7TH WCTR
SYDNEY AUSTRALIA
1995

TOPIC 34
URBAN PUBLIC
TRANSPORT

ESTIMATING AN INTERMODAL FLOW MATRIX IN A PUBLIC TRANSPORT INTERCHANGE THROUGH ADJECTIVE INFORMATION

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Abstract

Public transport interchanges are very important structural elements in the transport system of large conurbations. This paper presents a methodology developed to allow use of adjective information to estimate that intermodal flow matrix. A suite of mathematical programmes is written to solve this problem. The basic idea is that each adjective must correspond to a numerical interval of the ratios between flows.

INTRODUCTION

Public transport interchanges are very important structural elements in the transport system of large conurbations. When a new interchange is planned, it is frequently not clear what services will be offered, both on the outer side of the region and on the more central part of the conurbation.

The most frequent scenario is one in which there is a definition of modes that will be present on the interchange, and the total capacity of service by each mode (number of bus berths, railway tracks, parking spaces for private cars, etc), the actual services offered being defined at much later stages, and evolving during the life of the interchange.

Before the interchange is designed by the architects, it is essential that a matrix of flows of passengers between modes is produced, so that walking corridors and waiting areas can be dimensioned. Even if an O/D matrix of the region were known, it would be impossible to estimate such matrix through the usual assignment techniques, since they require detailed description of the public transport networks in the future. This description is normally available for present conditions, but many changes will happen in the future, which cannot be foreseen. The existence of the interchange will in itself be a factor for significant changes of the public transport networks.

What is required is not an exact estimation of flows (the real ones will change from day to day anyway), but a knowledge of the orders of magnitude involved in each connection, so that the essential input of demand is available to the architects.

Matrix estimation is a problem which has deserved widespread attention, although in a different context. For the case of a road network, using information from traffic counts, initial methods based on entropy maximization or minimization of additional information (Van Zuyl et al. 1980), have been followed by many papers, either reviewing the formulations for the same kind of objectives (Yang et al. 1994), or proposing new approaches (Xu et al. 1993). For that same problem, recent papers have been proposing the use of larger fields of information, not only traffic counts (Miles 1993). In a recent paper (Kikuchi et al. 1993) attention is drawn to a problem not too different from the one studied here—in that case, estimating an O/D matrix for a railway line—but the information used is the set of repeated observations of column and row totals.

This paper presents a methodology developed to allow use of *adjective information* to estimate a matrix of intermodal transfer flows in a public transport interchange.

The author is not aware of any previous paper dealing with the problem proposed here, in which the flows to be estimated are all related to a single node in the network. The methodology proposed can be used by itself (ie using *only* adjective information) but it should be relatively easy to adapt it for cases in which a more classical methodology would be used for the estimation of a base matrix, and adjective information used to estimate additional or shifted flows. Only the first type of problem is treated in this paper.

The first step is the estimation of row totals in that matrix—total flow coming out of each mode—which is relatively straightforward given the capacity of that mode and the sequence of stations on the same line. In many cases, a similar reasoning may be applicable to flows collected by each mode, in which case column totals may also be estimated.

Adjective information is then used to estimate the inside of the matrix. This information is supplied in the form of statements about the ratio of flows in a pair of matrix cells (for instance "identical", "much larger", etc). The providers of this information are transport experts, with a deep knowledge of the region and of the city, and the corridors served by each mode.

There is no "a priori" constraint on the minimal number of such statements. Some cells may be involved in one statement, some others in several, others not involved at all. In practice we find that it is relatively easy to extract this kind of information for a limited number of pairs of cells, and that, as the process goes on, the experts will try to avoid committing themselves to further explicitation.

Another type of important information for the estimation of the matrix is the identification of the cells with nil or very small flow expected (illogical or sometimes even impossible connections). This will in many cases concern only the diagonal cells, but also, depending on circumstances, some other inter-modal connections (for instance a connection between two given modes having much better conditions in a nearby station). In either case, this should be included explicitly in the model, as it lowers the slackness of the problem, and also prevents the occurrence of very visible mistakes in the final results.

STRUCTURE OF THE MATHEMATICAL FORMULATION

The basic idea is that there must be a correspondence between the adjective classes and a set (of equal dimension) of numerical intervals of ratios between flows. Thus, the value in each cell is constrained not only by its row and column totals, but also by its ratios with respect to other cells.

Without loss of generality, we can impose that ratios always be defined in the direction of the largest to the smallest element. A certain number of intensity classes (or levels) for the relationship "larger than" is defined. For instance, if we choose to have 6 classes, the following intensities could be used:

1 very weakly (ie the two elements are identical)

2 weakly

3 moderately

4 strongly

5 very strongly

6 extremely

This method defines a constructive and incremental process for the estimation of a matrix subject to those constraints, so that at each step, a measurement is made of the remaining "slackness" of the estimated flows.

Let the following terminology be adopted:

f(i,j) —flow in cell (i,j) of the matrix (non-negative)

$O(i) = \sum_j f(i,j)$ —sum of flows in row i of matrix

$D(j) = \sum_i f(i,j)$—sum of flows in column j of matrix

LS(n)—Upper bound of class n of ratios between flows

N—number of classes (ratio intervals) defined

We assume to know with good precision the values of one of the sum sets (rows or columns).

Since we often have to deal with zero (or almost zero) flow values in matrices, a transformed expression for the ratios has been adopted, which expresses the relative difference of the two values with respect to their mean, and produces no unpleasant results for any non-negative values of the flows (except with two zero values).

$$\frac{f(i,j) - f(k,l)}{\frac{1}{2}(f(i,j) + f(k,l))} \tag{1}$$

Since we assumed that the expression is written so that the largest element comes first, this relationship will take values between 0 (equal flow values) and 2 (f(i,j) very much larger than f(k,l)). This limited interval of variation will be very useful for the definition of aggregate objective functions, which would be much more difficult if we were using the direct ratios, varying between 1 and infinity.

The expression stating that the adjective applicable to the ratio between the flows in two cells is of order n is the following:

$$LS(n-1) \leq \frac{f(i,j) - f(k,l)}{\frac{1}{2}(f(i,j) + f(k,l))} \leq LS(n) \tag{2}$$

For simplicity, we shall define LS(0) as the algebraic symmetric of LS(1), which allows that in cases of identical values, in which we may not be sure about which is larger, the bounds of the constraint on the ratio be placed symmetrically around zero.

These constraints can be defined between two cells in the matrix (normally, but not necessarily in the same row or column), or between a cell in the matrix and the sum of its row or column.

The following constraints are defined on the interval limits LS(n) in order to ensure the consistency of the solutions obtained for the flows.

Non negativity of the upper bounds of the classes

$$LS(n) > 0, \ \forall n \geq 1 \tag{3}$$

Classes defined by growing order of their upper bounds

$$LS(n) \leq LS(n+1), \ \forall n \leq N-1 \tag{4}$$

Approximate values for the bounds of the classes, associating each of the adjectives with a typical ratio. Using the same example with 6 classes as above, we could have:

Table 1 Default definition of ratio classes

Class	Adjective (larger than)	Typical Ratio (up to)	Relative Difference
1	very weakly, ie identical	1.10	0.095
2	weakly	1.50	0.400
3	moderately	4.00	1.200
4	strongly	10.00	1.636
5	very strongly	50.00	1.922
6	extremely	infinity	2.000

This can be expressed by a set of pairs of inequations. If we call LST(n)the typical upper bound of class *n*, and allow it to float by some Δ (for instance 0.05), we can write

$$LS(n)-\Delta \ \leq \ LS(n) \ \leq \ LST(n)+\Delta, \ \ \forall n \leq N-1 \tag{5}$$

with a rigid upper bound of the last class

$$LS(N) = 2 \tag{6}$$

and finally, for symmetry,

$$LS(0) = -LS(1) \tag{7}$$

As we can see, the use of these typical values and a small enough Δ (as the one suggested) effectively covers the non-negativity and monotonicity constraints mentioned above.

OBJECTIVE FUNCTIONS

We wish to obtain a stable set of values of f(i,j) that satisfies the sum constraints as well as the declared ratio constraints. In the presence of a given set of ratio constraints, subject to the structure laid above, there may be or not a set of flows that satisfy all those constraints.

It will often happen that the set of ratio constraints defined be incoherent (no solution), but a frequent case will also be that the constraints defined allow a wide field of variation of the flow values.

The intention is to drive the estimation through a constructive process, along which knowledge of the implications of the adjective statements over the set of flow values is improved.

The final goal is to reach a set of ratio constraints that is coherent and produces a flow matrix with little slack (possible fluctuations) in its values. For this, a sequence of 4 problems is defined, along which the set of applicable ration constraints may have to be changed.

In all these problems, the constraints of:

- non negativity of flows f(i,j) and of class bounds LS(n)
- row and column sums
- cells with very small flow
- structure of ratio intervals (listed above)
- ratios between cell values

are applied, bearing in mind that the latter may be transcribed differently in successive problems, and be "adjusted" along the process, either because a ratio needs to change class to restore consistency to the set of constraints, or because a new ratio constraint must be added to the problem to decrease the remaining slackness in flow values.

Check for inconsistencies in the set of constraints

We wish to check whether there is a set of flows that fully satisfies the set of ratio constraints (1)

$$LS(n-1) \leq \frac{f(i,j) - f(k,l)}{\frac{1}{2}(f(i,j) + f(k,l))} \leq LS(n) \tag{8}$$

and also, if this proves impossible, to obtain a measure of the degree of inconsistency.

Let us consider a parameter $C \geq 0$ (level of disrespect of the constraints), and rewrite the double inequation above in the following way:

$$LS(n-1)-C \leq \frac{f(i,j) - f(k,l)}{\frac{1}{2}(f(i,j) + f(k,l))} \leq LS(n)+C \tag{9}$$

or, if we call the lower bound LI, the upper bound LS, and the expression defining the ratio R((i,j), (k,l)) or, simpler still, R:

$$LI - C \leq R \leq LS + C \tag{10}$$

With the introduction of this parameter, these constraints can always be satisfied. The level of disrespect will be larger as C grows, which means we wish to minimize C such that all constraints are satisfied. If the solution is C=0, this means there is no inconsistency in the statements made, and we may proceed directly to Problem 3.

We may thus write that the Objective Function of Problem 1 is **Min C**

An alternative way to write the ratio constraints is:

$$C \geq LI - R \text{ and } C \geq R - LS \tag{11}$$

which allows the synthetic expression:

$$C = Max(\{LI - R\}, \{R - LS\}) \tag{12}$$

keeping unchanged the objective **Min C.** Let us call CMIN the optimum value of C obtained in this problem.

Identify minimal changes in the ratio statements

If the value of CMIN obtained in Problem 1 is non-zero, at least one of the ratio constraints could not be satisfied without recourse to this parameter. But there may be more than one constraint in which this case happens, and the solution of Problem 1 gives no clue towards finding how many and which those constraints are.

To get this information, we have to create two non-negative parameters specific to each ratio constraint, one additive, and another subtractive, so that the free variation of the expression is permitted.

If we refer the synthetic terms by the order of the constraints (and no longer by their indices in the matrix and the level of the ratio class), the following transformed expressions are obtained:

Instead of	$R(m) \leq LI(m)+C$	we have	$R(m)=LS(m)+\beta(m)-\chi(m)$	(13)
	$R(m) \geq LI(m)-C$		$R(m)=LI(m)-\alpha(m)+\delta(m)$	(14)

Since each ratio constraint was composed by two inequations, we now have two equations. In the case of the equation relative to the upper bound, an impossible satisfaction of the initial constraint will imply that $\beta>0$, whereas $\chi>0$ simply means that the ratio does not reach the upper level of the interval. Identically for the equation relative to the lower bound, an impossible satisfaction of the initial constraint implies $\alpha>0$, whereas $\delta>0$ means that this constraint is not active.

Since we already know that solutions with levels of disrespect of the constraint above are not necessary, we should impose that $\alpha(m) \leq CMIN$ and $\beta \leq CMIN$, $\forall m$.

We now look for the set of minimum disrespect, ie a set of values of α and β with a minimum sum. The objective function will thus be:

$$F = Min \sum_{m} \left(\alpha(m) + \beta(m)\right) \tag{15}$$

In the case of ratio constraints where $\alpha(m) > 0$ we must declare them (at least) one level down. We may check whether a change of more than one class will be needed by comparison of the value of that $\alpha(m)$ with the width of the next class. Similarly, in the constraints with $\beta(m) > 0$ we should increase the ratio class by at least one level.

These minimal changes must be made only if they do not violate the perceptions of the transport specialist. If there is a conflict in this direction, other changes (non minimal) may be introduced, constraints may be added or removed, and then Problem 1 run again.

After these adjustments have been made, we must check run Problem 1 again to check whether it is now possible to obtain CMIN = 0. If not, Problem 2 must be run again to locate the new adjustments needed.

When a set of constraints that generates CMIN = 0 is reached can we move on to the next phase, where the ratio constraints are stabilized and interval bounds become constant values, the decision variables then being the flows in the matrix cells.

Finding stable flow estimates compatible with the ratio constraints

Once the interval bounds have been stabilized, there will be most likely a possibility of fluctuation of the flow values. In principle, this fluctuation will be larger when the number of ratio constraints used is smaller.

It is thus important to measure the slackness in the flow estimates, so that the analyst may have a notion of how stable are the values with which he must work on his dimensioning job. As long as he feels that the estimates are to "loose", he will keep asking for (or generating himself) more ratio constraints. He will stop when he feels that the lack of firmness in the ratio statements becomes larger than his discomfort in working with the slacks of the current flow estimates.

For the measurement of those slacks, two antagonic problems are solved, with the same set of constraints, one minimizing and the other maximizing the same objective function.

The constraints used are those of non-negativity of flows, and strict respect for the ratio intervals (with the fixed bounds obtained previously). Using again the synthetic notation and the index *m* for the order of the constraint, we have:

$$LI(m) \leq R(m) \leq LS(m) \quad (16)$$

The function chosen for objective is the (sum of) heterogeneities of the numeric values of function in the same ratio interval. By looking for a minimum of this heterogeneity, we wish that the numeric translations of the same adjective (expressing a relationship between two matrix cells) be as similar as possible.

To measure this heterogeneity the sum of square deviations of the flows ratios in each class (with respect to their average) was used, the sum being then accumulated over all classes. If we call this measure HET:

$$HET = \sum_W \left[\sum_m [R(m)-\mu(R(m))]^2, m \in \Phi(W) \right] \quad (17)$$

in which $\Phi(W)$ is the set of constraints defined for class *W* and $\mu(R(m))$ is the average of the numeric values of the function *R* in the constraints of order *m* in that set.

The total number of parcels in this double sum is always equal to the total number of declared ratio constraints.

The final solution will be obtained with Problem 3—***Min* HET**, but it is wiser to consider it final only after the set of flows thus obtained is close enough to the set of flows obtained by solving Problem 4—***Max* HET**.

The discrepancy of flows between the solutions of these two problems may be measured for instance through the same function *R* (difference relative to the mean), computed for all homologous cells in the two estimated matrices. As long as these are considered too different, more ratio constraints should be added and the resulting remaining slacks measured.

A SMALL EXAMPLE

Problem description

Let us consider a small example taken from a real case involving a new interchange being built in Lisbon (Cais do Sodré), in which suburban rail and ferry boats will bring passengers into the city for distribution by the Metro, the urban buses and walking (this interchange is very close to the traditional center of the city). The real problem is a bit more complicated, since there are also taxis and a new light rail system, but a smaller dimension of the matrix simplifies the understanding of the example.

Modes have been numbered as follows:

Table 2 Numbering of modes in example

1	2	3	4	5
Suburban Rail	Ferries	Metro	Bus	Walk

and the same 6 classes defined above for intensity of the relationship "greater than" have been adopted.

A total of 10 order relation statements have been used:

1 (From Rail to Metro) versus (from Rail to Bus) → Class 4: strongly larger
2 (From Rail to Bus) versus (from Rail to Walk) → Class 1: identical
3 (From Rail to Metro) versus (from Rail to Ferries) → Class 6: extremely larger
4 (From Ferry to Metro) versus (from Ferry to Rail) → Class 3: moderately larger
5 (From Ferry to Metro) versus (from Ferry to Walk) → Class 3: moderately larger
6 (From Ferry to Metro) versus (from Ferry to Bus) → Class 5: very strongly larger
7 (From Metro to Rail) versus (from Metro to Ferries) → Class 2: weakly larger
8 (From Bus to Rail) versus (from Bus to Ferries) → Class 2: weakly larger
9 (From Walk to Metro) versus (from Walk to Rail) → Class 2: weakly larger
10 (From Walk to Rail) versus (from Walk to Ferries) → Class 1: identical

Many more relationships could be defined (for 20 non-zero cells in the matrix, their total is 190), but this was considered a "non-controversial" set. Expansion of the set would be made only if needed.

Based on previous studies, it was possible to estimate with reasonable accuracy the flow brought into the interchange by each mode (row totals in the matrix). Column totals were defined by mode capacity, but assumed unknown. The model thus takes a "=" sign for the row totals and a "≤"sign for the column totals.

The values considered for such totals were:

Table 3 Inbound and maximum outbound flows per mode

Mode	Rail	Ferries	Metro	Bus	Walk
Row Total	12000	6000	3000	3000	1000
Column Total	12000	6000	10000	4000	

As can be seen, Rail and Ferries and working at capacity on the inbound direction. There is no point in constraining the capacity for the walking mode, so the total of column 6 is left unconstrained.

It was imposed that all values in the flow matrix were non negative, and that the diagonal be all with zero values. The variations of the default class bounds (taken as the typical values described above in section 2) were allowed up to a maximum of 0.05 in either direction.

All the calculations referred hereafter have been performed using the EXCEL spreadsheet SOLVER tool. Since the problems to solve are non-linear, the risk exists that the computed solution is only a local optimum. Indeed we have found this to be the case, which has lead to the adoption of a scheme whereby repeated runs of the SOLVER were made using newly generated random values for the free variables in each of the problems. For each problem a series of 10 runs was made, in which typically three of four times the best of the optima would be achieved.

Consistency checking

For consistency checking, we have solved Problem 1 as defined above. Maybe due to the fact that this is a small matrix, and a small number of statements was made, a value of zero could be found for CMIN, so there was no need to run Problem 2 or to change the level of any of the ratio statements.

The flow matrix thus obtained is already a feasible solution for the estimation problem:

Table 4 First feasible flow matrix

	.Sub_Rail	Ferry	Metro	Bus	Walk	Sum_Row
Sub_Rail	0	228	7581	2020	2172	12000
Ferry	1682	0	2396	249	1673	6000
Metro	786	578	0	798	837	3000
Bus	1484	1137	0	0	379	3000
Walk	21	23	23	933	0	1000
Sum_Col	3974	1966	10000	4000	5060	

The limits obtained for the classes for the ratio relationships were those in Table 5. As can be seen, the upper bounds of classes 2 to 5 were all lowered by the maximum allowed change (0.05), the upper bound of class 1 suffering also a change of the same direction but of much smaller magnitude.

Table 5 Ratio class limits before and after optimization

	Default Values		Optimized Values	
	Quotient	Relative Difference	Quotient	Relative Difference
LS(1)	1.10	0.095	1.10	0.093
LS(2)	1.50	0.400	1.42	0.350
LS(3)	4.00	1.200	3.71	1.150
LS(4)	10.00	1.636	8.67	1.586
LS(5)	50.00	1.922	30.15	1.872
LS(6)	infinity	2.000	infinity	2.000

We can verify that the flows in the matrix respect the ratio constraints: for instance, on the first constraint:

(From Rail to Metro) versus (from Rail to Bus) $\rightarrow$ Class 4

These flows are respectively 7581 and 2020, for which the quotient is 3.753 and the relative difference is 1.158. This value is located between LS(3) and LS(4) as it should. The same verification could be made for any other constraint.

Finding a homogeneous solution and measuring its stability

After a feasible solution has been found, we must run problems 3 and 4 (*Min* HET and *Max* HET), trying to find solutions that, whilst verifying all constraints, differ as much as possible between themselves.

For that purpose, constraints must be rearranged in the spreadsheet, sorting them by class of relationship to ease the computations of intra-class variances. Again we should run the SOLVER several times, starting from different initial random flows. But here the class limits are rigidly defined.

The following table (Table 6) shows the results obtained for those two problems (multiple solutions exist with the same extreme values of **HET**, both at the minimum and at the maximum, but they always present very small changes among themselves). The vector format is chosen here to simplify the perception of the differences between flow values in homologous cells. Along with the flows, the values of relative difference and absolute difference are shown:

In this table we can see that the differences are most smaller than 100 (passengers per hour) in absolute value, and that in all such cases the corresponding relative difference is smaller than 20%. Several cases remain with relative differences above 20%, but they all correspond to matrix cells with very little flow (maximum flow in such cases is 108).

Table 6 Flow values in extreme heterogeneity solutions

Mode_in	Mode_out	Min HET Flow	Max HET Flow	Relative Difference	Absolute Difference
Rail	Rail	0	0		0
Rail	Ferries	249	249	0.00	0
Rail	Metro	7501	7501	0.00	0
Rail	Bus	2024	2024	0.00	0
Rail	Walk	2226	2226	0.00	0
Ferries	Rail	1672	1672	0.00	0
Ferries	Ferries	0	0		0
Ferries	Metro	2381	2381	0.00	0
Ferries	Bus	275	275	0.00	0
Ferries	Walk	1672	1672	0.00	0
Metro	Rail	736	838	0.13	101
Metro	Ferries	671	588	0.13	83
Metro	Metro	0	0		0
Metro	Bus	593	525	0.12	67
Metro	Walk	1000	1049	0.05	49
Bus	Rail	1304	1480	0.13	176
Bus	Ferries	1188	1039	0.13	149
Bus	Metro	17	0	2.00	17
Bus	Bus	0	0		0
Bus	Walk	491	481	0.02	11
Walk	Rail	50	98	0.65	48
Walk	Ferries	55	90	0.47	34
Walk	Metro	55	108	0.65	53
Walk	Bus	840	704	0.18	135
Walk	Walk	0	0		0

These levels of inaccuracy can be taken as good enough for the subsequent dimensioning of the interchange. If this was not the case, more ratio statements would have been requested in order to diminish the possible variability of the estimated flows.

For the purpose of dimensioning, the matrix obtained in the *Min* **HET** problem is used.

Table 7 Final estimated flow matrix

	Sub_Rail	Ferry	Metro	Bus	Walk	Sum_Row
Sub_Rail	0	249	7501	2024	2226	12000
Ferry	1672	0	2381	275	1672	6000
Metro	736	671	0	593	1000	3000
Bus	1304	1188	17	0	491	3000
Walk	50	55	55	840	0	1000
Sum_Col	3762	2163	9954	3732	5389	

CONCLUSIONS

A methodology has been presented to estimate flow matrices in cases for which very little quantitative information is available, namely at best the row and/or column totals. But judgment about the likely ratio of flows in pairs of cells can be used in connection with those constraining totals.

The method presented here shows how to produce such a matrix, not only as a "best" estimate, but also measuring an indication of the stability of that estimation.

An example for a public transport interchange was presented, corresponding to a 5 × 5 matrix. Real examples have already been performed for matrices up to dimension 8, and the level of difficulty in reaching the final solutions has always been relatively small.

This method seems very promising for use in wider contexts, since it can be applied on top of any level of previous quantitative information for matrix estimation, and filling in the gaps with the adjective information extracted from the experts. Further research is going on in this direction.

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