7TH WCTR
SYDNEY AUSTRALIA
1995

TOPIC 1
TRANSPORT AND
LAND USE (SIG)

MUSSA MODEL: THE THEORETICAL FRAMEWORK

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Abstract

The Bid-Choice theory of urban land market is developed here for the Santiago land use model MUSSA. Extensions to the original framework are: dwelling and lot size choices, suppliers dynamic behaviour, market equilibrium conditions and speculative behaviour of locators. Other extensions are interactions with the transport model and a population and firms growth model.

INTRODUCTION

MUSSA is a land use model of the city of Santiago developed for the Chilean Government. It was designed to interact with the four stages transport model of the city, called ESTRAUS, following the proposal of a five stages land use-transport interaction model 5-LUT (Martínez 1992a). The Santiago model also contains a sixth stage which feeds MUSSA with forecasts of total (non located) population and economic activity in the city, which is based on a microeconomic input/output model.

This paper describes the theoretical framework of MUSSA, discussing economic and statistical issues. It provides a framework to study the urban land market, including dwellings, assuming a competitive interaction among residential and firm locators.

THE BID-CHOICE THEORY

In a previous article (Martínez 1992b), the Bid-Choice theory was presented for the land market, which is a theory for the location choice and for the multi-locators equilibrium in the urban market. There, the equivalency between the random utility (McFadden 1978 and Anas 1982) and the random bidding (Ellickson 1981) approaches is demonstrated, producing a unified urban economic framework called the Bid-Choice model. Similar models have been obtained by Hayashi and Doi (1989) and Botchie et al. (1994) from a linear programming approach, and by Miyamoto (1993) by combining both approaches.

This paper presents an extension of the original Bid-Choice model in order produce a theoretical approach which deals with urban location choices taking into account land consumption, dwelling preferences, accessibility and attractiveness (or access) advantages, as well as environmental quality. This is called the dwelling-location model.

The deterministic dwelling-location framework

Let us call U_{hvi} the household h's indirect utility obtained from enjoying the use of a property with a dwelling type v and located at zone i, then $U_{hvi}=U_h(d_v, z_i, y_h - r_{vi}, \overline{P})$. Vectors d and z contain the set of attributes that properly describe dwelling types (including land size) and zone advantages (access and environment) respectively, y_h is the household's income, r_{vi} is the property rent (or cost of use) and $\overline{P}$ is the price of a composite good. Following Rosen (1974), it is possible to obtain the household h's willingness to pay to enjoy the use of the property (v,i), WP_{hvi}, achieving a utility level U^*_h, as the inverse of the indirect utility function in the property rent. Then:

$$WP_{hvi}=WP_h(d_v, z_i; U_h^*, y_h, \beta_h)$$

with β_h a vector of household's valuation of d and z attributes; they indeed represent hedonic (or implicit) prices of attributes. There is one function for each household and it is possible to derive a similar WP function for each firm, except for income and dwelling attributes. In the absence of subsidies, the domain of WP is [0,y], to comply with income constraints.

One property of the WP function is that, by definition, income appears in a linear form, independently of the utility function, which is a direct consequence of the linear relationship between rent and income in the income constraint. Then, we can write:

$$WP_{hvi}=y_h-\overline{WP}_h(d_v, z_i; U_h^*, \beta_h)$$

It can be shown that $\overline{WP}$ represents the expenditure function and, secondly, that the difference between the WP and rent values of a given location represents the household (consumer) surplus (CS) obtained from enjoying the use of that property.

Market equilibrium is attained as the spatial distribution of activities that accomplishes with the following conditions simultaneously, assuming H locators and Ω available locations.

1. Each household is located in a dwelling/zone option that maximizes its utility or surplus (CS) at an exogenous rent (r), across alternative location options S.

CONSUMERS $$\underset{(v,i)\in\Omega}{\text{Max}} \; CS_{hvi} = \underset{(v,i)\in\Omega}{\text{Max}} \; (WP_{hvi} - r_{vi}) \qquad \forall h\in H \qquad (1)$$

which must be verified by the location choice of every household/firm in the market.

2. In order to maximize the owner's profit, the consumer finally located in a given lot must be the maximum bidder. This best bidder rule assures that owners obtain the maximum rent for the property. Then, assuming that WP and bids differ by a locator speculative factor $w_g \in [0,y]$, rents are given by:

OWNERS $$r_{(v,i)} = \underset{g\in H}{\text{Max}} \left[WP_{gvi} - w_g\right] \qquad \forall (v,i)\in\Omega \qquad (2)$$

which is verified for each location (v,i) in the market supply Ω.

Then, each consumer chooses, among Ω alternatives, that location which maximizes his/her surplus; each owner chooses the best bidder among H consumers. Both conditions must be satisfied simultaneously obtaining the following equilibrium of the urban dwelling/land market. Replacing (2) in (1):

EQUILIBRIUM $$\underset{(v,i)\in S}{\text{Max}} \; CS_{hvi} = \underset{(v,i)\in\Omega}{\text{Max}} \; \left(WP_{hvi} - \left[\underset{g\in H}{\text{Max}} \; WP_{gvi} - w_g\right]\right) \qquad \forall h\in H \qquad (3)$$

Equation (3) represents the equilibrium equation of the activity system and is called the deterministic version of the dwelling/location Bid-Choice model. Writing one equation for each consumer, a system of equations of simultaneous solution in WP's parameters is obtained.

The equivalence of the bid approach (Alonso 1964) and the choice or maximum utility approach (McFadden 1978; Anas 1982) is observed here. Indeed, if consumer h is best bidder at (v,i) then $CS_{hvi} = w_h$, otherwise $CS_{hvi} < w_h$; therefore, the maximum CS occurs at a location where both utility and bid are at the maximum. If the speculative factor is zero, then WP equal bids and CS=0 at equilibrium for all competitive locators everywhere; this case implies full capitalization of location benefits by the landowner.

Corollary: If a consumer is the best bidder in a given lot, then that is his/her optimal (maximum utility) location. In a competitive case (CS=0) landowners obtain full capitalization of location advantages (see details in Martínez 1992b).

The stochastic disaggregate model

A more operational model can be developed by assuming that locators' WP can not be completely identified by observing their preferences, hence if the modeler accepts a level of ignorance on locators' behaviour, then the deterministic model is no longer valid.

In the stochastic model, willingness to pay for the use of a property (v,i), which is described by dwelling and zone attributes (d,z), is given by:

$$WP^*_h\,(d_v, z_i) = WP_h\,(d_v, z_i, y_h, w_h) + \varepsilon_h$$

where ε_h represents the random term, associated with the level of ignorance, and WP_h is the systematic part of the WP* function. MUSSA assumes errors terms as being identically and independently distributed Gumbel, a distribution which is highly convenient since it preserves the same distribution under the maximization procedures, ie the maximum of Gumbel variables is also a Gumbel variable; ie it is closed for the maximum value operator.

The model is disaggregated both in consumers (h) and property (v,i), in contrast to the usual classification of consumers in certain types and locations by zones. Nevertheless, model formula are presented here in a general form allowing some level of aggregation. Zone attributes are used in the model to describe environmental characteristics of the property, which does not imply the aggregation of supply into zones.

The probability that a household h chooses a location option (v,i) from the set of alternatives S, called the *choice version*, is given by the probability that (v,i) offers to h the maximum utility or maximum consumer surplus compared with other alternative locations in S (Martínez 1992b). The choice probability is:

$$P_{vi/h} = \frac{f_{vi}\exp(\mu[WP_{hvi}-r_{vi}])}{\sum_{(v'i')\in\Omega} f_{v'i'}\exp(\mu[WP_{hv'i'}-r_{v'i'}])} \tag{4}$$

where size factors $f_{v'i'}$ stand for the number of similar units v′ available in zone i′, which allows for the aggregation of property units into supply types; in the fully disaggregated model f factors are equal to one.

In addition, the probability that a consumer h makes the highest bid in a given location (v,i), having H alternative bidders, called the *bid version*, is:

$$P_{h/vi} = \frac{f_h\exp(u'[WP_{hvi}-w_h])}{\sum_{h'\in H} f_{h'}\exp(\mu'[WP_{h'vi}-w_{h'}])} \tag{5}$$

where size factors $f_{h'}$ stand for the number of homogeneous households in a given cluster h′, which, allows for aggregation of the population into groups. Ellickson (1981) proposed a similar equation for the competitive case (w_h=0). Note that scale factors μ and μ' are associated with WP and bids random terms respectively.

Some authors assume that bid and WP functions are different, eg Miyamoto and Kitazume (1989) and Hayashi and Doi (1989), arguing that consumers behave in a speculative manner in the market. For the sake of presentation, and also because it is a relevant case in calibration, we shall present the competitive case; the extension to the speculative market follows below.

The rent function

Additionally, according to the bid auction process, rents are the maximum bid (or WP*) for the property. Since WP* is distributed independent and identically Gumbel, rents are also random variables distributed Gumbel:

$$r^*_{vi}=r_{vi}+\varepsilon_r = \frac{1}{\mu}\ln\left[\sum_{h\in H} f_h\exp(\mu WP_{hvi})\right]+\varepsilon_r \tag{6}$$

with an expected value given by:

$$\overline{r_{vi}}=E(r^*_{vi})=\frac{1}{\mu}\ln\left[\sum_{h\in H} f_h\exp(\mu WP_{hvi})\right]+\frac{\gamma}{\mu} \tag{7}$$

with γ the Euler's constant (= 0.57). Equation (7) represents the rent model of the urban location market, which is endogenous in the Bid-Choice model; ie it represents the probabilistic version of rent equation (2).

A relevant characteristic of the rent equation (7) is that it is a function of WP for dwelling and zones attributes, called a hedonic function. Hence, rents are specific for each dwelling/zone (v,i) and does not provide a unique land rent per square meter in a zone; a unique land rent is can only represent an average value across all land lots in the zone. Secondly, the hedonic rent function do have an underpinning functional form derived from the location model.

Noting that the term in parenthesis in equation (7) is identical to the denominator of equation (5), the location model can be written in a form involving rents explicitly:

$$P_{h/vi}=f_h \exp\left[\mu(WP_{hvi}-\overline{r_{vi}})\right] \tag{8}$$

and represents a linear expression for the bid model (see Martínez 1991). With this formulae it is easy to demonstrate the corollary in the stochastic model which states the choice and bid approaches are equivalent, which is the heart of the Bid-Choice model (see Martínez 1992b).

Note that the Bid-Choice model assumes *endogenous and deterministic* rents ($\bar{r}$) in the choice version, under otherwise, if rents are random variables (r^*) the difference with WP^* would make CS to distribute logistic, not Gumbel, in the choice probability formulae (equation 4); which makes the model highly complex.

Bidders choice set

An important assumption in the above model is the number of bidders for each property; theoretically, potential bidders are all households and firms. According to equation (7), rents are dependant on the number of actual bidders N; consider the case where WP for a given property is equal for all bidders, then:

$$\overline{r_{vi}}=WP_{vi}+\ln(N)+\frac{\gamma}{\mu} \tag{9}$$

This results shows that as the population increases, N dominates the explanation of rents; which is caused by the increase in the likelihood of the random term ε_h to reach, for some bidders, higher values in the distribution; hence, the higher is the value of the maximum WP. This is a property of the maximum value operator. The effect of this issue may be reduced if actual and potential bidders have different distributions, eg if the supplier faces a set of bidders which depends on the characteristics of the property on rent. In that case, the expected number of bidders is given by the number of potential bidders multiplied by their probability to "show up" at the auction (ϕ_{hvi}), if we assume that bids are independent. This means that equations (5) to (8) should be generalized by replacing factors f_h by the modified factors ($f_h \cdot \phi_{hvi}$). In this modified model, the example of equation (9) becomes:

$$\overline{r_{vi}}=WP_{vi}+\ln\left(\sum_{h\in H} f_h\phi_{hvi}\right)+\frac{\gamma}{\mu} \tag{10}$$

which makes rents still dependent on the number of potential bidders but softened by the "show up" probability. Moreover, this argument also affects bid probabilities (equation 5), as f_h factors should be replaced by the modified factors ($f_h \cdot \phi_{hvi}$).

Suppliers behaviour

In MUSSA, the suppliers' model is introduced as a dynamic element, calibrated using time series data set in order to introduce in the model observed tendencies of development and speed of change in the city. We believe that, as far as suppliers is concern, their role is not relevant in the static model because it provides little extra information to the Bid-Choice equilibrium. However, dynamic tendencies, which may be largely captured by observing the developers market, provides independent information to the static consumer's behavior model. Thus, MUSSA was developed as an hybrid static-dynamic model while keeping the concept of static equilibrium at every stage in time. Further research is required to produce a fully dynamic equilibrium model.

Developers provide dwellings for households and land lots for households and firms; in terms of the model notation, suppliers define the number of locations (v,i), denoted as f_{vi}. They are assumed to produce in a competitive market such that they are an homogeneous group, supplying location options for households and firms in return for a rent. This include the hypothetical case of selfconstruction of houses and owners, where the developer/owner rents himself the house, since otherwise he may obtain the same rent from another household.

The assumption of the suppliers' behaviour is that they choose what to supply in each zone in order to maximize profit, given rent values, supply costs and land availability. Let us denote by G^*_{vi} the developer's profit obtained from supplying a property type v in zone i, assumed to be a random variable with a random term distributed identical and independent Gumbel. Then, the suppliers' probability of offering an option v in a given zone i, denoted as P_{vi}, is:

$$P_{vi} \frac{\exp(\lambda G_{vi})}{\sum_{v' \in \Omega_i} \exp(\lambda G_{v'i})} \tag{11}$$

where G_{vi} is the systematic term of the profit function, λ is the usual scale factor and Ω_i is a set of feasible supply options in zone i which includes land use and building regulations.

The profit function G is defined as the revenue minus costs of developing a supply option. Revenue is given by the option rent r_{vi}, while costs depends on dwelling and zone characteristics; therefore:

$$G_{vi} = r_{vi} - C(d_v, z_i) \tag{12}$$

Rents are exogenously given by equation (7), following the best bid rule, so the suppliers' behaviour is to maximize profit subject to rents. The cost function involves building cost, whose variation may be limited within the same urban area, plus legal and financial costs. A priori, one can only say that the profit function depends on the rent, floor space, dwelling quality and land size, etc., plus other legal items.

Note that the supply function is applicable for both residences and firms locations, depending on the specification of the cost term. Then, the developer faces the choice of using the available land in a zone to supply dwelling options for households, offices, retail floorspace, land for manufacture industry, etc.; indeed all options in the feasible set Ω_i are choices for the developer.

Equilibrium conditions

These conditions are meant to hold simultaneously at every stage in time, providing an static equilibrium in the location market. However, the static equilibrium is not imposed simultaneously for the relationship between location and transport, which is treated in a dynamic approach as a sequence of delayed impacts.

Location equilibrium

The location equilibrium can be stated as: *every locator should be located*. This means that every household and firm should be located at their maximum utility location which is also the location where they are the highest bidders. This condition implies that supply options are equal to the demand for each option. Then:

$$f^t_h P_{vi/h} = f^t_{vi} P^t_{h/vi} \qquad \forall h,v,i \tag{13}$$

which reproduces the Bid-Choice equivalence established in Martínez (1992b) under the assumption of endogenous rents. This is the primitive condition that implies the following conditions normally used in land use modelling:

i) The accumulated number of located households and firms, best bidders, is equal to the population and total number of firms respectively:

$$\sum_{(v,i)\in\Omega^t} f^t_{vi} \cdot P^t_{h/vi} = f^t_h \qquad \forall h \tag{14a}$$

ii) Demand for dwelling/location options at each zone is satisfied by supply, which can be expressed as:

$$\sum_{h\in H^t} f^t_h \cdot P^t_{vi/h} = f^t_{vi} \qquad \forall (v,i) \tag{14b}$$

These equations hold at every stage in time *t*. Replacing equation (8) in (14a) we obtain:

$$w^t_h = \frac{1}{\mu} \ln\left[\sum_{(v,i)\in\Omega} f^t_{vi} \exp\left[\mu(WP^t_{hvi} - r^t_{vi})\right]\right] = 0 \tag{15}$$

which is interpreted as, at location equilibrium, the expected maximum consumer surplus obtained by (every) household and firm, given by w^t_h, is zero. This result reproduces the conclusion obtained earlier in the deterministic model for the competitive case; it also represents the economic counterpart of the more physical constraint of equation (14a). It is worth to emphasize that this result holds if and only if rents are endogenous and WP values represent actual bids, which are the assumption implicit in equation (8). The case where $w_h>0$ represents excess of demand for the supply available, while $w_h<0$ represents excess of supply; both under the competitive assumption.

Additionally, the rent equation (7) may be written as:

$$w^t_{vi} = \frac{1}{\mu} \ln\left[\sum_{h\in H} f^t_h \exp\left[\mu(WP^t_{hvi} - r^t_{vi})\right]\right] = 0 \tag{16}$$

where w_{vi} is interpreted as the maximum expected surplus that may be obtained at location (v,i). Then, endogenous rents (or the maximum bidder rule), also implies that, under the competitive case, the maximum consumer surplus at equilibrium is zero at every location. Again, the condition $w_{vi}<0$ implies that at this location rents are higher than the expected maximum bid, hence this location is not occupied, which represents a case of excess of supply; while $w_{vi}>0$ implies rents below the maximum bid, which is a case of excess of demand.

The conclusion is that in the Bid-Choice model equations (14) to (16) are equivalent and consistent with the corollary stated in Section 2.1. Secondly, urban disequilibrium is described, in the competitive case, by w_h and w_{vi} different from zero. Lastly, note that in equation (13) assures that total population and firms, given by the sum of f_h, equals the total location options, given by the sum of f_{vi}.

Land market equilibrium

The fact that *urban land supply is non elastic* imposes further constraints to the urban location equilibrium associated with the use of available land. This constraint simply states total land used by supplied location options in a given zone, Ω_i, should not exceed the available total zone land Q_i. That is:

$$\sum_{v\in\Omega^t_i} f^t_{vi} \cdot q_v \leq Q_i \qquad \forall i \tag{17}$$

which must be satisfied for every zone in the city.

Location externalities

This kind of externality can be stated as: *location is affected by others' location*. Indeed location of activities create some of environmental characteristics of a zone, defining the neighborhood quality. As some activities, eg residential, do perceive neighbourhood quality as a relevant attribute in their location choice, then location technical externalities are present.

This implies that some location attributes are endogenous to the location process, hence they are endogenous attributes in MUSSA. Examples of such attributes are: agglomeration of activities, zonal average income of residents, etc. This defines a type of fixed point problem in location probabilities analytically expressed as:

$$P^t_{h/vi} = P_{h/vi}(X^t_{hvi}\,(P^t),\, Y^t_{hvi},\, \beta_h,\, y^t_h, U^*_h,\, w^t_h) \tag{18}$$

where X^t and Y^t represents vectors of endogenous and exogenous attributes at any given time respectively. The term P^t in parenthesis in the right hand side is the vector of probability functions for every household and firm; ß, *y*, U* and *w* were previously defined. The solution of the fixed point problem is a condition for location equilibrium in static analysis.

The speculative market

The results obtained above are consistent with the assumption that locators' bids are identical to their WP and rents are given by the maximum bid. Let us now assume, that bids are speculative and different from WP, so that:

$$bid_{hvi} = WP_{hvi} - w_{hvi} + e_h \tag{19}$$

with w_{hvi} a general speculative function and e_h a random term which we assume as distributed identical and independent Gumbel with scale factor μ'. The speculative rent equation is obtained as the expected maximum bid:

$$\overline{r_{vi}} = \frac{1}{\mu'} \ln\left[\sum_{h \in H} f_h \exp(\mu'[WP_{hvi} - w_{hvi}] + \gamma)\right] \tag{20}$$

and the speculative bid probability is:

$$P_{h/vi} = f_h \exp\left[\mu'(WP_{hvi} - w_{hvi} - \overline{r_{vi}})\right] \tag{21}$$

The choice probability expression remains unaffected because it does not involve bids. Replacing this probability in equation 13 expression we obtain:

$$w_{hvi} = \frac{1}{\mu'} \ln\left[\sum_{(v,i) \in \Omega} f_{vi} \exp[\mu(WP_{hvi} - \overline{r_{vi}})]\right] + (1 - \frac{\mu}{\mu'})(WP_{hi} - \overline{r_{vi}}) \tag{22}$$

which represents the expected speculative consumer surplus across location alternatives. Note that if $\mu = \mu'$, the expression of the left side takes the same value despite the subindex ***vi*** in the right side, so in that case the location equilibrium imposes a constant speculative power across the city for each locator; that is:

$$w_h = \frac{1}{\mu} \ln\left[\sum_{(v,i) \in \Omega} f_{vi} \exp[\mu(WP_{hvi} - \overline{r_{vi}})]\right] \tag{23}$$

which, compared with equation (15), it shows that in the speculative case the consumer surplus is no longer equal to zero. Then, the conclusion is that *the speculative consumer surplus is greater than zero and it is constant across the city for the same locator if scale factors are equal.* This result is highly intuitive since it implies that a locator bids as to obtain a given surplus or utility level, which yields the locator indifferent to the result of the auction. Nevertheless, different scale factors is also a plausible case if bids and WP random terms have different variance.

Note that a constant locator surplus implies that bid functions are identical to WP functions except for the constant parameter, a result which is relevant for the calibration stage in static analysis because w_h is implicit in WP constant. Secondly, note that for the aggregate model the speculative consumer surplus w_h is interpreted as the expected value of the consumer surplus across households of group h.

FORECASTING PROCEDURE

Inputs to MUSSA

MUSSA receives two types of inputs: WP parameters obtained in the calibration procedure and, secondly, the updated total number of households and firms (denoted as f_h).

This last input is generated by an input/output model (I/O) exogenous to MUSSA which forecasts the growth of non-residential activities (industry, services, retail, education and other activities) for each forecasting year ***t***. The I/O includes labor as a sector and adjusts growth of activities to a population tendency exogenous model; alternatively, it is also possible to exogenously forecast basic industry growth and adjust other sectors and population, allowing for migration effects; or a mixed population-basic employment exogenous constraint. The modeler has these options to forecast different growth scenarios.

Population growth is then splitted into households growth based on exogenous forecasts of households income, car ownership and household size (number of members). The I/O model and the distribution procedure are called the growth model and constitutes the sixth stage of the land use-transport model.

The optimization problem

Updated totals of households and firms are then used by MUSSA as locators (f_h^t), whose expected location and rents is estimated. This is an equilibrium procedure which finds out the required supply to cope with demand under land constraints and following the supply tendency. In other words, it finds updated values for (f_{vi}^t)which fulfil equation (13 to 16) constrained by equation (17); additionally, all supply factors (f_{vi}^t) should be consistent with equation (11) and locators' totals (f_h^t) must be modified by the show up factor ϕ_{hvi} in bid expressions (equations 5 and 8).

The growth of population and firms will increment demand for available supply which will impact on rents due to land constraints and dwelling supply inelasticities. Hence, the equilibrium solution may require a significant change on the demand side in terms on the achievable utility level or consumer surplus in that year t; therefore, w_h^t may be greater, equal or less than zero. Then, the analytical problem is expressed by:

$$\sum_{(v,i)\in S^t} f_{vi}^t \cdot P_{h/vi}^t = f_h^t \qquad \forall h \tag{30a}$$

$$\sum_{v\in \Omega_i^t} f_{vi}^t \cdot q_v \leq Q_i \qquad \forall i \tag{30b}$$

$$f_{v/i}^t = \varphi(G_{vi}(r_{v/i}^t)) \qquad \forall v,i \tag{30c}$$

where the supply equation (11) is expressed here as a general expression of rents (30c). Additionally, the bid location probability is:

$$P_{h/vi}^t = f_h^t \phi_{hvi}^t \exp\left[u'(WP_{hvi}^t - w_h^t - r_{vi}^t)\right] \tag{30d}$$

which is subject to the fixed point or location extemalities effect.

The equilibrium is defined as an optimization problem (OP) which seeks to find (f^t_{vi}) and w^t_h which best fulfil equations (30a) and (30c), subject to comply with equations (30b). Theoretically, the problem may be specified in a single vector of unknowns, say w^t, since f^t_{vi} may be expressed as $f(w^t)$ (see equation 14b) but, as far as the authors know, it is not possible to obtain a treatable specification due to non-linearities in some expressions.

The OP problem has multiple solutions, location probabilities are insensitive to a (positive) scaling factor in *w* vector. As rents are defined by *w* through equation (20):

$$\overline{r_{vi}}=\frac{1}{\mu}\ln\left[\sum_{h\in H} f^t_h \phi^t_{hvi}\exp(\mu\left[WP^t_{hvi}-w^t_h\right])+\gamma\right]$$

an scaled *w* vector induces a flat increase in rents everywhere; hence, a criterion should be introduced to identify a unique solution. In MUSSA the criterion is that the minimum change in locators consumer surplus (loss) is the most likely to occur, based on the argument that higher rents will reduce consumer surplus down to unnecessarily low levels, so locators are expected to make better choices as to reduce the loss. Then the criterion may be stated as *minimization of consumers' loss.*

Changes in consumer's surplus should be understood as changes in the maximum utility level of the household or firm (U_h*), which represents a change in life standards. The interpretation of such changes in this model is that, as time passes and population increases, rents have to rise according to new demand, thus utility levels are reduced; nevertheless, differential changes in *w* across socioeconomic groups may also be explained by a change in their relative speculative power. These social effects could be treated explicitly in this framework, if exogenous information on social developments (eg equality, information technology, democracy, etc) is provided; thus, one could analyse social policies leading to minimise differentials in the speculative power across population groups.

Additionally, this framework also allows us to specify land use regulations and policies through:

- the definition of the choice set Ω^t_i. For example, dwellings or firm types constraints in a particular zone i can be modelled by reducing the number of altematives in the choice set.
- the introduction of additional constraints explicitly in the OP problem, for example, limits to the zonal population density.

MAIN ISSUES

The theoretical framework herein presented provides bases for the analysis of some relevant issues in location modelling.

i) The interpretation of land rents as maximum WP, with WP defined for a given utility level, induces the interpretation of rent changes as either the result of the capitalization of location amenities or a change in the reference utility level (or locator surplus) caused by land constraints or dwelling supply constraints.

ii) The presence of speculative power has an explicit treatment in the model, which enables the modeler to identify required assumptions in the operational model and to specify the model accordingly; this is particularly sensitive in the precise specification of stochastic terms and their associated scale factors.

iii) The relationship of rents with consumer surplus and capitalization effects have a clear interpretation and can be identified properly with this model.

iv) The role of the number of bidders in the rent function becomes clear in the stochastic model: the more the number of bidders the higher the expected rent, given WP fixed.

v) The fact that the forecasting model requires an exogenous criteria to identify a unique solution (ie a consumer surplus *w* vector should be identified from a set of feasible solutions) is an interesting theoretical issue. It links the economics of the location model with the wider issue of social values and speculative power.

ACKNOWLEDGMENTS

We thank FONDECYT for partial funding of this research and SECTRA for allowing publication of the MUSSA framework.

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