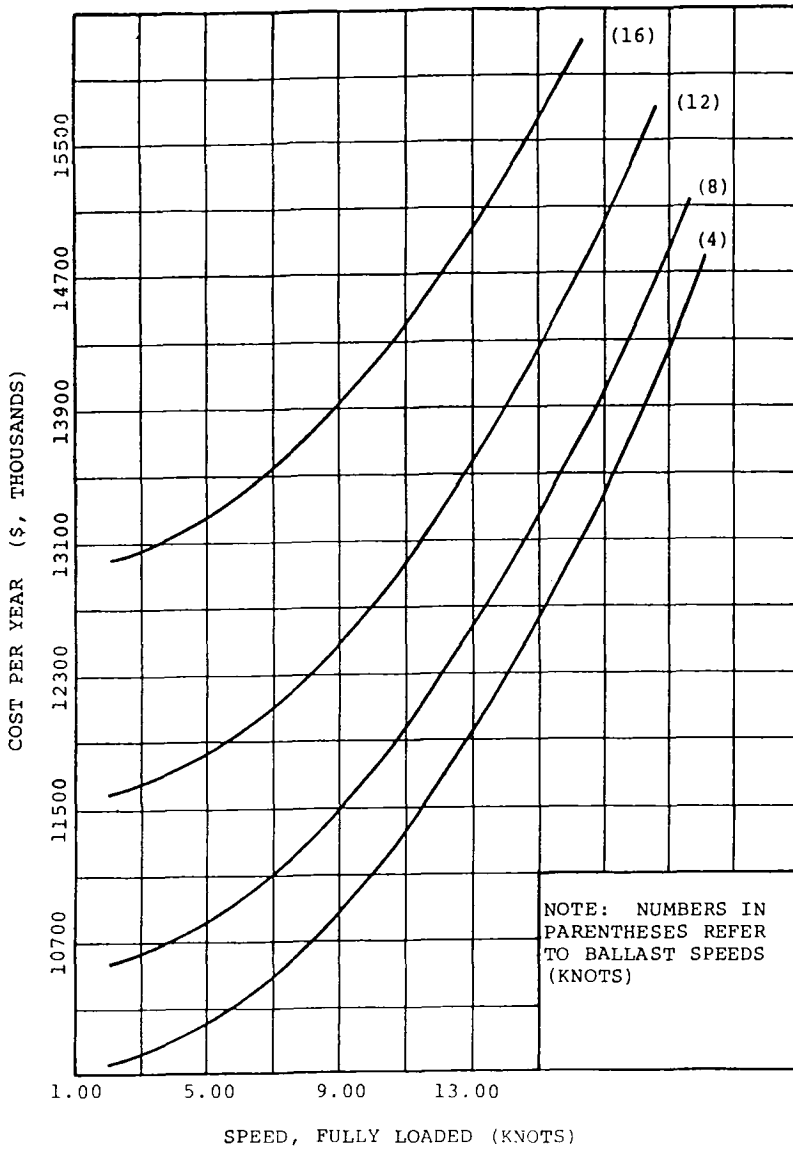


Figure 2 Typical plot for the total operating cost per year as a function of ship full load and ballast speeds

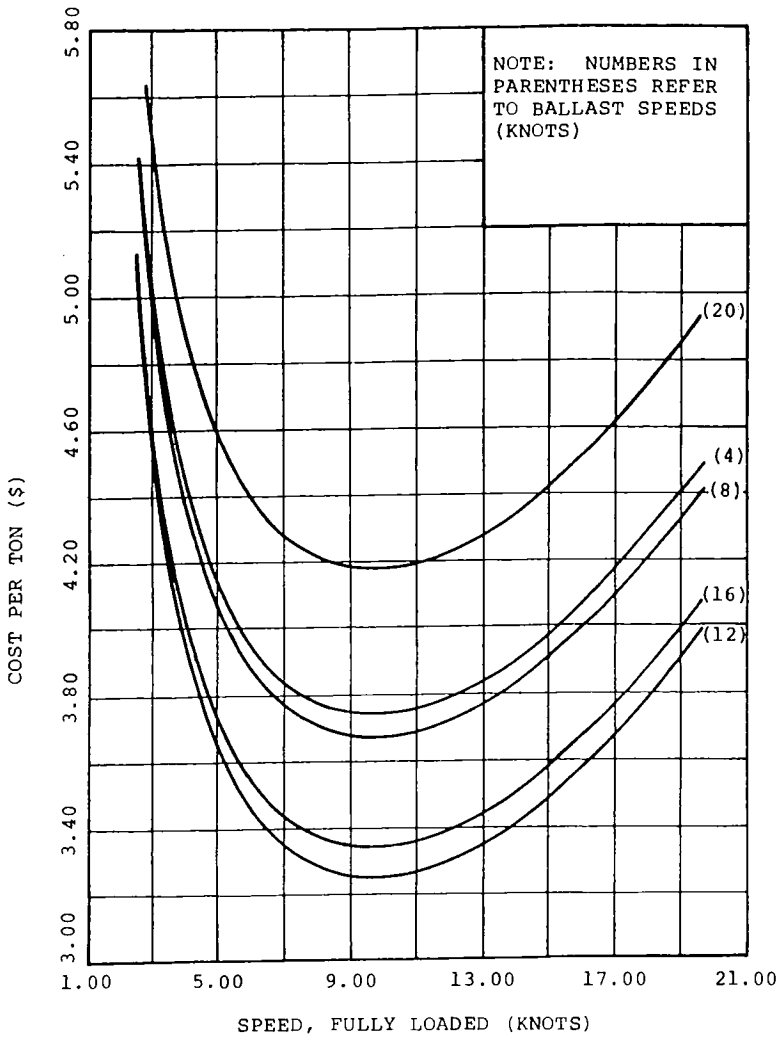


Figure 3 Typical plot for the total cost per ton as a function of ship full load and ballast speeds

Introducing the linear inequality constraints on the speeds complicates the problem solution considerably. In the first part of this research, an External Penalty Technique (EPT) has been combined with the Nelder and Mead Simplex Search Technique to solve our optimization problem. The purpose of a penalty function method is to transform a constrained problem into an unconstrained problem which can be solved using the coupled unconstrained technique.

A computer program has been written to solve this problem using the techniques and the formulation mentioned above. The solution returned consists of the ship speeds, for those vessels specified for analysis, that will minimize the total mission operating costs and fulfil the cargo transport obligation.

For the lay-up option, it is shown in the technical report that for even moderate numbers of ships in a fleet, it is rather too time-consuming to use an exhaustive enumeration scheme. Instead, a dynamic programming-like sequential optimization approach is developed, significantly reducing the computational burden. If  $Z$  is the number of ships in the fleet, the maximum number of fleets we will have to examine using this approach is  $M_{\max} = Z(Z+1)/2-1$ . The *actual* number of fleets which we will have to consider will be significantly smaller than  $M_{\max}$ , due to the elimination of several fleets as infeasible and much smaller than the upper limit of total possible cases. The above scheme has been implemented and referred to in the following as the operating cost without rerunning the program. This property holds for a given fleet and not in cases when one or more ships are laid-up or chartered to a third party (on top of the changes in the cost components).

The fleet deployment problem with time-varying cost components was also studied. A time horizon in this formulation is any interval within which cost components are constant but at least one of them is different than its value in another interval. In other words, our cost components are given "staircase" functions of time. In the case of rapidly changing costs, resulting in rather short intervals where these costs are constant, the problem of non-integer number of round trips per interval could be crucial. A heuristic approach was developed to find the nearest "integer" solution corresponding to the non-integer solution generally provided by the SIMPLEX algorithm.

Further details may be found in Perakis and Papadakis (1985 and 1987a, b) and the associated user's documentation (Perakis et al. 1985), where a more extensive multi-page flow-chart is presented.

The fleet deployment problem for the case when some of the cost components are random variables with known probability density functions was finally considered (Perakis and Papadakis 1987b). We note that the minimum of the possible mean values of the total annual operating costs,  $C_{\min}$  and the variance of  $C_{\min}$  can be found relatively "easily." However, this approach has not yet been implemented on a computer and probably will not prove very useful: The inputs to the problem, ie the user-supplied probability density functions, can have any particular theoretical or experimental form, thus discouraging the development of any general computer code for this problem.

### **The multi-origin, multi-destination fleet deployment problem**

The problem of minimum-cost operation of a fleet of ships which has to carry a specific amount of cargo from several origin ports to several destination ports during a specified time interval was next examined. During the season any vessel can be loaded in any source port (S) and unloaded at any destination port (D) provided that these ports belong to a subset I or J (respectively) of the total set of ports, such that draft and other constraints for the corresponding vessel are satisfied. Under this assumption for each vessel the number of possible routes (number of possible sequences of S-D ports) is quite large. The full load and ballast characteristics of each ship on each route are assumed to be known.

This nonlinear optimization problem consists of a nonlinear objective function and a set of five linear and two integer constraints. The objective function to be minimized is the total fleet operating cost during the time interval (shipping season) in question. The following constraints have to be satisfied:

- a) for each vessel, the total time spent in loading, traveling from origins to destinations, unloading and traveling from destinations to origins plus the lay-up time has to be equal to the total amount of time available for each ship in the shipping season.
- b) The total amount shipped to a particular destination  $j$  must be equal to the amount of cargo to be delivered to  $j$  during the shipping season (in tons).
- c) The total amount of cargo loaded from a particular source  $i$  must be less or equal to the cargo available at  $i$ .
- d) For each ship, the number of trips to destination  $j$  must be equal to the number of trips out of  $j$ .
- e) Same as (d) for all source ports  $i$ .



- f) The full-load and ballast operating speeds have to be between given upper and lower limits.
- g) The numbers of full load and ballast trips for each vessel, origin and destination combination must all be nonnegative integers.

The constraints presented above are linear except constraints (a) and (g). The maximum number of unknown variables (if all source and destinations ports are accessible by any ship of the given fleet) is  $(4 \cdot I \cdot J + 1) \cdot Z$ . The number of the associated constraints is  $2Z + (1 + J)(Z + 1) + 4 \cdot I \cdot J \cdot Z$ . For a case with  $I=4$ ,  $J=6$  and  $Z=10$  we have 970 variables and 1090 constraints. Using today's personal computers, it is clear that we cannot use any classical nonlinear optimization technique, since the expected computation time would be too long.

In the case of the multi-origin, multi-destination fleet deployment problem, it was seen that the linear programming approaches to the literature do not take into account significant nonlinearities of the relevant cost functions and may lead to very suboptimal decisions. The iterative procedure we developed uses a linear programming software in an algorithmic scheme that takes into account these nonlinearities and produces accurate results. This approach is ideally suited for a personal computer due to the reasonable running times of the LP software for almost any practical situation. A second, nonlinear approach to solve the multi-origin, multi-destination problem was also implemented, using the available MINOS nonlinear optimization package.

In Papadakis and Perakis (1989), the fleet deployment problem for a fleet of vessels operating between a set of *several loading and unloading ports* under certain time and cargo constraints was examined. Full load and ballast voyage costs were treated as nonlinear functions of the ship full load and ballast speeds, respectively. An optimization model, appropriate for bulk carrier fleets, minimizing the total operating cost, was formulated. The existence of a coupling between the optimal speed selection and the optimal vessel allocation on the available routes was demonstrated, and conditions leading to the decoupling of these problems were established. Considerations referring to the structure of the optimal solution resulted in a substantial reduction of the dimensionality of the problem. We found that in cases of low-to-moderate fleet utilization, linear programming may be applied to derive the optimal solution, while in cases of higher fleet utilization, use of nonlinear optimization may become necessary. The potential benefits of our approach were demonstrated by several examples.

Finally, we would like to note that the algorithms and the computer codes, developed for both the one-origin one-destination and for the multi-origin, multi-destination fleet deployment problem can be easily used to find not only the optimal fleet deployment policy within the given time horizon but also to help the fleet operator to make decisions in case unexpected events like strikes or accidents occur. In such a case the programs can be re-run for the remaining time interval and an optimal decision can still be obtained. Other plans, such as renewing or improving a part of the fleet and selling or chartering decisions may also be evaluated.

## FLEET DEPLOYMENT MODELS FOR LINER SHIPPING

In Perakis and Jaramillo (1991a) we have reviewed the relevant work on liner shipping deployment and described current industry practices. Our objectives and assumptions were then presented. A model for the optimization of the deployment of a liner fleet composed of both owned and chartered vessels was formulated. The determination of the operating costs of the ships in every one of the routes in which the company operates was carried out by a means of a realistic model, providing the coefficients representing voyage cost and time required for the input of the linear program presented in Perakis and Jaramillo (1991b). A method for determining the best speeds and service frequencies was also presented; the fixing of those two groups of variables was required to linearize the deployment problem as formulated there. The overall optimization method was described in detail, and a real life case study was presented, based on the co-author's company (FMG, Flota Mercante Grancolombiana) operations, in Jaramillo and Perakis (1991b).

In an upcoming journal article (Powell and Perakis 1995), we extend and improve on the above. An Integer Programming (IP) model was developed to minimize the operating and lay-up costs for a fleet of liner ships operating on various routes. The IP model determines the optimal deployment

of the existing fleet, given route, service, charter, and compatibility constraints. Two case studies were carried out, with the same as above extensive actual data provided by FMG. The optimal deployment was determined for their existing ship and service frequency requirements.

The inputs to the optimization model presented in Powell and Perakis (1995) are based on the existing cost estimation model provided in Perakis and Jaramillo (1991), including ship daily running costs, voyage costs, costs at sea, costs at port, daily lay-up costs.

The optimization model in Perakis and Jaramillo (1991) is given as:

$$\text{Minimize } \left( \sum_{k=1}^K \sum_{r=1}^R C_{kr} X_{kr} + \sum_{k=1}^K e_k Y_k \right)$$

where

$C_{kr}$  = operating cost per voyage for a type k ship on route r

$X_{kr}$  = number of voyages per year of a type k ship on route r

$e_k$  = lay-up cost for a type k ship

$Y_k$  = number of lay-up days per year for a type k ship

In Perakis and Jaramillo (1991) and Jaramillo and Perakis (1991) a Linear Programming (LP) approach was used to solve this optimization problem. Using an LP formulation required the rounding of the number of ships allocated to each route. The rounding led to some variations in targeted service frequencies and to sub-optimal results. An Integer Programming formulation is used in Powell and Perakis (1995) to eliminate any rounding errors in the previous LP solution.

### **Integer programming problem formulation**

#### *Decision Variables*

$N_{kr}$  = the number of a type k ship operating on route r

$Y_k$  = the number of lay-up days per year of a type k ship

for  $k = 1$  to  $K$  and  $r = 1$  to  $R$ ;  $K$  is the number of ship types and  $R$  is the number of routes.

#### *Objective function*

The objective function in the model minimizes the sum of the operating costs and the lay-up costs. The objective function in terms of the decision variables is:

$$\text{Minimize } \left( \sum_{k=1}^K \sum_{r=1}^R C_{kr} N_{kr} + \sum_{k=1}^K Y_k e_k \right)$$

where

$C_{kr}$  = operating costs of a type k ship operating on route r

$e_k$  = daily lay-up cost for a type k ship

#### *Constraints*

*Ship availability.* The maximum number of ships of type k operating cannot be greater than the maximum number of ships of type k available. Therefore:

$$\sum_{r=1}^R N_{kr} \leq N_k^{\max} \quad \text{for each type } k \text{ ship}$$

where

$N_k^{\max}$  = maximum number of type  $k$  ships available

*Service frequency.* Service frequency is the driving force in liner shipping. With all rates being set by conferences, the main product differentiation is on service. To ensure that minimum service frequencies are met, the following constraint is included:

$$\sum_{k=1}^K t'_{kr} N_{kr} \geq M_r \quad \text{for all } r,$$

where

$t'_{kr}$  = yearly voyages of a type  $k$  ship on route  $r$  and:

$$t'_{kr} = t_{kr}/T_k$$

$T_k$  = shipping season for a type  $k$  ship

$M_r$  = number of voyages required per year in route  $r$

By finding the highest load level for any given leg of route  $r$  and comparing this with given ship capacity, we find the minimum required number of voyages per year for a specific route.

*Ship/route incompatibility.* Some ships may be unable to operate on a given route due to cargo constraints, government regulations, and/or environmental constraints. It is necessary to eliminate these ships from the model. Therefore:

$N_{kr} = 0$ , for given  $(k,r)$  pairs

*Lay-up time.* The lay-up time in our models is equal to the time a ship is not operating during the year. This includes dry-docking and repair time.

$$Y_k = 365N_k^{\max} - T_k \sum_{r=1}^R N_{kr}$$

*Non-negativity.* The decision variables must be non-negative.

$$N_{kr} \geq 0$$

## Software application

The software package used to run the above example was "A Mathematical Programming Language" (AMPL) (Holmes 1992 and Fourer et al. 1992) and OSL, a mathematical program solver. See Powell and Perakis (1995) for more details. The output file from AMPL gives the following information:

- (i) optimal value of objective function
- (ii) value of objective function with LP relaxation
- (iii) number of iterations to find solution
- (iv) values of variables at the optimal solution

The values of the  $N_{kr}$  variables will show how many type  $k$  ships should be allocated to each route  $r$ . The  $Y_k$  variable will indicate the number of days for which type  $k$  ships must be laid-up.

**Optimization examples**

The following two examples are for the fleet deployment for FMG. The fleet consists of six types of owned ships and five types of chartered ships (one long-term charter and four short-term charters). The data used to calculate the coefficients for the optimization model is taken from Perakis and Jaramillo (1991b). The cost and time coefficients used are transformed from per voyage units to per ship values.

*Example 1*

The first example optimizes the FMG fleet deployment for their current shipping conditions. This example uses FMG's existing service frequencies and the number of ships available of each type. The current allocation is shown in Table 3.

**Table 3 Current ship allocation**

		Route							Total
		1	2	3	4	5	6	7	
<i>(Owned)</i>	1	3			3				6
	2						2		2
	3				2		1		3
	4							1	1
	5		1						1
	6			1					1
<b>Ship Type</b>									
<i>(Chartered)</i>	7						1		1
	8		1	1					2
	9								0
	10								0
	11					2			2
<b>Total</b>		3	2	2	5	2	4	1	19

*Example 1 Results*

The IP optimal allocation is given in Table 4. The minimum objective function yields a total operating cost of \$91,831,000. This is compared with \$93,148,000 for the current allocation. This corresponds to a reduction in total operating costs of 1.4% (a savings of \$1,317,000 per year).

**Table 4 Resultant ship allocation (Example 1)**

		Route							Total
		1	2	3	4	5	6	7	
<i>(Owned)</i>	1	3			3				6
	2			1				1	2
	3				1		2		3
	4		1						1
	5		1						1
	6						1		1
<b>Ship Type</b>									
<i>(Chartered)</i>	7			1			1		2
	8								0
	9								0
	10					2			2
	11				1				1
<b>Total</b>		3	2	2	5	2	4	1	19

Analyzing the resulting allocation shows that all owned ships ( $k = 1$  to 6) and the long-term charter ( $k = 7$ ) are in use for the entire shipping season. This is due to the high lay-up costs associated with these ship types.

None of ship type 9 are allocated. This ship type has the highest operating cost of any of the short-term charters.

*Example 2*

Example 2 uses the frequency constraints of the LP model presented in (Jaramillo and Perakis 1991). The resultant allocation of the LP model is contained in Table 5. This example compares the results and highlights the advantages of the IP model versus the results of the LP model.

**Table 5** Linear programming allocation

		Route							
		1	2	3	4	5	6	7	Total
<i>(Owned)</i>	1	1			5				6
	2			1				1	2
	3						3		3
	4		1						1
	5		1						1
	6						1		1
<b>Ship Type</b>									
<i>(Chartered)</i>	7			1					1
	8								0
	9								0
	10					2			2
	11	2							2
<b>Total</b>	<b>3</b>	<b>2</b>	<b>2</b>	<b>5</b>	<b>2</b>	<b>4</b>	<b>1</b>	<b>19</b>	

*Example 2 Results*

The IP optimal allocation of ships is given in Table 6. The minimum objective function gives a total operating cost of \$99,400,000.

**Table 6** Integer programming allocation (Example 2)

		Route							
		1	2	3	4	5	6	7	Total
<i>(Owned)</i>	1	1			1		4		6
	2				1			1	2
	3				3				3
	4		1						1
	5		1						1
	6	1							1
<b>Ship Type</b>									
<i>(Chartered)</i>	7	1			1				2
	8		1						1
	9								0
	10					2			2
	11			2					2
<b>Total</b>	<b>3</b>	<b>3</b>	<b>2</b>	<b>6</b>	<b>2</b>	<b>4</b>	<b>1</b>	<b>21</b>	

The resulting allocation of the IP optimization model maintains all of the target frequencies. Routes 1, 3, and 5 exactly meet the target frequencies while on routes 2, 4, 6, and 7 the frequency is improved. The improvement ranges from 1.3 days to 3.3 days.

For the LP comparison example presented, the optimal objective function of the IP model is \$99,400,000. Although the cost produced by the LP model is substantially smaller, it is important to note that the service frequencies are compromised in the 1991 LP solution, which leads to sub-optimal allocation. Table 7 shows the comparison between service frequencies of the IP optimization model and the LP model.

**Table 7 Comparison of frequencies**

	Route						
	1	2	3	4	5	6	7
Target Frequency	14	14	21	15	30	23	35
IP Model	14	10.7	21	12.9	29.2	20.4	33.7
Difference	0.0	-3.3	0.0	-2.1	0.0	-2.6	-1.3
LP Model	14.7	16.1	18.9	16.1	29.2	19.2	33.7
Difference	0.7	2.1	-2.1	1.1	-0.8	-3.8	-1.3

Since service is a priority in liner shipping, it is necessary to meet the target frequencies. The IP optimization model ensures that all target frequencies are met. The LP model violates the target frequency for routes 1, 2 and 4. This is an average increase in service time of 1.3 days or 9.1%.

Using Integer Programming to solve integer problems always produces the optimal solution for the given constraints. No manipulation of results is necessary. Using Linear Programming to solve IPs requires manipulations of the results to make the decision variables integer numbers. This leads to sub-optimal solutions and constraints being violated.

Substantial savings may be achieved by applying our IP optimization model for the fleet deployment of a liner shipping company. The first example in (Powell and Perakis 1995) compares our IP model against the existing fleet deployment of a liner shipping company. This example shows a reduction in operating costs of 1.5%. The second example compares our IP model with the LP model contained in (Perakis and Jaramillo 1991). The results of the IP model are optimal and meet all service frequency constraints. The LP model violates the service constraints in three routes by an average of 9.1%.

The solution indicates that all owned and long-term charter ship types should be operated for their entire shipping season, due to the high lay-up cost associated with these ship types. Short-term charters should only be used if the owned ships and long-term charters can not meet the cargo and service frequency constraints.

## SUMMARY

Due to the very different degree of competition in the bulker and liner markets, and also due to the very dissimilar constraints on their respective operations, optimal fleet deployment is quite different for each one. Over the past ten years, we have provided "exact" and "approximate" algorithms for realistic, single or multi-origin and destination problems for bulker fleet deployment, including optimal slow-steaming lay-up decisions, under conditions of certainty or uncertainty for the various cost components. We then also solved problems in optimal strategic planning and ship-route allocation for a major liner company, presenting independent models for fixing both the service frequencies in the different routes and the speeds of the ships, using at first linear and subsequently integer programming. Several insights from a review and comparative study of the above were presented in this paper, starting from the proper problem definition (constraints artificially imposed have resulted in 15% higher costs in early literature on this

problem) and ending with the benefits of optimal integer solutions to the liner fleet deployment problems we studied.

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