

TOPIC 23 RAIL SECTOR TRANSPORT

# A METHOD OF CAPACITY COMPUTATION FOR COMPLEX RAILWAYS SYSTEMS

### LIVIO FLORIO

Department of Transport Systems and Mobility Polytechnic of Milan P.za L. da Vinci 32, 20133 Milano ITALY

## LORENZO MUSSONE

Department of Transport Systems and Mobility Polytechnic of Milan P.za L. da Vinci 32, 20133 Milano ITALY

# Abstract

The capacity of railway systems is usually calculated by different methods for lines (see UIC and DB methods) and for stations. The analytical method we propose, computes, simultaneously and without using time-tables, the maximum number of trains and their percentage occupation in the network for a determinate time interval, for a particular composition of train classes and for each element of the system.

### INTRODUCTION

Rail Administrations are used to calculating rail line capacity through empirical or analytical methods. The most widespread analytical methods for the evaluation of rail line capacity have been developed by Union General de Chemins de Fer (UIC 1988) and by German railways (DB, 1974).

The researchers in this field have recently provided very many patterns for single and double track rail line simulation (Petersen and Petersen-Taylor 1982; Yokota 1980), also introducing values of the potential capacity (number of trains) of the rail lines.

However, these methods do not allow any calculation of the potential capacity of the tracks at rail junctions and at stations, therefore separate methods are usually employed with this regard. The problem of the junction capacity has been first approached through analytical methods by the German Potthoff (Potthoff 1970) who treated the generic complex rail node into a simple node, in which the possible average number of passages may take place at the same time. The capacity of the tracks in the station is usually determined separately, on the basis of the average time they are occupied.

This paper proposes a single method allowing the simultaneous calculation of a generic "rail system" (scheme of tracks) which is meant to be a set of lines, stations and junctions.

The scheme in Figure 1 provides a good example: circles mark the so called "nodes" or "areas of conflicts between trains" (the set of tracks on which the run of the train may be affected by a lateral interference from other trains).

The "nodes" identify more "paths" which are meant as the starting and ending points of a given movement of the train. At a station or at a complex junction, several passages may take place simultaneously (compatible passages) or at different times (incompatible passages). The tracks joining junctions or stations (usually several kilometres long) are called "lines", whereas the tracks beside the platform at which a train can stop for passengers to board or alight or for goods service, are called "station tracks". The set of the latter constitutes the "station", which may be enclosed between two junctions: the entry and the exit one.

The definition of the "capacity of a rail system" implies the calculation of the highest number of passages allowed to occur in a given rail scheme within a definite time interval. This capacity is to be meant as the maximum number of trains which are allowed to pass on each point of a rail system or scheme, at a given time interval and in the presence of conflicts among trains. It is understandable that such potential conflicts are ruled through the right of way which is given to trains according to FIFO (first in—first out) rule or to train category.

The conflicts are always likely to occur because of the flow itself (as it happens in any other type of transport/system), although, in railway systems, train timetable has the sake of eliminating such conflicts. However, as a matter of operating policy, trains often do not respect the timetable because of several reasons.

In this paper it isn't taken into account the existence of a timetable, therefore the capacity of a scheme is determined on the basis of the number of running trains which can take place on that scheme, under pre specified traffic hypotheses, and in presence of conflicts among trains.

This approach allows the association of a given rail scheme to the capacity, namely to the number of trains traversing each point of that scheme, at a given time interval and in presence of pre specified traffic hypotheses. This is the solution of the timetable project. This method is alternative to that which verifies applicability of an assigned timetable and does not depend on the complexity of the analysed scheme. Since it is totally general, it may be used on both single and double track lines and on any type of station or rail node. It is based on the "probability of interference" which may occur among trains and in analytical terms it represents a search of a optimum with constraints.

RAILWAY SYSTEM CAPACITY FLORIO & MUSSONE

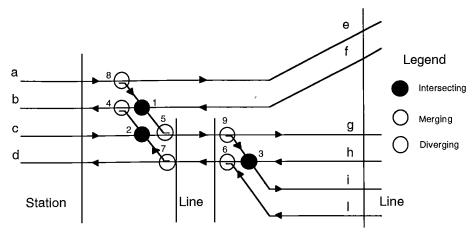


Figure 1 Scheme of a railway system

### THE CAPACITY OF A SIMPLE JUNCTION

The simplest case which may be considered, is that of an intersection between two lines which are not endowed with any points, as shown in Figure 2. For this exposition, only two paths are considered here (1 and 2) which are part of the same lines 1 and 2, and which are run by trains of a single category (for example only freight trains or fast trains).

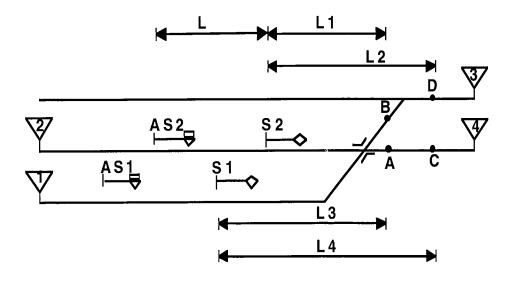


Figure 2 Simple railway junction

AS1 and AS2 denote the warning signals on the lines 1 and 2, whereas S1 and S2 denote the safety signals of the intersection. B and A denote the points at which the intersection is released,

namely the sites at which a train following the route 1 or 2 is allowed to enter the intersection soon after the rear carriage of the preceding train has passed.

D and C denote the points at which a line is released, when a train is allowed to follow a second train on the same line (1 and 2 respectively) after the signals AS1 and AS2 allow the train to enter the line.

In normal practice, the intersection is considered occupied, for example by a train running on line 1, from the instant at which the signal AS1 allows the train to enter the intersection until the instant at which the rear carriage of the train passes over the point B. At this moment, a train on line 2 may be allowed to enter the intersection. The signal AS2 and the point A then regulate the passing of the second train.

In the case of two trains running on the same line, the second train is not allowed to enter the intersection until the rear carriage of the first train has passed over the point D (or C), thus the signal AS1 (or AS2) is given.

When the intersection is "blocked" by a train running on line 2, a further train coming on line 1 can run up to the signal S1 and stop here until the entrance to the intersection is allowed. In this case, the time required to cross over the node is calculated from the moment at which the standing train is allowed to enter the intersection by the signal S1 until the moment at which the rear carriage of the same train passes over the point B, then running along a tract L2+L (where L is the length of the train).

While running from AS1 to S1, it may not be possible for a train to enter the intersection since the signal S1 does not allow the entrance, or on the contrary it may happen that the signal S1 allows the train to accelerate instead of halting. In this case, the time required to cross the intersection turns out to be different. However the application is approximated on the basis of two single possibilities: no stops during the run or a stop at S1.

However the time required to cross the intersection might be given a conventional value, such as the longest possible time or any other value which represents the average time among all possible times required to cross the intersection. The first case will be called "regular link trip time" (the train 1 running from AS1 up to the point B) and the second case "irregular link trip time" (the same train running from S1 up to the point B).

On the basis of the diagrams related to the running of trains, the following parameters are calculated:

- $t_1 = regular link trip time needed by a train running on line 1;$
- $t_1^* =$  irregular link trip time needed by the same train;
- t2 = regular link trip time needed by a train running on line 2;
- $t_2^*$  = irregular link trip time needed by the same train.

For reasons of safety a time interval is required between trains running on lines 1 and 2. Since such time interval is  $\Delta t_1$  on line 1 and  $\Delta t_2$  on line 2, the following conditions should be respected:

$$t_l, t_l^* \ge \Delta t_l \tag{1}$$

and

$$t_2, t_2^* \ge \Delta t_2 \tag{2}$$

Where  $\Delta t_1$  and  $\Delta t_2$  are the equivalent constraints on the line capacity (minimum distance between trains). They normally correspond to the distance between trains running from AS1 to D (or from AS2 to C).

In absence of timetable (absolute stochastic) and rules regulating the admission of trains to the links (so that the first arrived passes first), within an observation interval T (minutes) the link will be traversed by:

 $n_1 = \text{trains with regular link trip time on 1};$ 

 $n_1^* =$  trains with irregular link trip time on 1;

 $n_2 = trains$  with regular link trip time on 1;

 $n_2^* = train with irregular link trip time on 2.$ 

The question over the capacity of the simple junction is then reduced to the search for the maximum number of trains

$$\max(n_1 + n_2 + n_1^* + n_2^*) \tag{3}$$

The definition of the link trip time considers the following condition of congruence (observation interval longer than or equal to the sum of the link trip times):

$$T \ge n_1 t_1 + n_2 t_2 + n_1^* t_1^* + n_2^* t_2^*$$
(4)

Moreover, the definition of the link trip time takes into consideration all the conditions related to further irregular link trip times, if any, owing to the absence of timetables or order of priorities and given by:

$$p_{1} = \frac{n_{2}t_{2} + n_{2}t_{2}}{T}$$
(5)

and

$$p_2 = \frac{n_1 t_1 + n_1 t_1}{T}$$
(6)

where

$$p_l + p_2 = l \tag{7}$$

All the variables are  $\geq 0$ .

The optimization problem (as in equation 3) for the unknown parameters  $(n1, n1^*, n2, n2^*)$  remains unsolvable. The capacity of the intersection is given only by adding a further condition to two of the four unknown parameters of the kind:

$$n_1 = a n_2$$
 (8)

or

$$n_1 = b n_1^*$$
 (8 bis)

where a and b are constants.

By varying the values related to the number of trains  $(n1, n1^*, n2, n2^*)$  according to the four bounds 4), 5), 6), 7), 8) thus solving the optimization problem 3), it is possible to define the capacity of the examined elementary intersection, namely to calculate the number of trains allowed in T.

When considering trains of different categories (eg fast, freight, etc.) trip times (regular and irregular) shall be calculated by each category and according to the criteria stated above. Thus the formulas will be:

$$max(\sum_{i,k}n_{i,k} + \sum_{i,k}n_{i,k}^{*})$$
(3bis)

$$T \ge \sum_{i,k} t_{i,i} n_{i,k} + \sum_{i,k} t_{i,i}^* n_{i,k}^*$$
 (4bis)

where i = route

k = train category

Therefore, if train k runs along the route i, and is likely to be affected by the interference of other trains (of the same category) running along a route which is incompatible with i, the value of the generic probability of irregular link trip for train k is given by

$$p_{i,k} = \frac{\sum_{i,k} n_{i,k} t_{i,k} + \sum_{i,k} n_{i,k} t_{i,k}}{T}$$
(5bis)

Also in this case the solution to the problem of optimum requires the addition of further conditions of traffic (or relationship of traffic) on some of the lines or routes of absolute or relative type (8ter and 8quater respectively):

$$a \le \sum n_{ik} + \sum n_{ik,ir} \le b$$
 (8ter)

$$a \leq \frac{\sum n_{ik} + \sum n_{ik, jr}}{\sum n_{sd} + \sum n_{sd,hm}} \leq b$$
 (8quater)

The first equation implies that the traffic on one or more routes (or only by some train categories) is higher or lower than a prefixed numeric value. The second one, implies that the ratio between the total number of trains (regular + irregular) for a given category and for a given route and the total number for another category (or the same) and for the same (another) route varies between two limits.

The amount of conditions of the type 8ter) or 8quater) should be that which is likely to give a definite solution to the problem of the general optimum.

## **REDUCTION OF COMPLEX NODES INTO ELEMENTARY NODES**

Each complex rail node can be decomposed into elementary nodes. Capacity values will then be searched for each elementary node, according to the principles mentioned above.

The overall capacity is meant as the highest number of trains circulating within a rail system in the observance of the limits imposed on rail traffic.

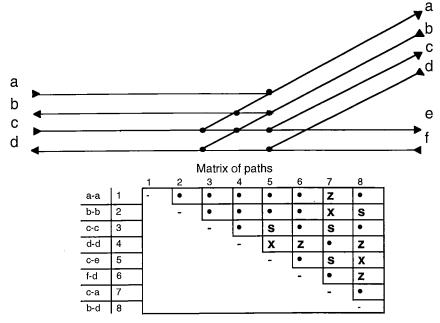
In such case, the overall capacity is strictly connected with simple nodes, since the latter are part and parcel of a complex node. Consequently, the values related to the number of trains which may take place on each single path are linked to the capacity of each simple node.

In other words, the complex rail node may be regarded as the equal of a cross-roads endowed with traffic flows. In the latter, the capacity and the "saturation rate" are calculated separately for each lane group which has been previously determined (HCM 1985), while in the former, the nodes are more complex and not feasible paths are usually more linked together than are the lanes converging into a cross-roads.

In practice, the identification of the simple nodes is performed through the "matrix of incompatibility" and, moreover through the "tree of incompatibilities among paths". Consider the scheme reported in Figure 3 as example.

The "matrix of the incompatible paths" reported on the bottom of the figure, results from the analysis of the conflicts between paths which have been considered two at a time. From this

matrix, consecutive comparisons among paths lead to the construction of the "tree of the incompatibilities" reported in Figure 4.



Legend:

- compatible
- z merging
- s diverging
- x intersecting

## Figure 3 Scheme of a complex junction as combination of basic junctions

The outermost bounds of the tree, which are not included into a group of higher level, represent the "groups of incompatible paths" or "simple node", which may be both real and fictitious.

Therefore, a "simple node" represents a single way system (with a train on a single path) whereas the trains following other paths which are incompatible with the former, give rise to the "waiting queue".

Any change in the number of trains running on a path, implies a decrease or an increase in the amount of occupation of those groups of paths in which it is included, creating a "concatenation" among simple nodes. Similarly the values of number of trains resulting from the maximum occupation of a simple node (outermost bound of the tree) are as large as both the number of categories of trains and the number of paths including that node. This number of trains must be the same in the other nodes or groups it belongs to. For each simple node the same service conditions are applied in the form of analytical equations (3bis), (4 bis), (5bis).

The "capacity of the complex node" is the vector  $(n_{i,r})$ , namely the set of number of trains on a generic path i of category r compatible with the capacity of all the simple nodes, given the hypotheses on the traffic sharing.

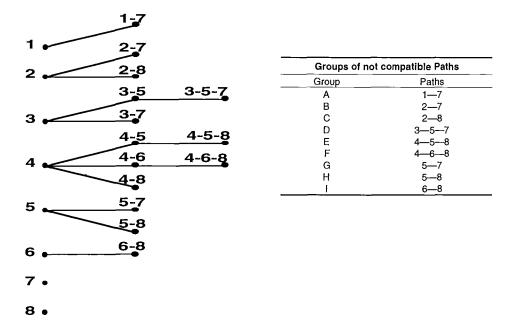


Figure 4 Tree of incompatible paths

## PROBLEM FORMULATION TO COMPUTE RAILWAY SYSTEM CAPACITY

### The criterion of optimum

Let  $n_{ik}^{c}$  be the number of trains of category k, found on node c, running on path i with no interferences from other trains. Let  $n_{ik,jr}^{c}$  be the trains of the same category on the same path, which are affected by interferences from the trains of category r on path j, which is clearly incompatible with i.

The problem of the calculation of the capacity implies the search of the total number of trains in the railway system (objective function):

$$\max \sum_{c} \left( \sum_{ik} n_{ik}^{c} + \sum_{ik,jr} n_{ik,jr}^{c} \right)$$
(9)

with traffic flows  $n_{ik}^{c} \ge 0$  and  $n_{ik,ir}^{c} \ge 0$  and with the constraints already described.

Further criteria of optimum formulation are alternative to the 8): for example the maximization of the number of trains on one or more nodes, or the minimization of the overall waste of time of trains on system.

Such criteria, all of which can be translated into analytical terms, do not search the "capacity of the system", since they only find a sub-optimum working in order to meet the particular needs.

### Constraint equations

Constraint equations are of seven types:

Non-negativity of all the variables (number of trains)

$$(\mathbf{n}_{\mathbf{i}\mathbf{k},\mathbf{j}\mathbf{r}},\,\mathbf{n}_{\mathbf{j}\mathbf{r}}\geq\mathbf{0}\,)\tag{10}$$

#### Probability of interference

On each simple node, the probability that the generic train of category k, running on path i may be interfered (conflict) (right of way given to the train arrived as first) results from:

$$p_{ik,jr} = \frac{\left(\sum_{jr} n_{jr} t_{jr} + \sum_{jr,fg} n_{jr,fg} t_{jr,fg}\right)}{T}$$
(11)

where:

 $t_{ir}$  = trip time (regular) on a simple node taken by the train of category r on path j (evidently incompatible with j);

tir.fg= same amount of time in case of irregular trip, due to a train of category g running on path f, which is incompatible with j.

For exposition, the value of 11) for all possible combinations of cases, can be translated into a matrix. This latter is the same size as the matrix related to the irregular trip times. Obviously the sum of its terms by lines and columns equals the unit (sum of all possible cases). Consequently, the number of trains of category k on path i, interfered by trains r on path j can be written as:

$$\mathbf{n}_{ik,jr} = (\mathbf{n}_{ik} + \mathbf{n}_{ik,jr}) \mathbf{p}_{ik,jr}$$
(12)

Along with 11) (which gives the mean probability) it may separately be considered the case pik.ir = 1, which is the "worst case". In this case all trains k on path i are supposed to be interfered by the trains entering the node and belonging either to the same or to a different category. This corresponds to the hypothesis that trains r on path j are always given the right of way. The railway engineer can therefore evaluate the effect that the right of way among trains exerts on the capacity. Such case gives a rough indication of the potential capacity which is closest to the reality.

#### Flow continuity in the stations

For every platform at station, b, the number of trains entering during T must equal the number of trains which are exiting. This can be written as:

$$\sum_{ik} n_{ik}^{(b)} + \sum_{ik,jr} n_{ik,jr}^{(b)} = 0$$
(13)

where:

 $\sum n_{i\nu}^{(b)}$ is the number of regular trains on all paths leading to or out of track b (the sum is performed by paths leading to (or out of) and by train categories);

 $\sum n_{ik}^{(b)}$  indicates the same for irregular trains.

Generally, the sign + (plus) indicates the number of trains entering the station, whereas the sign -(minus) indicates the number of trains exiting.

#### Capacity of tracks at stations

For each track at station—b—the number of trains crossing the track must be lower than or equal to the capacity of the track itself. This can be written as:

$$\sum n_{ik}^{(b)} t_k^{(b)} + \sum n_{ik,jr}^{(b)} t_k^{(b)} \le T$$
(14)

where

 $\sum n_{ik}^{(b)} t_k^{(b)}$  is the number of trains on all paths (with regular trip times) which interest the generic

track at the station—b—with a stop time or crossing time equal to  $t_k^{(b)}$ ;

 $\sum n^{(b)}_{ik,jr}$  and  $t_k^{*(b)}$  denote the same values for trains k which show irregular trip times for entering, exiting or both.

Time  $t_k^{(b)}$ , introduced into 12), is composed by three terms:

- dwell time of a train at a station as it is hypothesized, or trip time on the track;
- trip time on the link leading to the station;
- the analogous amount of time the path (and therefore the track) is busy during the departure of the train

Similarly time t  $k^{*(b)}$  is composed by three elements:

- dwell time of a train at the station or trip time on the track at the station;
- trip time on the entry path in case of interference (if any) by a second train;
- trip time on the exit path (added to or replacing the entry path).

#### Congruence in each node

For each simple node the sum of regular and irregular trip times must be lower than or equal to the interval T:

$$\sum n_{ik}^{(H)} t_{ik}^{(H)} + \sum n_{ik,jr}^{(H)} t_{ik,jr}^{(H)} \le T$$
(15)

where

nik<sup>(H)</sup> denote the trains on path i (passing through simple node H) with regular trip times.

nik.ir<sup>(H)</sup> denote analogous trains with irregular trip times;

#### Capacity of lines

For each line track—L—found in the system under consideration, the number of trains running along the track in the time interval T must be lower than or equal to the capacity of the same line  $(P_{L}(N))$ :

$$\sum n_{ik}^{(L)} + \sum n_{ik,jr}^{(L)} \le P_L^{(N)}$$
(16)

where:

 $(\sum n_{ik}^{(L)})$  denotes the number of trains belonging to category k and which take regular trip times on the paths leading to line track L

 $(\sum n_{ik,jr}^{(L)})$  denotes the analogous number of trains which take irregular trip time.

The capacity of the line,  $P_L^{(N)}$ , is then calculated according to the total number of trains N entering the line, according to the method UIC (UIC, 1978) or alternatively, according to DB (DB, 1974).

The capacity of a line track depends on the traffic composition (train categories) which characterizes such track (UIC, 1978). In fact  $P_L$  (N) can be written as:

$$P_{L}^{(N)} = \frac{T}{t_{fm} + t_{Zl}}$$
(17)

where

T = period of reference

 $t_{fm}$  = average distance between trains

 $t_{711}$  = mean range within which each train time can vary.

 $t_{fm}$  and  $t_{zu}$  can in turn be obtained as average values according to the composition of the traffic found on the line, therefore the constraint 15) acts interactively according to the number and category of the trains running on the track.

The following matrixes must be set up:

- matrix of headways between trains (all train categories included on lines—{t<sub>k,r</sub>}—(included the case in which trains of the same category run on the same line). Such headways must include the extension time foreseen for each train;
- matrix of headways probabilities between trains—{p<sub>k,r</sub>}—function of the number of trains (by category) admitted on the line;
- matrix of the sum of the above mentioned matrixes  $\{t_{k,r}^* p_{k,r}\}$ , whose sum by lines and column gives the average headway between trains  $t_{fm} + t_{zu}$  which is to be introduced into 12).

### Traffic constraints

Constraints on traffic must be of two types: absolute and relative of the kind 8ter) and 8quater), which must be properly generalized in order to be applied not only to the simple nodes but also to the complex node.

Obviously, the number of added conditions must be sufficient so as to give a solution to the problem of the general constrained optimum

### THE SOLUTION TO THE PROBLEM OF OPTIMUM

The problem of optimum for a railway system, synthesized in the equations from 9) to 17), is of the non linear kind, since the constraints on interference probability (11)) are non linear.

The linearisation of the problem, however, can be achieved through successive iterations by fixing the probability values given by 11) for each case of interference. Then the number of trains is calculated by using the previous method. The values obtained are then used to recalculate the probability values 11) and applied to the problem in order to obtain the second approximation values. The iterations terminate when the solution is convergent, namely when the vector of the number of trains  $\{n_{ik}\}$  remains unchanged for each further iteration. The procedure of the calculation is performed according to the logic-operative diagram presented in Figure 5.

## **EXAMPLE OF APPLICATION**

Let consider the "railway system" reported on the scheme in Figure 6, which consists of three lines (ef, gh, il) which lead to four platforms a,b,c,d, through two complex nodes.

In this scheme nine simple nodes can be easily recognized without using "trees". Such nodes are marked with the numbers from 1 to 9 and can be grouped three by three according to their type: intersecting, merging and diverging. The observation interval is defined as T=240 min.

The following conditions are set up:

- a. or the entry paths (a,c,h,f,l) n<sub>R</sub>=n<sub>L</sub>=n<sub>R</sub> (number of fast trains=number of local trains=number of freight trains);
- b. absence of freight trains from a to e and from f to b;
- c. absence of fast trains from g and to h;
- d. number of trains, from 8 to 5 and from 7 to  $4, \ge 1$ ;
- e. dwell times at platforms:
  - 5 minutes for local trains,
  - 3 minutes for express trains,
  - 3.2 minutes for freight trains with regular dwell time,
  - 5.2 minutes for freight trains with irregular dwell times.

The capacity of the lines is calculated by using the values of  $t_{fm}$  and  $t_{zu}$  (15)) as reported in Figure 6. The trip times at nodes are calculated by path and train category, on the basis of standard curves of motion. The resulting regular and irregular trip times are reported in Table 1.

To set up the method it is necessary to impose:

- · conditions of non negativity on all variables;
- integer values on all variables;
- six equations of congruency on nodes;
- 10 constraints on the capacity of the lines;
- 17 equations expressing the probability of irregular trip times (eq 9);
- 4 equations of the type 11) and 4 of the type 12);
- 2 traffic conditions on trains (condition d);
- 4 conditions of congruency for the time occupation at platforms (12));

The application of the method by using consecutive iterations (for the example 8 iterations have been needed) provides the values of  $\{n\}$  reported in Table 2. The percentages related to the occupation of all the elements of the railway system (lines, stations and nodes) are calculated too (Table 3).

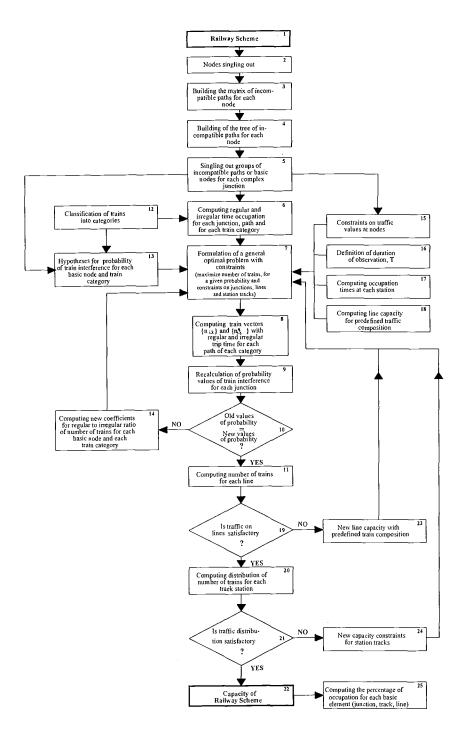


Figure 5 Flow-chart to determine capacity of a complex railway system

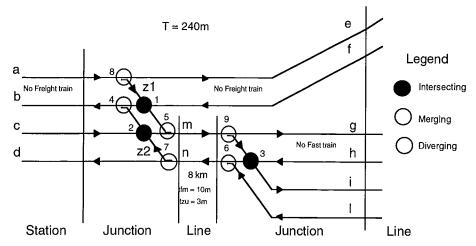


Figure 6 Scheme of Figure 1 with values for junction, station and line parameters of the example carried out

			Occupati	on times [s]			
Path	Length (m)		Regular Trair	าร		rregular Trai	าร
		Fast	Local	Freight	Fast	Local	Freight
tae	1600	96	144	192	105	198	312
tam	1200	72	108	144	81	162	264
tcm	1600	96	144	192	105	198	312
tmg	1600	96	144	192	105	198	312
tmi	800	48	72	96	57	126	216
tfb	1200	72	108	144	81	162	264
thn	800	48	72	96	57	126	216
tnd	1600	96	144	192	105	198	312
tnb	800	48	72	96	57	126	216
tin	800	48	72	96	57	126	216

Table 2	Solution of the optimal problem
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				Number	of trains				
		Fast Train	s		.ocal Trair	าร	F	reight Trai	ns
Entry Path	Reg.	Irreg.	Tot.	Reg.	Irreg.	Tot.	Reg.	Irreg.	Tot.
а	29.42	0.58	-30.00	28,55	1.45	30.00	-	•	-
b	10.45	19.55	30.00	10.45	19.55	30.00	-		-
с	3.27	1.73	5.00	3.27	1.73	5.00	3.27	1.73	5.00
d	0.00	4.00	4.00	0.00	5.00	5.00	0.00	6.00	6.00
е	29.00	0.00	29.00	27.50	0.00	27.50	-	-	-
f	9.92	19.08	29.00	9.92	19.08	29.00	-	-	-
g	-	-	-	2.16	3.18	5.34	1.64	-	1.64
h	-	- 1	-	0.53	0.47	1.00	0.00	1.00	1.00
i	3.69	2.31	6.00	2.16	-	2.16	1.64	1.72	3.36
1	0.53	4.47	5.00	0.00	5.00	5.00	0.00	5.00	5.00
m	3.69	2.31	6.00	4.32	3.18	7.50	3.28	1.72	5.00
n	0.53	4.47	5.00	0.53	5.47	6.00	0.00	6.00	6.00
z1	0.42	0.58	1.00	1.05	1.45	2.50	-	-	-
z2	0.53	0.47	1.00	0.53	0.47	1.00	-	-	-

Entry Path	% of occupation	Type of track		
a	100.00	Station		
b	100.00	Station		
с	24.77	Station		
d	28.41	Station		
е	46.83	Line		
f	44.73	Line		
g	8.72	Junction		
ĥ	2.18	Junction		
i	6.89	Junction		
1	13.82	Junction		
m	100.00	Line		
n	92.08	Line		
z1	3.23	Junction		
z2	1.04	Junction		

#### Table 3 Percentage of occupation for each track

### FINAL REMARKS

It is the opinion of the authors that all phenomena, connected to railway systems, have to be dealt analytically in an unitary manner avoiding any differentiation among nodes, stations and lines.

Any separation of these elements actually does not allow to take into account all the aspects which are strictly interconnected and which characterize a railway system, above all with regard to their potential capacity. In fact very often, capacity values—in number of trains—provided by several methods (UIC) for urban and extra-urban railways could not be applied in practice because of the existence of ramification or succeeding stations.

Nevertheless those methods such as UIC or DB are to be considered formally valid and applicable to rail way lines, when it is integrated with the check of nodes capacity, which has been here formulated in terms of a maximization problem.

On the other hand, simulation approaches should not be accepted, since the situations they analyse at a time has its own peculiarities, and cannot be taken as valid example for all possible situations. The obtained solutions, also with regard to capacity, are susceptible to revision through consecutive simulations performed over the time. The method reported here, has a range of many possibilities since it may be applied to several problems connected to rail way systems and to transport systems in general.

Among these possibilities, here are remembered:

#### a) A better configuration of timetable, after the capacity has been defined

The next step of the present method is the identification of the "optimal timetable", namely the one which reduces conflicts between trains to the lowest degree. The procedure is that of considering the interactions between trains running on a link, and then formulating a method for the optimization (minimization) of the wasted times of the trains. The procedure although analytical, is not different from the method set out here.

#### b) The choice of the most suitable rail scheme to meet the traffic demand (number of trains)

The consecutive application of the proposed method to several schemes, gives the values related to the trip times on tracks and lines in the observed time interval, by holding constant the train traffic flows on lines and at stations. Among these schemes will be preferred the one with equal flows but the least occupation times.

c) The identification of the measures to be actuated on specific areas of the rail scheme in order to better train circulation

Once the method has been applied and the link trip times have been obtained, it may be profitable to decrease the trip times on those nodes or links which result to be particularly busy, by reducing the number of trains on some paths, thus reducing irregular trips as well (this possibility is provided by 5bis).

The method has demonstrated its feasibility to any kind of situations concerned with rail system. Any increase in the complexity of the scheme corresponds to an increased number of equations for each node and constraint. Although such equations may be hundreds, the task can be easily afforded by the existing computers.

It may be interesting to set up an Artificial Intelligence software capable of exploring all the elements of a rail system, in order to define the constraints on the lines, the paths and in the stations. This would provide a good tool for engineers who may therefore avoid errors and hard tasks.

As it is comprehensible, the research on the capacity of rail way systems could be largely developed, in order to give a solution to problems which are particularly urgent nowadays. The authors wish themselves to be given the opportunity to treat the argument in detail, by reporting the results of further applications.

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