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OPTIMAL LINER FLEET ROUTING STRATEGIES

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Abstract

This paper provides two optimisation models for routing strategies of liner fleets, a linear programming model for profit maximisation, and a mixed integer programming model for cost minimisation. The models provide optimal routing mix over the planning horizon, and also optimal capital investment alternatives related to ship owning or chartering.

INTRODUCTION

Despite the fact that world liner shipping accounts for only about 10% of total seaborne ton-miles, its freight revenue amounts to about one half of the freight revenue of total world shipping industry (Jansson and Schneerson 1987). We can therefore expect that a typical liner shipping company can benefit a lot from improving routing or scheduling by systematic methods. The objective of this paper is to suggest practical optimization models for routing strategies for liner fleets.

Many useful routing and scheduling problems have been studied in the transportation literature. A comprehensive survey of vehicle routing and scheduling problems can be found in Bodin (1983) and in Laporte (1992). As for ship scheduling or routing problems, relatively less effort has been devoted, in spite of the fact that sea transportation involves large capital and operating costs. Among major reasons of this lack of research are the major uncertainties involved in shipping services due to such factors as weather conditions, mechanical problems, or strikes, and the volatility of the international shipping markets. These, and other reasons, have frequently made the routing or scheduling problems for ships less structured than those for other transportation modes, and accordingly, made it difficult to build analytical models for sea transportation problems. A more detailed discussion and a survey of many relevant studies are found in the papers of Ronen (1983, 1993).

There have been some studies on optimization models for routing or scheduling problems in sea transportation, but the majority have been on industrial carriers, bulk carriers, or tankers. On liner fleet management, some heuristic approaches rather than analytic optimization models have been dominant. For example, Boffey et al. (1979) developed a heuristic optimization model and an interactive decision support system for scheduling container ships on the North Atlantic route. Olson et al. (1969) used a simulation model to obtain regular schedules for a fleet of cargo ships involved in a liner trade. But it should be mentioned that, unlike the exact optimization methods, the heuristic methods or simulation models can pick up a best solution only among a finite set of alternatives. Only quite recently, a few analytic optimization models have been attempted to routing and scheduling problems for liner fleet. Perakis and Jaramillo (1991), and Jaramillo and Perakis (1991) developed a linear programming model for a routing strategy to minimize total fleet operating and lay-up cost during a planning horizon. They assumed several predetermined routes (sequences of ports of call) and developed a model to assign each ship to some mix of the predetermined routes. Rana and Vickson (1988, 1991) presented nonlinear programming models. They tried to maximize total profit by finding an optimal sequence of ports of call for each ship. For solution methods, they used Lagrangean relaxation (Fisher 1981) and decomposition methods.

Since the route of a container ship, once determined, is hard to be altered for a certain period of time, the initial routing decision should be made after a thorough study. It is also highly desirable to rearrange the whole system of routes by some analytical methods, periodically, to adjust for changing shipping environments, such as changes in primary cargo demand, in freight rates, or in international regulations. A slight improvement of routes could yield substantial additional profits or cost savings.

The optimization models developed so far seem to be of limited usefulness to real-life applications. The first model of Rana and Vickson (1988) is developed for only one container ship, and only time charter decision making. The second (1991) is rather complicated by its nonlinearities in both objective function and constraints. The model of Perakis and Jaramillo (1991) is easier to use for a realistic situation, but does not take into consideration the cargo demand forecasts that arise between pairs of ports in the model.

The models developed in this paper are aimed not only to be easy to use in a real life situation, but also to systematically include future cargo demand forecasts. To achieve a tractable model, the nonlinearities have been avoided, and the models have been developed only within the area of linear programming and mixed integer programming with 0-1 variables. This has made it possible to obtain the solutions of the models of practical size with the aid of well known commercial software now available for personal computers. To systematically connect the cargo demand forecasts with the routes under consideration, we have devised the concept of flow-route incidence matrix and used it in the models. This is expected to become a general formulation tool for many other optimization models for ship routing or scheduling problems.

The remainder of this paper is organized as follows: In the next section the routing problem treated in this paper and its assumptions are discussed. The section following that introduces the concept of flow-route incidence matrix and gives some illustrations and potential uses. In the final two sections, two optimization models for strategic routing problems of a container liner fleet are provided. One uses linear programming for profit maximization, and the other uses mixed integer programming for cost minimization. A numerical example and brief discussions of some specific considerations for the solution methods are also added.

PROBLEM DESCRIPTION AND ASSUMPTIONS

Managers of shipping companies typically respond to change by using insights acquired through experience. In particular, if they have only a few ships available for their operation, they might do well without any help from analytical models for ship routing or scheduling problems. But as their fleet size and involved shipping routes increase, the decision making problems should consider more and more complex factors that humans alone cannot process simultaneously, and the number of feasible alternatives also increases beyond human ability to handle. The important thing is that a slightly better alternative than their best insights that would be suggested by analytical models could get them a larger amount of additional benefits, the larger and the more complicated the problem at hand is. The optimization models in this paper are developed to help managers of shipping companies under this situation to make better decisions in liner fleet routing problems.

Since a liner route, once determined, cannot be changed for a certain period of time in practice, the routing problem of a container liner fleet is similar to the strategic production planning problem of a manufacturing company. Therefore, we need, as important preliminary data for the routing problems, the cargo demand forecasts for any markets the shipping company plans to serve. In light of the above, we assume that the shipping company has all the required demand forecasts, d_{ij} (the demand forecast of cargo from port i to port j), for the planning horizon.

With this data, we want to assign each ship (or each type of ship) k, (k=1,..., K) to some mix of routes r (r=1, ..., R) among a finite set of candidate routes considered by the shipping company, to optimize our objective. At the same time, we want to determine approximate service frequencies on each route. Additionally, we might also want to decide which ship to add to the existing available fleet, ie which ship to charter in for the planning horizon, which ship to build or purchase among a finite set of capital investment options. This kind of problem corresponds to the capital investment planning problem of a manufacturing company, and the mixed integer programming model later in this paper will be of help.

In this paper the following assumptions are imposed for the models and the following notation is used:

(a) The demand of cargo (number of containers) from port i to port j, d_{ij}, over the planning horizon is deterministic, known, and occurs uniformly during the planning horizon.

(b) A ship can be operated on more than one route, if needed.

(c) The managers of the shipping company can suggest a finite set of candidate routes for their liner fleet, old or new, derived from common sense, their past experience, or their view of future main cargo flows. We could also think of a model which determines the sequences of ports of call, from the very start, without any a priori set of routes, just like the one of Rana and Vickson (1991). But it seems more practical to assume that the managers have their own sense of the routes, and the routes determined through the analytical models should not be very different from these.

Notation

(a) d_{ij} : Cargo demand (number of containers) from port i to port j; we simply call it the flow from port i to port j.
(b) m_{ij} : Minimum required number of trips from port i to port j to satisfy the flow d_{ij}.
(c) c_{rk} : Expected operating cost of ship k on route r per round trip.
(d) π_{rk} : Expected profit from a round trip on route r by ship k.
(e) t_{rk} : Total travel time for ship k on route r per round trip, which is the sum of sailing time and the time spent at all ports on route r.
(f) t_k : Maximum time ship k is available during the planning horizon.
(g) h_k : Lay-up (idle) cost of ship k per unit time.
(h) f_k : Fixed cost of ship k during the planning horizon; total charter amount for a newly chartered ship, or total capital cost incurred for a new ship built or purchased.
(i) $a_{ij,rk}$: A component of the augmented flow-route incidence matrix.
(j) x_{rk} : Number of round trips of ship k on route r over the planning horizon.
(k) y_k : Lay-up time of ship k during the planning horizon.
(l) $(i,j) \in r$: Route r contains a path from port i to port j either in outbound direction or in inbound direction.

FLOW-ROUTE INCIDENCE MATRIX

For the models suggested in the next two sections, we introduce a matrix which is expected to serve as a generally useful formulation tool for routing or scheduling problems of ships. We call this matrix the flow-route incidence matrix, for it links the cargo demands to candidate routes.

Suppose there are N pairs of ports (i,j) where flow d_{ij} (> 0) occurs from port i to port j over the planning horizon, and there are R candidate routes considered. By $(i,j) \in r$, we indicate that route r contains a path from port i to port j either on its outbound direction or on inbound direction, ie a flow from port i to port j can be fulfilled by a finite number of round trips on route r.

Definition 1

A matrix $A = (a_{ij,r})_{N \times R}$ called the *flow-route incidence matrix* with N flows and R routes, if,

$$a_{ij,r} \left\{ \begin{array}{ll} 1, & \text{if } (i,j) \in r; \\ 0, & \text{otherwise} \end{array} \right\}$$

The following example gives a simple illustration of the flow-route incidence matrix.

Example 1

Suppose there are flows d_{12}, d_{13}, d_{24}, d_{41} and two candidate routes 1 - 2 - 4 - 1, and 1 - 3 - 4 - 1, where port 1 and 4 are the common end ports of both routes. Then, the corresponding flow-route incidence matrix becomes as follows,

$$A = \begin{array}{c} \\ (1,2) \\ (1,3) \\ (2,4) \\ (4,1) \end{array} \begin{array}{c} (1,2) \\ \begin{pmatrix} 1\,0 \\ 0\,1 \\ 1\,0 \\ 1\,1 \end{pmatrix} \end{array}$$

The above flow-route incidence matrix seems to have various potential uses for modeling both strategic routing problems and operational scheduling problems of ships. For an instance, the following simple 0-1 integer programming model, in fact an instance of a *set covering problem*

(Padberg 1979), is used to find a way of minimum number of routes to satisfy all the future cargo demands (flows):

$$\min\left\{x_1+\cdots+x_R \mid Ax\geq 1,\ x_j\in\{0,1\}\right\}$$

where $x = (x_1,\cdots,x_R)^T$, and **1** is the column vector every component of which is 1.

For another example, we can think of a problem to find a feasible set of ship schedules to satisfy some temporary cargo demands that have occurred unexpectedly. Let c_r indicate the cost incurred to run schedule r, then the best set of schedules can be found by solving the following similar problem:

$$\min\left\{cx \mid Ax\geq 1,\ x_j\in\{0,1\}\right\}$$

where c is the row vector of the costs c_r, and each column of A should represent each candidate schedule rather than a route. This application is actually found in a bulk cargo ship scheduling model (Fisher 1989), and in a tanker scheduling model (Bremer and Perakis 1992; Perakis and Bremer 1992).

The next two sections use an extended version of the above matrix defined as below. Other formulations also could lead to different variations. For each route r, let K_r denote the set of available ships that can be assigned to route r during the planning horizon. In practice, we may also start by setting K_r=K for any route r, if we have no a priori estimate for the contents of set K_r.

Definition 2

An *augmented flow-route incidence matrix* is a matrix $\overline{A} = (a_{ij,rk})$ of N rows and $\sum_{r=1}^{R} |K_r|$ columns ($|K_r|$ denotes the cardinality of K_r), where $a_{ij,rk}$ is the component in the row corresponding to the flow of (i,j) and in the column corresponding to route r and ship k ($k \in K_r$), and

$$a_{ij,rk} = \begin{Bmatrix} 1 & \text{if } (i,j) \in r, \\ 0 & \text{otherwise} \end{Bmatrix}$$

Example 2

Continuing with Example 1, suppose there are two ships 1, 2 available during the planning horizon, and ship 1 can be assigned to both routes while ship 2 cannot be assigned to route 2 because of a certain government regulation. Then the resulting augmented flow-route incidence matrix becomes

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

A LINEAR PROGRAMMING MODEL

In this section, we present a linear programming model for the routing strategy of a liner fleet by using the augmented flow-route incidence matrix $\overline{A}$. Unlike the model in the next section, it is assumed that all the ships available during the planning horizon are known and fixed; the shipping company has already decided about what ships to add to and what ships to delete from the existing fleet for the planning horizon, ie the shipping company has already developed a fixed plan about what ships to charter in, what to charter out, and what to build or purchase.

Given this capital investment plan and demand forecasts, the decision making problem of this section is to find a routing mix of the liner fleet among candidate routes to maximize the expected profit of the liner operation during the planning horizon.

Objective function

Let e_{rk} be the expected revenue per voyage on route r by ship k, which could be estimated from past experience data of operation, or extrapolated or guessed at in case r is a new route. Let c_{rk} be the estimated operating cost of the ship k per voyage on route r. This could be calculated by such methods as suggested in (Perakis and Jaramillo 1991). Then an estimated profit per voyage on route r by ship k, π_{rk}, is determined by

$$\pi_{rk} = e_{rk} - c_{rk}$$

A natural objective function would then be:

$$\max \sum_{r=1}^{R} \sum_{k \in K_r} \pi_{rk} x_{rk} \tag{1}$$

Constraints

The augmented flow-route incidence matrix is used to make a set of constraints that the selected routes and service frequencies should be enough to satisfy all the required flows d_{ij}. Let w_{ij} be the average amount of cargo (average number of containers) that has been transported per voyage from port i to port j from past experience data. (In case d_{ij} is a newly required flow, we could compute w_{ij} from other sources such as data from other companies or just use a guess provided by the decision maker.) Let x_{rk} be the decision variable denoting the number of voyages of ship k on route r during the planning horizon, and $a_{ij,rk}$ as defined in Definition 2. To ensure that the total cargo demand over the planning horizon is met, the following set of constraints should be satisfied:

$$w_{ij} \left(\sum_{r=1}^{R} \sum_{k \in K_r} a_{ij,rk} x_{rk} \right) \geq d_{ij} \quad \forall (i,j).$$

Again, if we set

$$m_{ij} = \frac{d_{ij}}{w_{ij}}$$

the above constraints can be expressed equivalently as follows:

$$\sum_{r=1}^{R} \sum_{k \in K_r} a_{ij,rk} x_{rk} \geq m_{ij} \quad \forall (i,j). \tag{2}$$

From the derivation, the above constraints (2) can be regarded as cargo demand constraints, and the right hand side m_{ij} can be interpreted as an estimation of the minimum number of voyages on the whole network of routes required to satisfy the estimated demand flow d_{ij}.

The second group of constraints depicts the *time constraints* or *ship capacity constraints*. Let t_k be the total time that ship k is available for operation during the planning horizon, and t_{rk} the total travel time per voyage for ship k on route r. See Perakis and Jaramillo (1991) for typical values. If we are to maintain the same speed for all the ships assigned to a route for the regularity of service, we can simply set $t_{rk} = t_r$ for any k. For each ship k, let R_k be the set of routes r that ship k can be assigned to for the planning horizon, ie $R_k = \{r | k \in K_r\}$. Then the following set of constraints should be satisfied.

$$\sum_{r=R_k} t_{rk} x_{rk} \leq t_k \qquad \forall k=1,\cdots,K \tag{3}$$

The objective function (1), and the constraints (2), (3), with the nonnegativity constraints $x_{rk} \geq 0$ together, compose a linear programming model for the routing strategy.

Completed model and comments

The completed model can be represented in matrix form as follows:

(P1)

$$\max \pi x \tag{4}$$

$$\text{subject to } \bar{A}x \geq m \tag{5}$$

$$Tx \leq t \tag{6}$$

$$x \geq 0$$

where (4), (5), (6) are the matrix representations of (1), (2), (3) respectively with consistent dimensions of matrices. The above (P1) has no more than RxK variables, and exactly N+K constraints, and can be solved by standard linear programming packages. The larger part of the constraints can be completely determined by the augmented flow-route incidence matrix.

The optimal solution, $x^* = (x^*_{rk})$, indicate which candidate routes we should choose and which we should not. The optimal service frequencies on each route r can be found as $\sum_k x^*_{rk}$. As for the utilization of each ship, x^*_{rk} shows how many round trips should be made on each route r by ship k. We do not need to include the lay-up time and cost in this model since the objective function (profit) should be maximized. It is clear that the constraints (3) will be satisfied by equality at the optimal solutions. In this sense, there is in fact no difference if we replace the inequalities (≤) in (3) with equalities (=).

Since our routing problem is strategic, and therefore has a relatively long planning horizon, it is inevitable that the coefficients used in (P1) have a certain degree of uncertainty involved. However, we can easily treat these various uncertainties with the help of the well established post-optimality procedures of the linear programming. Minor fluctuations in operating costs, for example in fuel costs, or uncertainties of future freight rates, can be treated in the framework of objective function sensitivity analysis. Difficulties caused by uncertainties in the estimation of cargo demands d_{ij} can be alleviated by the aid of such methods as the right hand side sensitivity analysis.

Numerical example

Consider a small fleet consisting of 3 ships, and 5 candidate routes over a network of 5 ports. Let each node 1,...,5 denote each of the five ports. The 5 candidate routes over these nodes are shown in Figure 1. Suppose the managers consider (1,2), (2,3), (2,4), (3,2), (4,5), and (5,3) as important flows for the next planning period. These flows and corresponding cargo demand forecast (d_{ij}, number of containers), average number of containers carried per voyage (w_{ij}), and the resulting required number of voyages for each flow (m_{ij}) are shown in Table 1. The estimated profits (π_{rk}) and travel times (t_{rk}, t_k) for each ship on each candidate route are given in Tables 2 and 3. In case a ship cannot be operated on some routes for some operational or institutional restrictions, it is marked by (–) in the tables. Then the augmented flow-route incidence matrix appears as follows:

$$\bar{A}=\begin{array}{c} (1,2)\\(2,3)\\(2,4)\\(3,2)\\(4,5)\\(5,3)\end{array}\overset{\begin{array}{ccccccccc}x_{11}&x_{12}&x_{22}&x_{23}&x_{31}&x_{33}&x_{43}&x_{51}&x_{52}\end{array}}{\begin{bmatrix}1&1&1&1&0&0&0&0&0\\0&0&1&1&1&1&0&0&0\\1&1&0&0&0&0&1&1&1\\0&0&0&0&1&1&1&1&1\\1&1&0&0&0&0&0&1&1\\1&1&0&0&1&1&0&1&1\end{bmatrix}}$$

It should be noted that all flows in the candidate routes are not included in the model. Only the flows considered as important, for decision making purposes, are included in the model. Conversely, for example, we observe that flows (3,1) and (5,1) are in route 1, but are not included in the model.

The planning horizon of this example can be regarded as 1 year, and the time unit for t_{rk} and t_k as a day. For example, as is seen in Table 3, the total available travel time by ship 1 (t_1) can be viewed as 300 days, in fact shorter than 1 year by allowing for some predetermined maintenance or repair during the planning year. Using these data and the above augmented flow-route incidence matrix ($\bar{A}$), the ship routing model is made as follows:

$$\max\ 20x_{11}+24x_{12}+10x_{22}+10x_{23}+14x_{31}+14x_{33}+10x_{43}+18x_{51}+16x_{52}$$

subject to:

$$x_{11}+x_{12}+x_{22}+x_{23}\geq 32$$
$$x_{22}+x_{23}+x_{31}+x_{33}\geq 30$$
$$x_{11}+x_{12}+x_{43}+x_{51}+x_{52}\geq 42$$
$$x_{31}+x_{33}+x_{43}+x_{51}+x_{52}\geq 45$$
$$x_{11}+x_{12}+x_{51}+x_{52}\geq 33$$
$$x_{11}+x_{12}+x_{31}+x_{33}+x_{51}+x_{52}\geq 42$$
$$10x_{11}+8x_{31}+8x_{51}\leq 300$$
$$10x_{12}+7x_{22}+8x_{52}\leq 300$$
$$7x_{23}+7x_{33}+6x_{43}\leq 320$$

$$(x_{11},x_{12},x_{22},x_{23},x_{31},x_{43},x_{51},x_{52})\geq 0$$

The optimal solution (x^*_{rk}) for the above model is found as follows by a linear programming software (we used LINDO/PC):

$$x^*_{11}=2.0,\ x^*_{12}=30.0,\ x^*_{33}=45.7,\ x^*_{51}=35.0,\ x^*_{22}=x^*_{23}=x^*_{31}=x^*_{43}=x^*_{52}=0.0$$

and the optimal objective function value is found to be 2030.0. This optimal solution shows that candidate routes 2 and 4 are not so profitable, but only routes 1, 3, 5 should be run over the planning period to maximize profit. About the utilization of ships, this solution suggests using ship 1 on route 1 (for only 2 voyages) and on route 5 (for 35 voyages), ship 2 on route 1 (for 30 voyages), and ship 3 on route 3 (for 45.7 voyages). We can also compute the optimal frequencies of the selected routes 1, 3 ,5 as 32.0, 45.7, and 35.0 respectively.

Table 1 Cargo flows and required frequencies

Flows	(1,2)	(2,3)	(2,4)	(3,2)	(4,5)	(5,3)
d_{ij}	160	120	252	450	241	210
w_{ij}	5	4	6	10	7	5
$m_{ij} = \frac{d_{ij}}{w_{ij}}$	32	30	42	45	33	42

Table 2 Profit per voyage (π_{rk})

Route No.	Ship 1	Ship 2	Ship 3
1	20	24	-
2	-	10	10
3	14	-	10
4	-	-	10
5	18	16	-

Table 3 Travel time per voyage (t_{rk})

Route No.	Ship 1	Ship 2	Ship 3
1	10	10	-
2	-	7	7
3	8	-	7
4	-	-	6
5	8	8	-
Total time	300	300	320

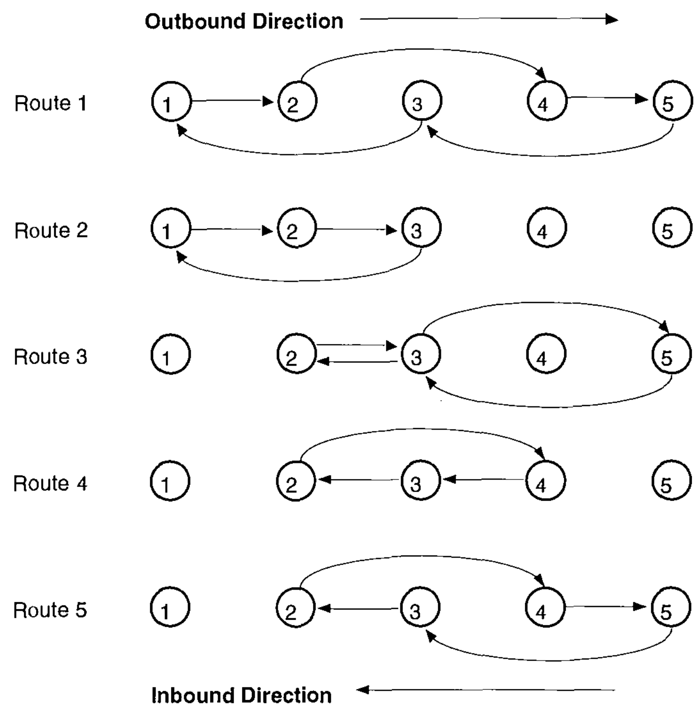

Figure 1 Candidate routes

Implementation under insufficient information of $\bar{A}$

Suppose the shipping company cannot determine, at first, the correct structure of $\bar{A}$, say, for example, the shipping company is not sure if ship k can be operated well on a newly suggested route r. By many practical reasons such as insufficient ship capacity, or government regulations against the ships of some flags, a ship k might be incompatible with a route r, ie $k \notin K_r$. But without complete preliminary consideration, ie without an exact idea of what K_r is like, (P1) cannot be solved at once. In this case the decision maker, at first, may be confronted with the linear program (P1) with the largest $\bar{A}$, ie the one with the objective function (a slight extension of (1)),

$$\max \sum_{r=1}^{R} \sum_{k=1}^{K} \pi_{rk} x_{rk} \qquad (1')$$

and the following constraints (2') and (3'),

$$\sum_{r=1}^{R} \sum_{k=1}^{K} a_{ij,rk} x_{rk} \geq m_{ij} \quad \forall (i,j) \qquad (2')$$

$$\sum_{r=1}^{R} t_{rk} x_{rk} \leq t_k \quad \forall k = 1,\cdots,K \qquad (3')$$

where $a_{ij,rk} = 0$ if $(i,j) \in r$, and 0 otherwise.

However, in order to get practical solutions, the decision maker, in addition, should consider the following implicit constraint that cannot be identified at first.

$$\sum_{r=1}^{R} \sum_{k \notin K_r} x_{rk} = 0 \qquad (7)$$

It is easy to see that (P1) can be equivalently reformulated as follows:

(P1)

$$\max \sum_{r=1}^{R} \sum_{k=1}^{K} \pi_{rk} x_{rk} \qquad (1')$$

subject to

$$\sum_{r=1}^{R} \sum_{k=1}^{K} a_{ij,rk} x_{rk} \geq m_{ij} \quad \forall (i,j) \qquad (2')$$

$$\sum_{r=1}^{R} t_{rk} x_{rk} \leq t_k \quad \forall k = 1,\cdots,K \qquad (3')$$

$$\sum_{r=1}^{R} \sum_{k \notin K_r} x_{rk} = 0 \qquad (7)$$

$$X_{rk} \geq 0 \qquad \forall r = 1,\cdots,R, \ \forall k = 1,\cdots,K$$

The decision maker should solve the linear programming problem determined by (1'), (2'), (3') and the implicit constraint (7). But since the implicit constraint (7) is not known at first, we can think of some iterative approaches that gradually expose it. A natural procedure, outlined below, is similar to some sort of a column deletion algorithm:

At first, we solve the linear program with (1'), (2') and (3'). If the optimal solutions obtained are found to satisfy (7), ie if the solution does not have any value $x^*_{rk} \geq 0$ where $k \notin K_r$, by some intuitive tests of the decision maker, the obtained solution may well be (supposed to be) a real optimal solution for (P1). Otherwise, ie if the solution has some values $x^*_{rk} \geq 0$ where $k \notin K_r$, we can delete the corresponding variables or equivalently the columns from the initial model and resolve the resulting reduced linear program. This step would be performed usually very easily, by a well known post-optimality analysis in linear programming. We can repeat the similar procedure until we are sure that the obtained solutions satisfy the implicit condition (7).

The above procedure ends in a finite number of repetitions, for we have a finite number of variables, and the usual number of repetitions is not likely to be very large.

A MIXED INTEGER PROGRAMMING MODEL

Unlike the linear programming model developed in the last section, this section assumes that the shipping company also has to make capital investment decisions for the planning horizon. To meet the expected increasing future cargo demands, the shipping company may consider some options for fleet capacity expansion such as building or purchasing new ships, or chartering some other ships. Since this kind of decision making problem usually requires a longer planning horizon than the one in the last section, a desirable suggestion for a decision criterion would be one not likely to be affected by growing uncertainties due to the longer duration. Hence the objective of the model adopted in this section is to minimize the total cost incurred from necessary operations to meet the cargo demands over the planning horizon.

Using the model in this section, the shipping company can get help in decision making problems, in addition to what has been mentioned in the last section, about what types of ships to build, to purchase, or to charter in to add to the existing fleet. To reflect these decision making concerns, 0-1 integer variables have been used in the model.

Objective function

The total cost of the objective function is taken as the sum of the operating cost, the lay-up (or idle) cost, and the (fixed) capital cost incurred during the planning horizon. Let K^0 be the subset of ships ($K^0 \subset \{1, \cdots, K\}$) that the shipping company considers for adding to the existing fleet. The resulting objective function becomes:

$$\min \sum_{r=1}^{R} \sum_{k \in K_r} c_{rk} x_{rk} + \sum_{k=1}^{K} h_k y_k + \sum_{k \in K^0} f_k z_k \tag{8}$$

where h_k denotes the lay-up cost of ship k per unit time, and y_k the lay-up time of ship k; f_k is the fixed capital cost involved by adding ship k to the existing fleet, and variable z_k is a binary variable to decide whether to add ship k ($z_k = 1$) or not ($z_k = 0$). The capital costs due to the existing fleet should be omitted in the model because they are not relevant to the decision making problem at hand.

Constraints

The cargo demand constraints are the same as in (2) or (5). The time constraints or ship capacity constraints are a little different from (3), by adding variables (y_k) for lay-up time, and can be split into the following two groups.

$$\sum_{r \in R_k} t_{rk} x_{rk} + y_k = t_k \qquad \forall k \notin K^0 \tag{9}$$

$$\sum_{r \in R_k} t_{rk} x_{rk} + y_k - t_k z_k = 0 \qquad \forall k \in K^0 \tag{10}$$

Constraints (9) are the usual time constraints that the operating time of a ship cannot exceed the total available time for that ship. Constraints (10) also maintain this, and moreover, require that any ship to be operated must be added to the existing fleet.

Completed model and solution methods

Now the completed cost minimization model appears as follows.

(P2)
$$\min \sum_{r=1}^{R} \sum_{k \in K_r} c_{rk} x_{rk} + \sum_{k=1}^{K} h_k y_k + \sum_{k \in K^0} f_k z_k \qquad (8)$$

subject to
$$\sum_{r=1}^{R} \sum_{k \in k_r}^{K} a_{ij,rk} x_{rk} \geq m_{ij} \quad \forall (i,j) \qquad (2)$$

$$\sum_{r \in R_k} t_{rk} x_{rk} + y_k = t_k \qquad \forall k \notin K^0 \qquad (9)$$

$$\sum_{r \in R_k} t_{rk} x_{rk} + y_k - t_k z_k = 0 \qquad \forall k \in K^0 \qquad (10)$$

$$x_{rk} \geq 0,\ y_k \geq 0,\ z_k \in \{0,1\}$$

The above model (P2) is a mixed integer programming model with no more than K binary variables. The continuous variables are no more than K x (R+1), and the number of constraints is N+K. The integer part of an optimal solution, (z_k^*), gives capital investment suggestions for fleet capacity expansion. If $z_k^*=1$ for some k, this can be interpreted differently depending on the situation: to build ship k, to purchase it, or to charter it in.

To solve the above problem in practice, one can use existing commercial software, since the number of integer variables is expected to be relatively small in general. But if we need a more efficient method to solve some extremely large problems, we also could consider using some heuristic methods. In particular, it seems that the Lagrangean relaxation approach (Fisher 1981) will go well with (P2); if we relax the constraints (10) with Lagrangean multipliers, the resulting Lagrangean subproblem is split into a linear program with constraints (2) and (9), and a trivial 0-1 integer program that can be solved by simple inspection. It also should be mentioned that the number of relaxed constraints (probably far less than K) would be quite small compared to the total number of constraints in many cases, which could imply a small Lagrangean duality gap.

CONCLUSIONS

This paper has suggested two optimization models that can be useful to liner shipping companies. One is a linear programming model of profit maximization, which provides an optimal routing mix for each ship available and optimal service frequencies for each candidate route. The other model is a mixed integer programming model with binary variables, which not only provides optimal routing mixes and service frequencies but also best capital investment alternatives to expand fleet capacity. The latter model is a cost minimization model.

While formulating the two models, we have suggested and used the concept of flow-route incidence matrix, and discussed its general usefulness for similar routing and scheduling problems. The most important merit of using this flow-route incidence matrix was that it linked various cargo demands to route utilization in a systematic and simple way.

The selected routes found as optimal solutions of the suggested models should seldom come out as a surprise to shipping managers, for the models choose the best combinations of routes among candidate routes from the existing ones and suggestions from experienced managers. By doing this, the suggested models can help to improve the existing network of routes or service frequencies in a generally acceptable way.

Regarding solution methods, the two models can be easily implemented with standard linear or integer programming software. Implementation with real data of shipping companies would be interesting for a practical future work.

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