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TOPIC 5
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STOCHASTIC FRAMEWORK FOR MODELING PAVEMENT PERFORMANCE

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Abstract

This paper describes the development of a highway pavement performance model. Concepts of failure time models and systems reliability are applied to developing a stochastic approach for modelling two simultaneous processes: pavement deterioration, and repair intervention and its effect on condition. Prediction accuracy of the model is proved to be better than other deterministic models.

INTRODUCTION

Over the past two decades, a large number of pavement performance models were developed and used mainly for rationalizing planning in the area of pavement maintenance and repair. Most of these models can be categorized as deterministic models in which performance can be determined by a certain function which directly relates pavement condition to other variables such as pavement age and traffic level. Such deterministic models do not explain the variation in pavement condition with age shown in Figure 1. This variation is attributable to the complex interactions of several uncertain factors, such as traffic, materials, construction, and environment, which result in various deterioration mechanisms difficult to express deterministically. As a result, a few stochastic models were recently developed in which deterioration is treated as a probabilistic phenomenon. These models apply techniques such as Markov chain and survivor curves; and employ transition matrices which describe the probability of transition between successive condition states. Results obtained using such models are usually better than those obtained using deterministic ones.

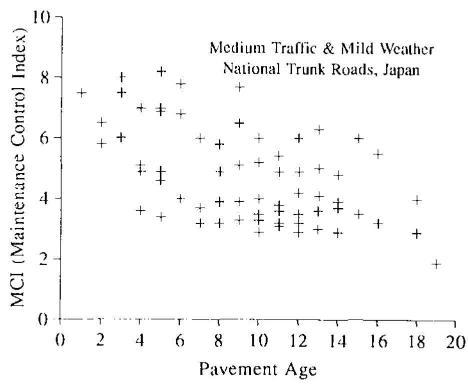

Figure 1 Variation of pavement condition with age

Review of the available examples of stochastic models (eg Carnahan et al. 1987 and 1988; George et al. 1989; Takeyama et al. 1990), however, reveals the following points which need more consideration:

- The transition rate between any two successive condition states is assumed to depend only on the preceding state of the pavement. The effect of pavement age on the transition rates is ignored despite the proven fact that age is the most significant factor influencing deterioration, and
- In applying these models for performance prediction, repair intervention is treated as a deterministic event based on pavement condition. However, according to actual records of past repairs, sections selected for repair are not always the worst ones. This disturbances in repair timing/logic is due to the subjective factors which have to be considered in the repair decision such as strategic, technical and financial constraints. Consequently, a stochastic approach for predicting repair intervention timing is more realistic.

STOCHASTIC MODELING OF PAVEMENT PERFORMANCE

The main objective of this paper is to develop a stochastic pavement performance model that takes the above points into consideration. The model treats performance as the outcome of the interaction between two simultaneous processes; namely, 1) pavement deterioration and 2) repair intervention. Review in other engineering fields indicates that the concepts of failure time models (FTMs) for analyzing life expectations of mechanical and structural elements subject to random failure could be particularly suitable for the aimed objective. Therefore, these concepts are briefly introduced and then applied to develop the proposed model. Mathematical formulation of the model and some indicators for performance evaluation are also presented. Finally, the prediction accuracy of the developed model is evaluated and compared with results obtained using other available performance models.

Concepts of failure time models

An FTM is simply a probability distribution function of a non-negative continuous stochastic variable representing the failure time (age) of an individual from a homogeneous population group (Lewis 1987; Kalbflesch and Prentice 1980). The term "failure" does not necessarily mean physical failure. Rather, it refers to the occurrence of a predefined event. For instance, such an event might be defined as the occurrence of a specific change in pavement condition or taking a certain action. Thus, referring to the time required for this event to occur as T, then the FTM is the probability distribution function of T, given that T is stochastic. In FTM, usually time is the only explanatory variable for failure. Thus, for accuracy, FTM should be constructed for homogeneous populations (other explanatory variables, eg traffic and pavement type, are treated as fixed parameters).

Suitability of FTM for pavement deterioration and repair intervention modeling

The suitability of FTM for modeling pavement deterioration and repair can be concluded in the following points:

- First, while several factors interact together to result in pavement deterioration and need for repair, the overall effect of all these factors on pavement is inherent in the age (time) at which a specific condition or need for repair action is reached. Thus, for a homogeneous population, deterioration and repair can be sufficiently expressed using pavement age.
- Second, according to Figure 1, the age at which a specific condition is reached varies from section to another. Thus, for pavements, this age can be considered as a non-negative stochastic variable.
- Third, deterioration/repair can be defined as a group of successive events or, in other words, a multi-stage "failure". In deterioration modeling, these stages can be, for example, stage I: transition from excellent state to good state; stage II: transition from good state to fair state; and so on. As for repair, it can be represented by one stage, that is transition from a "repair not required" state to a "repair required" state.
- Fourth, due to the variation in condition with age and the effect of the strategic, technical and financial factors on repair timing, the occurrence of any "failure" stage may happen at any age. Thus the transition rates to successive states can be represented by continuous probability density functions.
- Finally, to satisfy the condition of population homogeneity, pavement sections under study can be divided into homogeneous populations based on the factors affecting the deterioration rate, such as traffic level and pavement type.

From the above points, pavement deterioration/repair can be modeled in the form of a group of FTMs in which pavement age is the only explanatory variable. In the rest of this paper, pavement age t, (t=1, 2, ..., A) is assumed to be measured from the date of construction or the last major repair work. It is also assumed that the road network is divided into classes of homogeneous populations k, (k = 1, 2, ..., K) and deterioration/repair is represented as transition of condition over L states. The transition between any two successive condition states, eg from state i to j, represents one failure stage f. Thus deterioration/repair can be expressed by L-1 failure stages f, (f = 1, 2, ..., L-1). This results in a group of (Kx(L-1)) failure time models for each of deterioration and repair, ie one model for each population class k and failure stage f.

Forms and applications of FTMs

The probability distribution function in FTM can be expressed in three different forms (Figure 2) which are particularly useful in survival applications:

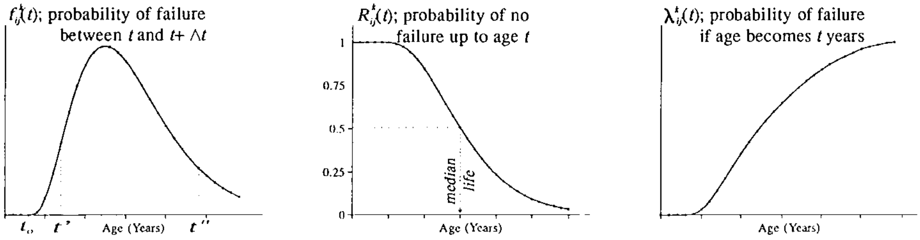

Figure 2 Typical forms of the probability distribution function in FTM

A probability density function (PDF) is the basic form of FTM for population k and failure stage f. This function can be in the form of any suitable standard probability distribution such as Gamma, Weibull or Erlang (Figure 2a). The general formula of this function is as follows:

$$f_{ij}^k(t) = P\left(t \le T_{ij}^k \le t+\Delta t\right)/\Delta t = \Phi\left(\alpha_{ij}^k, \beta_{ij}^k, t_{0ij}^k, t\right) \tag{1}$$

where, for population k, $P\left(t \le T_{ij}^k \le t+\Delta t\right)$ is the probability that failure stage f (transition from state i to j) will occur at the age T_{ij}^k which is between age t and $t+\Delta t$; Φ is any form of a theoretical probability distribution; t_{0ij}^k is the minimum age to failure; α_{ij}^k and β_{ij}^k are parameters. Historical data on T_{ij}^k can be used to estimate the parameters α_{ij}^k, β_{ij}^k and t_{0ij}^k. The meaning of the curve shown in Figure 2a can be explained in pavement deterioration context as follows: for a pavement section from population k and with condition state i: 1) failure stage f (transition to state j) is not expected to occur before age t_0; 2) at early ages (t_0~t') the probability of transition to state j, ie "fail", is relatively small since the section is still "young"; 3) as this section ages, the probability of "failure" gets higher with a peak probability somewhere between t' and t''; and 4) the probability of "failure" to take place beyond age t'' is relatively low since "failure" most likely would take place before reaching this age. In the context of a group of pavement sections, the above explanation can still be valid by replacing the word "probability" by "probable proportion".

The function $f_{ij}^{k}(t)$ is mainly used for deriving the formula for the other two forms explained below. It is also possible to use this function to estimate the upper and lower limits of pavement life expectancy in a specific state at any given confidence level.

A reliability function is the probability that a pavement from population k will survive above condition state j more than t years (Figure 2b). The general form of this function can be derived from Equation (1) as:

$$R_{ij}^{k}(t) = P\left(T_{ij}^{k}>t\right) = 1-\int_{0}^{t} f_{ij}^{k}(t)dt \qquad (2)$$

The area under the reliability curve can be thought as the expected service life of a pavement section above state j and thus can be used as a performance indicator. The slope of the curve also can be a good indicator for timing intervention actions. One should notice that the reliability curve resembles a typical condition-age relationship.

A failure rate function is the instantaneous rate of transition from state i to j at age t for road sections from population k (Figure 2c). The general formula can be derived from Equations (1) and (2) as:

$$\lambda_{ij}^{k}(t) = P\left(T_{ij}^{k}<t+\Delta t \mid T_{ij}^{k}>t\right)/\Delta t = f_{ij}^{k}(t)/R_{ij}^{k}(t) \qquad (3)$$

Where $P\left(T_{ij}^{k}<t+\Delta t \mid T_{ij}^{k}>t\right)$ is the conditional probability of "failure" occurrence before age t+Δt given that it did not take place up to age t. The meaning of $\lambda_{ij}^{k}(4) = 0.25$, for example, is; for a group of pavement sections from population k with condition i, 25% of these sections are expected to transfer to state j at the age of 4 years, while the remaining 75% are expected to survive in the same state i for longer time. Thus, this form of the probability function can be used in predicting the amount of yearly transitions between successive condition states, or in other words determining the transition matrices.

MODEL DEVELOPMENT

In this section, the concepts of FTM are applied to develop two submodels. The first is a deterioration submodel to predict the expected change in pavement condition with age. The second is a repair submodel which gives the probability of repair intervention.

The sample road network

Data on pavement condition and repair of the national trunk roads in Mie Prefecture, Japan, is used for model development. Selection of this study area is based on the availability of an up-to-date data base. The total length of the surveyed roads is about 355 km (1180 lane km). The whole sample network is located in an area with mild weather conditions. The data base contains various types of inventory and condition data given in increments of 100m sections or less for the years 1988 and 1991. Condition is given in terms of the value of the maintenance control index (MCI, an index developed by the Japanese Ministry of Construction which indicates pavement overall condition on a scale from 10 to 0, with 10 for excellent condition and 0 for a totally failed pavement) based on the amount of surveyed distresses. Construction and repair history are available for more than 30 years. No detailed data is available on the structural design of pavement layers nor on subgrade characteristics. Because only a small percentage of the sample network is concrete pavement, they are ignored. In developing the model, the data for 1988 is used for estimating model parameters while the data for 1991 is used for verifying it.

Deterioration submodel

The purpose of this submodel is to predict the change in pavement condition with age. Condition here is expressed in MCI ranges. These ranges are defined by dividing the possible MCI values (10-0) into 5 condition states (L=5) and thus 4 "failure" stages f, as shown in Figure 3.

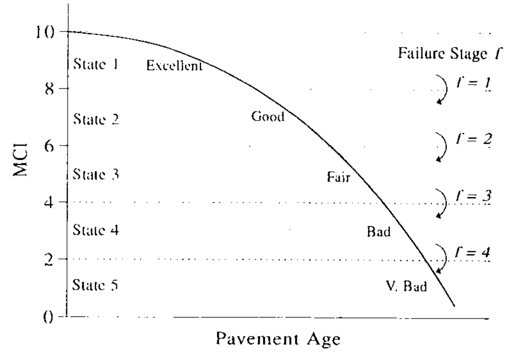

Figure 3 Condition states and failure stages

The sample data for 1988 is divided into 4 homogeneous population classes (K=4) based on two categories of traffic loads and two types of pavements as shown in the first column of Table 1. The sample data is not divided into 3 traffic categories in order to keep a sufficient sample size for each failure stage. The age of any section is calculated from the year of opening the road for traffic after construction for "new pavements" and from last repair for "old pavements" (rather than from the occurrence of the previous failure stage). The ages at which a transition in condition state has occurred for the sampled sections could not be directly calculated from the data base. Therefore, for any failure stage f, the age at which the MCI value of any section was in the range of $\pm v_f^k$ around the transition limit of this failure stage (eg MCI = 8 for stage 1) is considered as an age at which failure stage f has occurred. The values of v_f^k are taken as half the average yearly loss of MCI around the transition limit of each population class k and failure stage f. The resulting sample sizes are given in Table 1.

Several theoretical PDFs were then fitted to the calculated transition ages in each sample. The Weibull distribution, well known for modeling time-to-failure, is selected to represent since it gave the highest average significance level and since it is also more practical to use the same function form for all the samples. The estimated parameters $\left(\alpha_{ij}^k, \beta_{ij}^k, t_{0ij}^k\right)$ are given in Table 1. Since no data is available on failure stage 4, the parameters for this stage are estimated from the trends of the other stages. Average life expectancy (calculated from the year of opening the road for traffic) before the occurrence of each failure stage is also given. As shown, all the significance levels of the fitted PDFs are above 0.8 which we take as satisfactory.

The forms of Equations (1), (2) and (3) for the Weibull distribution are as follows:

$$f_{ij}^k(t) = \frac{\alpha_{ij}^k}{\beta_{ij}^k}\left(\frac{t-t_{0ij}^k}{\beta_{ij}^k}\right)^{\alpha_{ij}^k-1} \exp\left[-\left(\frac{t-t_{0ij}^k}{\beta_{ij}^k}\right)^{\alpha_{ij}^k}\right] \quad (4)$$

$$R_{ij}^{k}(t) = \exp\left[-\left(\frac{t-t_{0ij}^{k}}{\beta_{ij}^{k}}\right)^{\alpha_{ij}^{k}}\right] \tag{5}$$

$$\lambda_{ij}^{k}(t) = \frac{\alpha_{ij}^{k}}{\beta_{ij}^{k}}\left(\frac{t-t_{0ij}^{k}}{\beta_{ij}^{k}}\right)^{\alpha_{ij}^{k}-1} \tag{6}$$

Table 1 Parameters of the deterioration model

Population class (k)	Failure stage (f)	Sample size	Model parameters α_{ij}^{k}	β_{ij}^{k}	t_{0ij}^{k}	Mean life	Sig. level
New pavement B&C traffic	1	128	1.77	12.38	2	13.0	0.9
	2	317	2.67	15.64	4	17.9	0.9
	3	44	3.52	16.16	7	21.6	0.95
	4	—	4.27	16.72	10	25.6	—
New pavement D traffic	1	62	1.27	5.81	1	6.4	0.9
	2	132	1.73	8.18	2	9.3	0.93
	3	27	2.13	9.73	4	12.6	0.92
	4	—	2.50	11.64	7	17.3	—
Old pavement B&C traffic	1	551	1.63	4.54	1	5.1	0.9
	2	759	2.26	7.69	2	8.8	0.87
	3	127	2.71	8.66	3	10.7	0.93
	4	—	3.00	9.70	4	12.8	—
Old pavement D traffic	1	742	1.11	3.92	1	4.7	0.79
	2	948	1.58	6.65	1.5	7.5	0.83
	3	292	1.80	7.39	3	9.5	0.89
	4	—	2.00	8.22	4	11.3	—

Notes:
Traffic: B=250-1000 truck/day/dir; C=1000-3000; D=>3000
Pavement: New=never been rehabilitated before; Old=has been rehabilitated before

Figures 4 and 5 show the plots of the change in $R_{ij}^{k}(t)$ and $\lambda_{ij}^{k}(t)$ with age for each population class and failure stage. Figure 4 shows that, in general, for the same traffic level, the reliability of "new pavements" is higher than that of "old pavements". This also indicates higher life expectancy for "new pavements". A similar trend is observed from the failure rate in Figure 5 where lower rates and milder slopes are observed for "new pavements". This means that the performance of pavements after repair is inferior to that of original pavements. Regarding the effect of traffic, the reliability tends to be less for higher traffic loads. However, the rate of loss in reliability and increase in failure rate tend to be steeper for lower traffic loads as the pavement ages. This might be attributable to differences in the level of routine maintenance application, the effects of which would appear at later ages.

As for the failure stage, as expected, it can be seen that the rates of failure at early pavement ages are higher for first failure stages than those for last stages. The trend is reversed at later ages. This means that most of the first transitions are expected to occur at early ages while most of the last transitions are expected to occur at later ages.

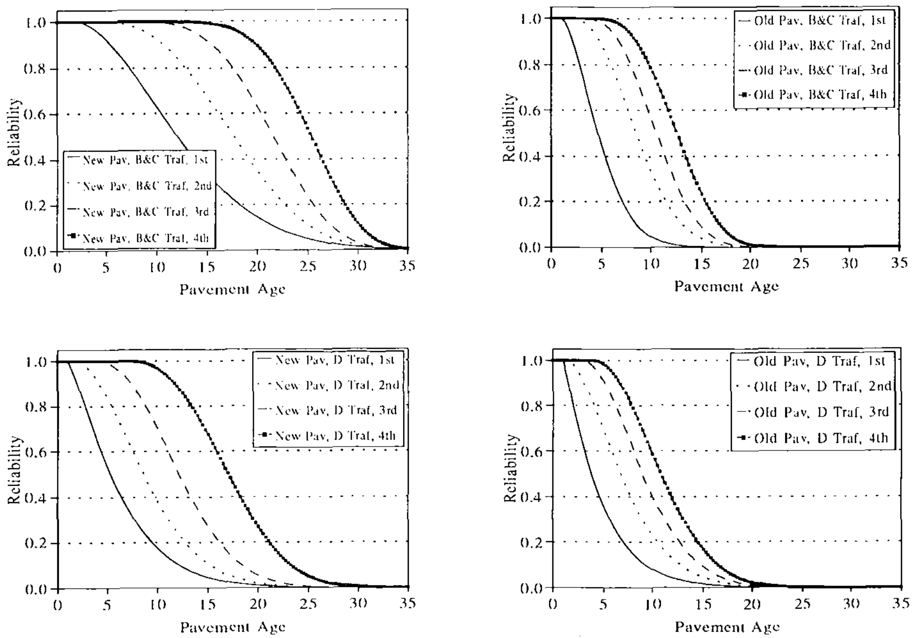

Figure 4 Deterioration submodel: Pavement reliability for each class and failure stage

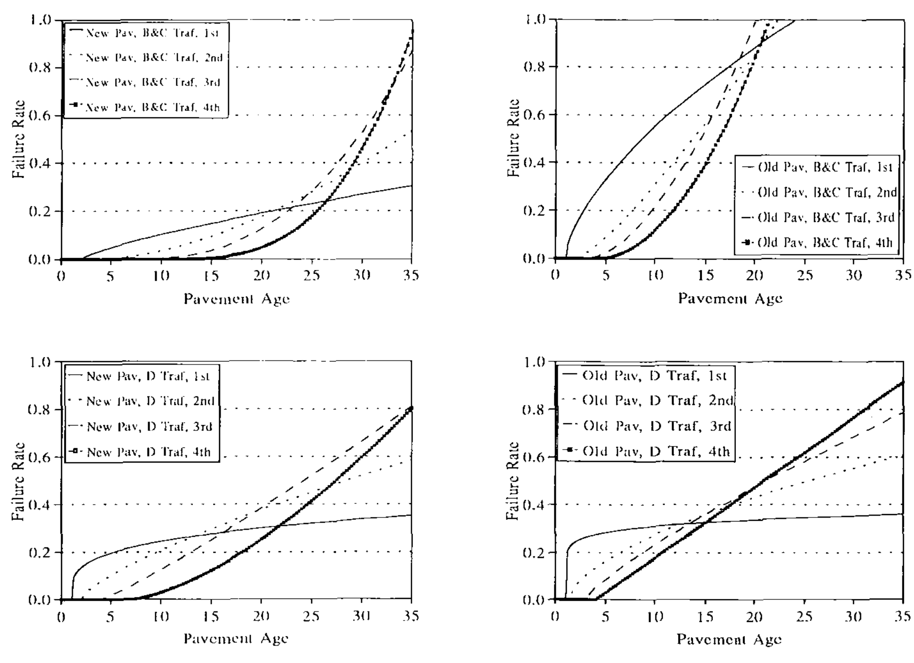

Figure 5 Deterioration submodel: Pavement failure rate for each class and failure stage

Mathematical representation of the deterioration submodel

Referring to Equation (6) of the developed model, K transition matrices, P^k, can be identified. Each matrix describes the transition rates between successive condition states for the sections from a certain population class k as a function of their pavement age. This results in a 3D matrix which can be expressed for any k as:

$$P^k = \left[p^k_{ijt}\right], \qquad i, j = 1, 2, \ldots\ldots, L; \quad t = 1, 2, \ldots\ldots, A \tag{7}$$

Figure 6 shows schematic representation of the 3D matrix P^k. As shown, this matrix can be thought as a group of layers of 2D matrices, each of which represent the transition rates of all pavement sections from population k and with age t. For the ease of purpose, we will refer to each layer as P^k_t.

The values of all elements of any matrix P^k_t (2D layer) are 0, except those elements on the principal diagonal (i=j), representing the probability of no transition at age class t, and on the upper diagonal (i=j-1), representing the probability of transition. In other words, for any t, the values of p^k_{ijt} can be written as:

$$p^k_{ijt} = \begin{cases} 0 & \text{for } i \neq j \text{ or } i \neq j\text{-}1 \\ 1\text{-}\lambda^k_{ijt}(t) & \text{for } i \neq j \neq L \\ \lambda^k_{ijt}(t) & \text{for } i = j\text{-}1 \\ 1 & \text{for } i=j=L \text{ (absorbing state)} \end{cases} \tag{8}$$

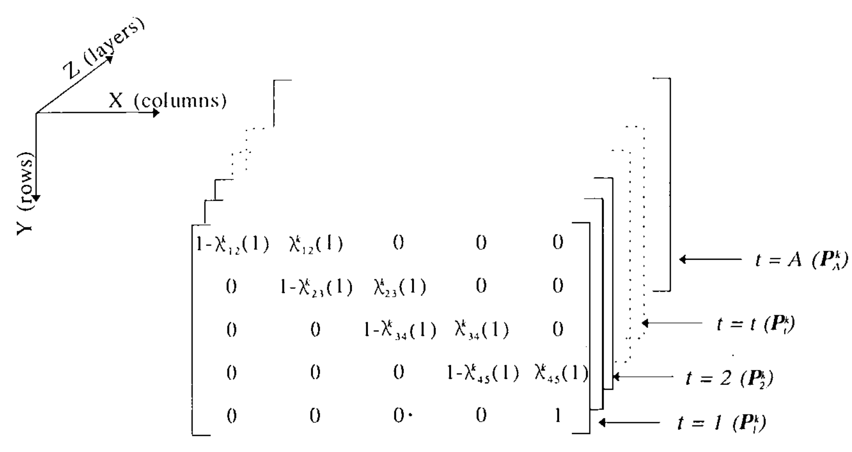

Figure 6 **Schematic representation of a transition matrix**

Prediction of condition change due to deterioration

The condition of a road network at any time t can be represented in matrix form by dividing the network's sections into cohorts based on the population class k, condition state i, and age class t of each section. This can be expressed for any k and t by a 3D matrix as follows:

$$N^{kt} = \left[n^{kt}_{1it}\right] \qquad i,j = 1, 2, \ldots\ldots, L; \quad t = 1, 2, \ldots\ldots, A \tag{9}$$

where, n^{kt}_{1it} is the number of those road sections with pavement class k, condition state i, and age t at time t. The subscript 1 in the equation indicates that the 3D matrix N^{kt} has only one row. As shown in Figure 7, any matrix N^{kt} can be thought as a group of layers (N^{kt}_t) of row vectors, each represents a certain age class t. These vectors can be easily constructed for current condition (t=0) for any network by referring to the corresponding condition survey.

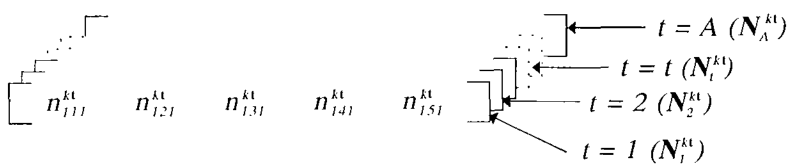

Figure 7 Schematic representation of road network condition matrix

Assuming that current condition of the network is given by the initial matrix, N^{k0} then, future condition of any pavement section from age class t can be estimated as follows:

$$N^{k1}_t = N^{k0}_t P^k_t \qquad \text{1st duty cycle}$$

$$N^{k2}_t = N^{k1}_t P^k_t = N^{k0}_t \left(P^k_t\right)^2 \qquad \text{2nd duty} \tag{10}$$

$$N^{kt}_t = N^{k0}_t \left(P^k_t\right)^t \qquad \text{t-th duty cycle}$$

Equation (10) is true under the condition that only routine maintenance is carried out between successive duty cycles. It should be noticed that the age class t of each pavement section will also change after each duty cycle. In the next section, a submodel for the prediction of repair intervention and its effect on condition is developed using the same data set of year 1988.

Repair intervention submodel

The repair intervention submodel is a special case of the deterioration submodel with only two condition states (L = 2) and, thus, one failure stage r. The purpose of this submodel is to predict the probability that a pavement section will be selected for repair. In this case, road data is divided into six homogeneous population classes, (K = 6), as shown in the first column of Table 2.

To estimate the model, the history (up to year 1988) of the ages at which repair works were applied is used. Such ages are calculated for "new pavements" as the time between construction and first repair, and for "old pavements" as the time between successive repairs. The Weibull distribution is selected here also to represent the Φ form for the same reasons mentioned before.

Estimated values of α^k_r, β^k_r and the significance levels are shown in Table 2. The parameter t^k_{0r} is given a value of zero since some sections were repaired twice in two successive years. For three of the classes, the significance level is lower than 0.8, and this might be explained by the relatively small sample sizes and the inhomogeneity of the samples due to the absence of necessary data on pavement structure. For the other classes, the significance levels are above 0.8 suggesting satisfactory fits. Further verification of the model is also given in a following section. The plots of the estimated reliability functions, $R_r^k(t)$ and failure rates, $\lambda_r^k(t)$ are shown in Figures 8 and 9. The figures show similar trends as those discussed in the case of the deterioration submodel.

Table 2 Parameters of the repair intervention model

Population class (k)	Sample size	Model parameter: Shape α_r^k	Model parameter: Scale β_r^k	Significance level
New pavement, B traffic	1132	4.5	22.0	0.84
New pavement, C traffic	365	2.7	13.9	0.65
New pavement, D traffic	1743	2.4	17.7	0.87
Old pavement, B traffic	437	3.0	14.3	0.70
Old pavement, C traffic	204	2.9	13.7	0.54
Old pavement, D traffic	1649	2.0	9.2	0.82

Notes:
Traffic: B=250-1000 truck/day/dir; C=1000-3000; D=>3000
Pavement: New=never been rehabilitated before; Old=has been rehabilitated before

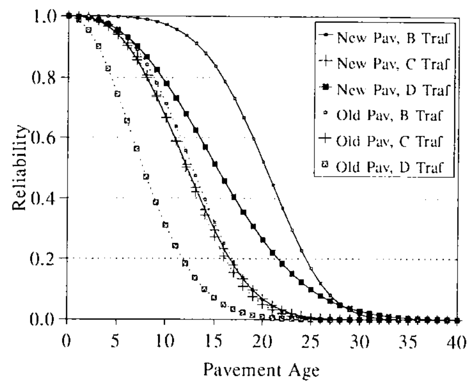

Figure 8 Repair submodel: reliability

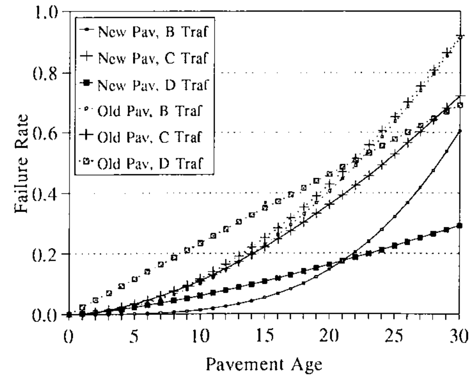

Figure 9 Repair submodel: failure rate

One should notice that the developed repair submodel directly reflects the past and current repair levels followed by the road authority. Direct application of this submodel would be a prediction of the future repair needs assuming continuation of the current repair strategy (budget, priorities, techniques and etc.). In this case, rate of repair of pavement sections from population k and age t years, $d_r^k(t)$, would equal the failure rate $\lambda_r^k(t)$. The effect of reducing the level of repair budget, for example, can be reflected by accordingly readjusting the rate of repair, $d_r^k(t)$, to a fraction of $\lambda_r^k(t)$.

Mathematical representation of the repair submodel

The probability of repair among pavement sections from population class k, D^k, can be expressed as a function of pavement age t using a 3D matrix as follows:

$$D^k = \left[d_{11t}^k\right] \qquad t = 1, 2, \ldots\ldots, A \tag{11}$$

where

$$d_{11t}^k = d_r^k(t) \tag{12}$$

The subscript 11t in the above equation indicates that the 3D matrix D^k has only one row in the Y direction, one column in the X direction, and A layers in the Z direction.

However, decision of repair intervention has to be associated with selection of a suitable repair type m, (m=1,2,....,M). Here we assume that repair type selection follows a certain logic and thus not stochastic. This logic is directly related to pavement state i and population class k at the time of taking intervention decision. In this case, the probability of selecting repair type m can be written for any k as a function of pavement age t and state i in the form of a 3D matrix D^{km} as:

$$D^{km} = \left[d_{i1t}^{km}\right] \qquad i = 1, 2, \ldots\ldots, L; \quad t = 1, 2, \ldots\ldots, A \tag{13}$$

where

$$d_{i1t}^{km} = \begin{cases} d_r^k(t) & \text{if m is the feasible repair type for state i and population k} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

Although the dimensions (i,1,t) are sufficient to express the repair rate matrix D^{km} (Equation (13)), it has to be re-written for mathematical reasons as follows:

$$D^{km} = \left[d_{ijt}^{km}\right] \qquad i, j = 1, 2, \ldots\ldots, L; \quad t = 1, 2, \ldots\ldots, A \tag{15}$$

where

$$d_{ijt}^{km} = \begin{cases} d_{i1t}^{km} & \text{for } i=j \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

Figure 10 shows an example of the matrix D^{km}. As before, we will refer to each layer in this matrix as D_t^{km}.

Prediction of the amount and cost of repair

The number of road sections in each condition state i which are expected to be selected for repair type m at time t (a certain year) can be expressed for any population k by a 3D matrix, $\dot{N}^{kmt}$ whose layers $\dot{N}^{kmt}_t$ can be determined by:

$$\dot{N}^{kmt}_t = N^{kt}_t D^{km}_t \tag{17}$$

Thus, the total amount of pavement sections getting repair type m at any year t, R^{mt} can be estimated by summing the elements $\dot{n}^{kmt}_{1it}$ of the matrix $\dot{N}^{kmt}_t$ (a row vector) over i, t and k. This can be written as:

$$R^{mt} = \sum_{k=1}^{K} \sum_{t=1}^{A} \sum_{i=1}^{L} \dot{n}^{kmt}_{1it} \tag{18}$$

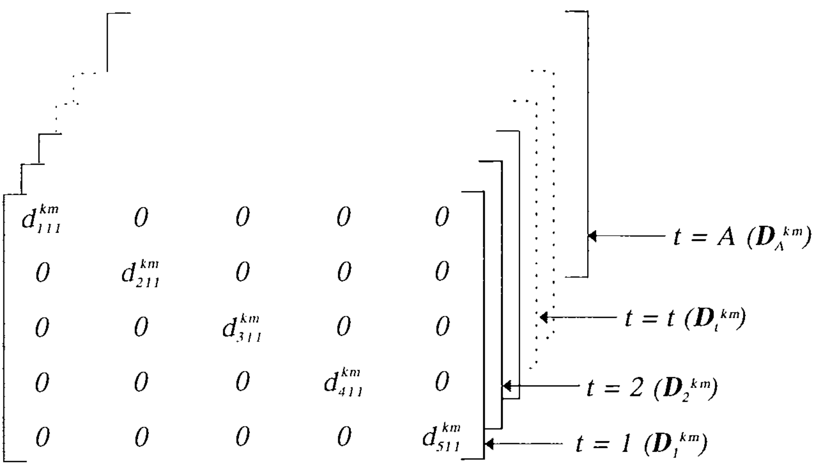

Figure 10 Schematic representation of a repair probability matrix

Assuming that the average repair cost per section is C_m, then total repair cost during year t; C^t, can be estimated as follows:

$$C^t = \sum_{m=1}^{M} R^{mt} C_m \tag{19}$$

Prediction of condition change due to repair

When a non-routine repair intervention action is carried out on a section, three changes take place: 1) change in surface type and thus population class, ie $k \rightarrow k'$; 2) change in pavement age, ie $t \rightarrow t'$; and 3) change in pavement condition state, ie $j \rightarrow i$. The outcomes of the first and second changes are certain while it is uncertain for the third. The first outcome is: repair always results in surface type "old pavement" unless the performed repair is reconstruction which results in "new pavement". The second outcome is: repair always results in a pavement with zero age ($t'= 1$) regardless of the repair type and the after repair state.

The effect of repair on pavement condition state (third outcome) is stochastic and depends on pavement class and the implemented repair type. A repair efficiency vector, which was introduced in Takayama (1990), was utilized to develop a suitable matrix to represent the expected effect of each repair type on pavement state. The efficiency vector can be written as:

$$E^{km} = \left[e_i^{km}\right] \qquad i = 1, 2, \ldots\ldots, L \qquad (20)$$

under the condition;

$$\sum_{i=1}^{L} e_i^{km} = 1 \qquad (21)$$

Where e_i^{km} is the probability that a road section from population class k will transfer to condition state i after repair m is performed regardless of its before-repair state. The values of e_i^{km} are given in Table 3.

Table 3 Probable efficiency of different repair types (partially after Takayama et al. 1990)

Traffic level	Repair type	Probability of moving to state i after repair 1	2	3	4	5
	Reconstruction	1.0	0.0	0.0	0.0	0.0
B	Over lay	0.773	0.227	0.0	0.0	0.0
	Surface dressing	0.551	0.381	0.051	0.017	0.0
	Reconstruction	1.0	0.0	0.0	0.0	0.0
C	Over lay	0.534	0.417	0.04	0.004	0.003
	Surface dressing	0.395	0.462	0.103	0.024	0.015
	Reconstruction	1.0	0.0	0.0	0.0	0.0
D	Over lay	0.240	0.451	0.240	0.051	0.017
	Surface dressing	0.145	0.591	0.199	0.054	0.011

This efficiency vector had to be re-formulated for two reasons: 1) to fit within the mathematical structure of the model and 2) to disallow the transfer to a worse condition state which is regarded as unreasonable. This is done as follows:

$$E^{km} = \left[e_{ji}^{km}\right], \qquad j, i = 1, 2, \ldots\ldots, L \qquad (22)$$

where

$$e_{ji}^{km} = \begin{cases} 0 & \text{for } j<i \\ \sum_{i}^{L} e_i^{km} & \text{for } j=i \\ e_j^{km} & \text{for } j>i \end{cases} \qquad (23)$$

under the condition;

$$\sum_{i=1}^{L} e_{ji}^{km} = 1 \qquad \text{for any } j \qquad (24)$$

Where e_{ji}^{km} is the probability that a road section from population class k with initial condition j will transfer to condition state i after repair m is performed. Figure 11 shows an example of the resulting efficiency matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ e_1^{km} & 1-e_1^{km} & 0 & 0 & 0 \\ e_1^{km} & e_2^{km} & 1-e_1^{km}-e_2^{km} & 0 & 0 \\ e_1^{km} & e_2^{km} & e_3^{km} & e_4^{km}+e_5^{km} & 0 \\ e_1^{km} & e_2^{km} & e_3^{km} & e_4^{km} & e_5^{km} \end{bmatrix}$$

Figure 11 Schematic representation of a repair efficiency matrix

The after repair condition of those sections selected for repair m given by the matrix $\dot{N}^{kmt}$ can be expressed by a 2D matrix, $\ddot{N}^{k'mt}$ can be determined as follows:

$$\ddot{N}^{k'mt} = \sum_{t=1}^{A} \dot{N}_t^{kmt} E^{km} \quad (25)$$

Where, the age class of all sections in $\ddot{N}^{k'mt}$ is 1 and k′ depends on before-repair population class k and the type of repair m as discussed before. In the case of applying routine maintenance, ie no repair, only pavement age changes so that the pavement section is aged by one year at the end of the duty cycle. Other characteristics, such as population class and state, remain unchanged. Based on this, the layers of the initial condition matrix of the existing road network at time t+1 can then be estimated as follows:

$$N_t^{kt+1} = \begin{cases} \sum_{m=1}^{M} \ddot{N}^{k'mt} & \text{for } k'=k \text{ and } t=1 \text{ (repaired sections)} \\ N_{t-1}^{kt} - \sum_{m=1}^{M} \dot{N}_{t-1}^{kmt} & \text{for } t>1 \text{ (remaining sections)} \end{cases} \quad (26)$$

Besides the above presented mathematical formulation of the model, a simulation system is also developed based on Monte Carlo simulation technique and is presented in Omar et al. (1993a and 1993b). Discussion on the advantages of each approach is also given in these references.

Application of the developed model for performance evaluation

The developed model provides three indicators for the evaluation of policy alternatives from the viewpoint of pavement performance. The first indicator is the condition state of the average section of the road network at time t; AS^t. This is similar to the common indicators which are provided by most of the previously developed models, eg average MCI, PCI, and etc. AS^t can be directly calculated from the matrices N^{kt} representing the condition of the road network at the end of any duty cycle t. This can be written as:

$$AS^t = \frac{\sum_{k=1}^{K} \sum_{t=1}^{A} \sum_{i=1}^{L} n_{lit}^{kt} \times i}{\sum_{k=1}^{K} \sum_{t=1}^{A} \sum_{i=1}^{L} n_{lit}^{kt}} \qquad (27)$$

This indicator, however, does not give a clear idea about repair needs of the network. The second indicator which becomes possible by applying the proposed model is the reliability of the average section of the road network; AR^t. This can be determined by referring to the reliability of each section, $R_r^k(t)$, based on the section's age at the end of the duty cycle t under consideration. This can be expressed as:

$$AR^t = \frac{\sum_{k=1}^{K} \sum_{t=1}^{A} \sum_{i=1}^{L} n_{lit}^{kt} \times R_r^k(t)}{\sum_{k=1}^{K} \sum_{t=1}^{A} \sum_{i=1}^{L} n_{lit}^{kt}} \qquad (28)$$

This indicator gives the probability that the average pavement section will need to be repaired during the following duty cycle. Thus, this directly indicates the expected repair needs of the network and can be a good measure for policy evaluation and budget estimation.

Neither of the above presented indicators takes into account the spatial structure (topology) of the network, ie link connectivity, redundancy, etc. A better indicator for evaluation would be the average flow reliability; AFR^t, between the important nodes of the network. Flow reliability can be defined as the probability that a trip between two certain nodes on the network can be done without passing on a section which needs repair. The proposed indicator can be determined by dividing the road network into a group of sub-networks, each represents the possible alternative routes between a pair of the important OD nodes on the original network. The reliability of each of these sub-networks can be calculated from the reliability of the sections composing the sub-network (component reliability), $R_r^k(t)$, by employing any algorithm for network reliability calculations. AFR^t can be then calculated as the average value of obtained sub-networks' reliability. This value is sensitive to where the repair budget is allocated over the network, unlike the previous indicators. For example, repair of a redundant link in the network would result in a lower increase in the AFR^t than if the repaired link is a part of a unique route. The other indicators may not be sensitive to such a change. Thus, the proposed indicator can be a good criterion for repair budget allocation over a network. An application example of this concept is given in Omar (1993a).

EVALUATION OF THE DEVELOPED MODEL

The purpose of this section is to further evaluate the developed model. As mentioned before, the statistical evaluation of the developed model is satisfactory (relatively high significance level). The observed trends of R_{ij}^k, λ_{ij}^k, R_r^k, and λ_r^k also suggest that the model is reasonable. However, performance prediction results obtained using the developed model has to be compared with actual performance and predicted performance using other models. To do this, data on the condition in 1988 contained in the data base were used for predicting condition in 1991 by employing three different performance prediction models. The first, "model 1", employs a deterministic model for deterioration prediction originally developed by the Japanese Ministry of Construction (Enomoto et al. 1987), and assumes a control limit criteria for repair intervention. The second, "model 2", is composed of the developed deterioration submodel and a control limit policy for repair intervention. The third, "model 3", is the developed performance model.

The predicted conditions in 1991 are then compared with the actual condition which is available in the road data base. Results of the comparison are shown in Figure 12 using the predicted versus actual state-structure (frequency distributions). It can be seen that condition prediction using the developed performance model is in close agreement with the actual conditions in 1991. In general, this agreement is better than that obtained using the other two models. Moreover, "model 2", which employs the developed deterioration submodel, also results in better predictions than "model 1". The stochastic repair intervention submodel, however, seems to contribute more to the prediction accuracy (compare the R^2 in the three cases). Such a result is another verification for the developed model and shows its higher accuracy in estimating future performance.

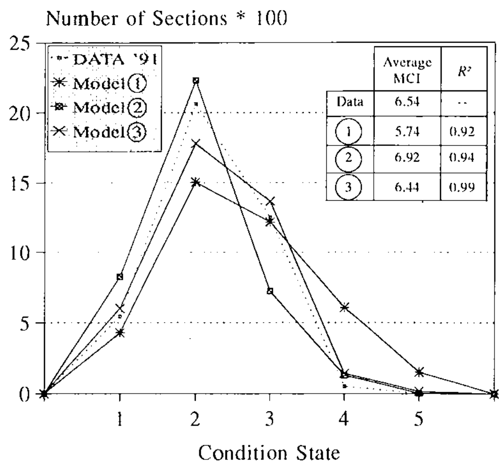

Figure 12 Real and predicted state-structure in 1991

SUMMARY AND CONCLUSIONS

This paper describes the development of a stochastic pavement performance model. The concepts of failure time models (FTMs) and systems reliability are applied to developing a stochastic approach for modeling two processes: 1) pavement deterioration with age and 2) repair intervention by the road agency and its effect on condition. Deterioration is represented as transition in condition over 5 successive condition states. Repair intervention, on the other hand, is represented by transition from a "repair not required" state to a "repair required" state. The model gives the probability of transition to a consequent state as a function of pavement class (ie original or rehabilitated pavement and traffic level), condition state, and, unlike other stochastic models, pavement age. Such transitions are treated as possible to occur at any age and condition. Performance is modeled as the outcome of a simultaneous interaction between the above mentioned two processes. Mathematical formulation is given for predicting performance. Statistical and intuitive evaluations of the developed model indicate a satisfactory model. Results of simulating change in condition and repair applications obtained by applying the developed model to a study road network show good agreement with actual conditions. Such an agreement can be mainly attributed to the following:

1. Treating the deterioration and repair intervention processes as stochastic phenomena, which is more realistic. This allows for consideration of variations in pavement condition with age which is usually ignored in deterministic models,
2. Considering the effect of pavement age on the transition rates between subsequent condition states unlike other stochastic models, and

3. Modeling repair intervention separately from deterioration so that the effect of factors other than condition on repair timing, eg strategic, technical and financial considerations, is accounted for in the model.

The developed model provides an indicator for pavement performance in terms of its reliability, that is, probability of no failure. Such an indicator is calculated from road age data which is easy and inexpensive to collect. Furthermore, reliability analysis may also be carried out while taking into considerations the network's spatial relations, eg, links connectivity and redundancy. This helps better allocation of repair budget throughout the network leading to higher traffic flow reliability.

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