A BASIC ANALYSIS OF THE DISTRIBUTION PATTERN OF WORK TRIPS

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1. INTRODUCTION

Finding the rules which control trip distribution patterns is an important subject in the field of urban transportation. Many studies on the trip distribution patterns to work, business, shopping, etc. have been made, and several models have been proposed. These models are the gravity model, the opportunity model, the logit model, etc. These models are useful in practice and used in the estimation of urban activity distributions. However, these previous models have not been based on what effects the travel distance produces on the economic activities in urban areas, but they have aimed at a faithful reproducing of the real urban activity distributions. Namely, the economic concept of the travel distance included in these models has not been defined. The real phenomenon that the trip frequency decreases with travel distance has just included in these models.

The gravity model is possibly used the most frequently in practical planning. A.G.Wilson gave a theoretical reason why the gravity model was formed, (1). He theoretically introduced the gravity model by using the entropy maximising models. At that time, he assumed that the sum of travel costs in a region at a given time was constant. He, however, made a mistake in his paper. It is the above assumption. It may be a fact that the sum of travel costs in a region at a given time is constant. However, this is not a cause but an effect. People are not distinguished between long trip persons and short trip persons under a social contract. The travel cost of a person is decided by himself irrespective of the travel cost of other persons. Therefore, it is considered that the above assumption is not valid.

As this writer mentioned in a paper,(2), a trip is not a natural phenomenon but a kind of economic behaviour. Therefore, the mechanism of trip generation should be considered in conformity with economic principles. When people intend to move from one place to another, they expect some benefit from their trip. Then we can broadly divide these trips into two categories. One is a trip which provides us with some benefit on the way, such as a leisure trip. The other is a trip which produces some benefit at the destination only. Most of the trips in our daily life come under the latter category. And these trips should be looked upon as dependent phenomena which are generated to accomplish their primary purposes at their destinations.

If now the work trips from a residential zone to a work zone are looked at from this angle, the work trips may be considered as the secondary demands which are derived from the primary demands of getting dwellings. Therefore, for the purpose of solving the problems of transportation and housing in urban areas, we must reveal the mechanism of housing demand which is a primary demand and the role that travel time to work plays in the mechanism.

Generally speaking, the amount of goods which will really be consumed is determined at the equilibrium intersection point where the supply and the demand match. However, this amount has two concepts. One is the size of goods. The other is the number of goods. These two concepts can be fitted in an equilibrium amount in the supply-and-demand of housing. Namely, one concept must be: Where do the households demand their dwellings and how large must they be ? The other concept must be : How many households want to get their dwellings in any given residential zone ? The former is a problem of the trade-offs between the floor area and the friction cost of the work trip. The results of research work on this problem have been published,(2). The latter is a problem that is related to the distribution of the residences or the work trips. The subject of this paper focuses on this point: the purposes of this study are to analyse empirically the distribution of work trips from possible residential zones to a particular work zone, and introduce theoretically the distribution pattern found there. This subject is directly related to the problems of residential distribution and the residential land values.

2. EMPIRICAL ANALYSIS OF WORK TRIPS DISTRIBUTION

In this study, the TOKYO Metropolitan Area where the railway netwrok is highly developed is chosen as the region being studies(see Fig.1).

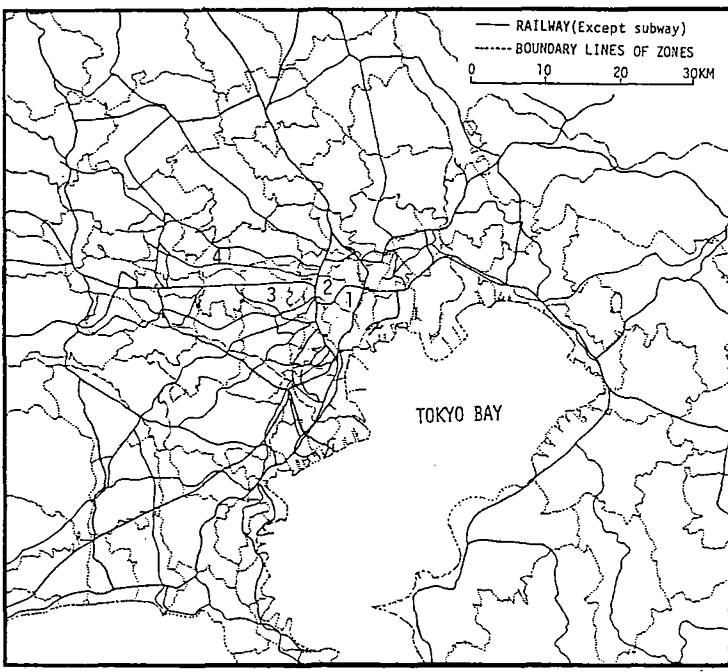


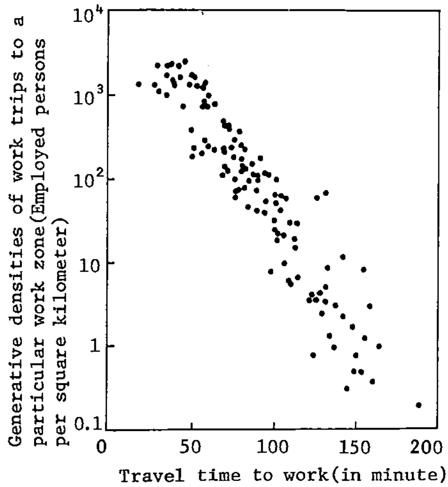
Fig.1 Zone Map of the TOKYO Metropolitan Area.
(Zone 1: Chiyoda, Zone 2: Shinjuku,
Zone 3: Suginami, Zone 4: Tanashi)

In this chapter, the volume of work trips going from possible residential zones to a particular work zone is analyzed. The data employed are the 1970 Population Census of Japan,(3). And the number of work zones which are considered is sixty-five. Four examples of them are shown in this paper.

Fig.2 shows the distribution of generative densities of work trips to a particular work zone(Chiyoda-ku: Zone 1 in Fig.1). Where, the generative densities of work trips to a particular work zone mean the volume of work trips going from unit land area available for housing at possible residential zones to a particular work zone,(4). And the travel time to work means the time required to travel between zone centroids.

Fig.2

The distribution of generative densities of work trips to a particular work zone(Chiyoda-ku: Zone 1 in Fig.1). (The number of employed persons at this work zone is 673,477).

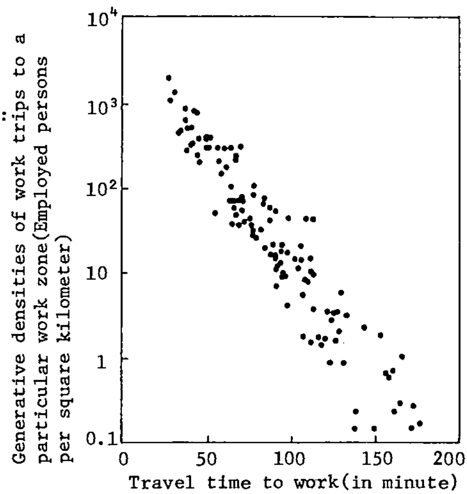


In Fig.2, an important characteristic can be found: the generative densities of work trips to the particular work zone decrease as the travel time becomes longer. The ordinate of this figure is graduated in a logarithmic scale. Therefore, it is considered that the distribution of the generative densities of work trips to the particular work zone takes the form of a nagative exponential distribution.

This phenomenon can be found not only on the generative densities of work trips to the CBD(Zone 1 is the CBD of the TOKYO Metropolitan Area), but on the generative densities of work trips to all other work zones. Figures 3,4 and 5 show respectively the distributions of the generative densities of work

Fig.3

The distribution of generative densities of work trips to a particular work zone(Shinjuku-ku: Zone 2 in Fig.1). (The number of employed persons at this work zone is 351,296).



trips to each particular work zone. Where, Shinjuku-ku(Zone 2) is 3.5Km away from the CBD, Suginami-ku(Zone 3) is 12Km away from the CBD and Tanashi-shi (Zone 4) is 21Km away from the CBD. As found in these figures, the gradients of the distributions are nearly equal, and the larger the number of employed persons at the work zones grows, the higher the generative densities of work trips to these work zones rise. However, not all dots on each figure are on a line. These dots are dispersed. It is supposed that this is caused by the differences in the quality of dwellings and the residential environments among the residential zones and the measurement errors of the travel time to work.

Fig.4

The distribution of generative densities of work trips to a particular work zone(Suginami-ku: Zone 3 if Fig.1). (The number of employed persons at this work zone is 152,937).

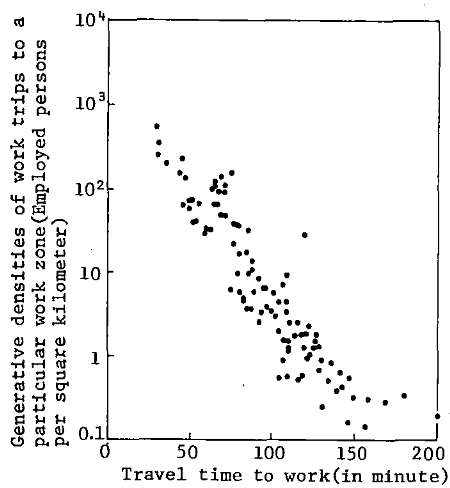
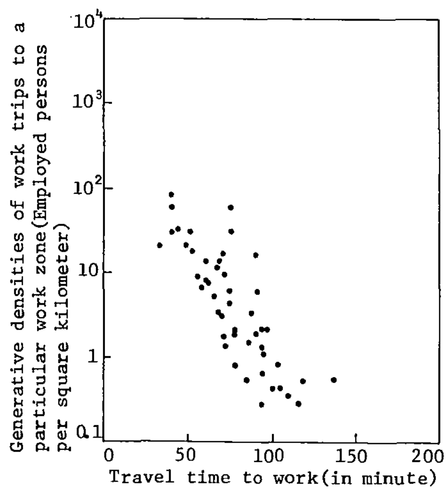


Fig.5

The distribution of generative densities of work trips to a particular work zone(Tanashi-shi: Zone 4 in Fig.1). (The number of employed persons at this work zone is 22,795).



In the following chapters, the phenomena that the generative densities of work trips to a particular work zone decrease, along a negative exponential function, as the travel time to work becomes longer will be introduced.

3. PREMISES FOR THEORETICAL ANALYSIS

Generally speaking, the equilibrium prices of goods or services are those at which the amounts willingly supplied and the amounts willingly demanded are equal. Housing is an economic commodity, too, so that, this principle can be applied to housing. There is a view point that an owned dwelling is not an economic commodity, because the household living in it doesn't pay house or land rent. However, there is no difference in the building expenses between an owned dwelling and a rented dwelling. Also, there is no difference in the residential land prices between them. An owned dwelling is a kind of accumlated capital. Therefore, it must be considered that the households living in the owned dwellings essentially pay rent for their housings and lands. For this reason, in this study, it is assumed that all the households live in rented dwellings(owned privately) and pay rent for their dwellings.

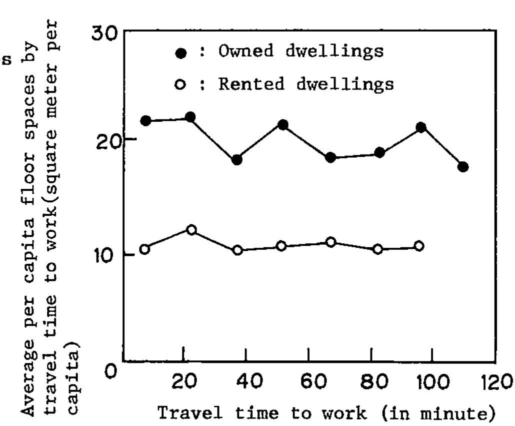
In this paper, the monthly bid rent for a rented dwelling(owned privately) is called the housing demand price, and the product of the building expense of a dwelling and the monthly interest rate is called the housing supply cost. Here, a payment for a residential lot is not included in the building expenses. Further, the friction cost of a work trip means the dissatisfaction incurred by any worker through time loss, money expense and human energy expenditure on his work trip. This friction cost of the work trip is given by the monthly total friction cost of a worker divided by the number of his household members.

All business firms actually try to maximize their profits. This is true of the housing suppliers, too. Thus, we can assume that all housing suppliers always pursue their maximum profits.

The number of workers per household is not the same in every household. There are some households which have several workers. The ways in which residential location decisions are made in multiworker households must be very complicated. Accordingly, in this study, it is assumed that every household has only one worker, in order to make the theoretical considerations easier.

Fig.6

Average per capita floor spaces by travel time to work.



As mentioned in the introduction, the main purpose of this study is to analyse theoretically the distribution of work trips to a particular work zone or the distribution of residences which the employed persons at a particular work zone ask for. It is assumed that the per capita floor spaces which all of the households ask for are constant, irrespective of the friction cost of the work trips. This assumption is based on the results of the motivation survey which have been published before,(2). That is, the reason why the assumption is set up is based on the facts that the average per capita floor spaces $\{\bar{A}\}$ by tenure of dwellings are nearly constant, independent of the travel time to work as shown in Fig.6.

In this figure, the average per capita floor space of owned dwellings is about 20 square meters per capita and that of rented dwellings(owned privately) is about 11 square meters per capita.

4. DEMAND AND SUPPLY FUNCTIONS FOR HOUSING

4.1 THE BASIC FACTORS COMPOSED HOUSING DEMAND MECHANISM AND THE MUTUAL RELATIONSHIPS AMONG THEM(Microscopic analysis)

The results of the microscopic analysis have been published before,(2). They are, in brief, as the follows:

It has been found that, other things being equal, the basic factors of which the housing demand mechanism are composed are the per capita income $\{I\}$, the per capita floor space $\{A\}$, the per capita housing demand price $\{P\}$ and the per capita friction cost of the work trip $\{T\}$. The mutual relationships among these basic factors have been represented by the following equations:

$$A = \alpha_0 \exp(\gamma I + \upsilon T + 1) \qquad (1)$$

$$I = (1/\gamma)\{\ln(A/\alpha_0) - \upsilon T - 1\} \qquad (2)$$

$$P = (\beta/\gamma)\{\ln(A/\alpha_0) - \upsilon T - 1\} - \beta I_0 \qquad (3)$$

$$P = \beta(I - I_0) \qquad (4)$$

Where, $\alpha_0 \exp(1)$ is the smallest per capita floor space which the households without any income ask for at the residential place where the friction costs of the work trips are zero.

β and γ are a coefficient.

I_0 is the per capita income without any housing expenditure.

And υ is

$$\upsilon = 1/\beta(I - I_0) \qquad (5)$$

Also, it has been confirmed that the per capita floor space $\{A\}$ has been substituted for the friction cost of the work trip $\{T\}$ when the per capita income has been fixed.

And, the friction cost of the work trip $\{T\}$ has been expressed as follows:

$$T = a\,t + (T_0 - a\,t_0) = a\,t + T_0' \qquad (6)$$

Where, a is the time value in the free time.

t is the travel time to work.

t_0 is the sum of the access time and the egress time.

T_0 is the friction cost which is taken with t.

And T_0' is

$$T_0' = T_0 - a\,t_0 \qquad (7)$$

Further, the relationships between the travel time to work $\{t\}$ and the other basic factors have been obtained as follows:

$$A = A_0 \exp(\gamma I + \eta t) \qquad (8)$$

$$I = (1/\gamma)\{\ln(A/A_0) - \eta t\} \qquad (9)$$

$$P = (\beta/\gamma)\{\ln(A/A_0) - \eta t\} - \beta I_0 \quad (10)$$

Where, $A_0 = \alpha_0 \exp(\nu T_0' + 1)$ (11)

$$\eta = \nu a \quad (12)$$

4.2 DEMAND FUNCTION FOR HOUSING

Generally the marginal demand price of a good tends to rise as a consumer's income increases, when the amount of the good consumed is fixed, for a high income household wants to purchase a superior good in quality. Vice, versa, the marginal demand price tends to fall as the amount of goods consumed increases, when the consumer's income and the quality of the goods are fixed, for the marginal utility of the good diminishes as the amount consumed increases. It is supposed that these principles can be valid to the social demand of the good, too. And these principles may be acting on the social demand of housing, because housing is an expensive economic good which is requisite to our lives. From this point of view, in this chapter, the housing demand function for the persons who are employed at a particular work zone is introduced.

Now, when the friction cost of the work trip is fixed, it is assumed that the marginal demand price per capita with respect to the amount demanded of housings(the number of housings) $\{\partial p/\partial x\}$ rises in proportion to the housing-cost-bearing capacity per capita $\{\Pi\}$ and falls in inverse proportion to the amount demanded of housings per unit land area available for housing in possible residential zones $\{x\}$. Then, $\partial p/\partial x$ can be represented as follows:

$$\frac{\partial P}{\partial x} = -\beta'' \Pi / x \quad (13)$$

Where, β'' is an index that is determined by the coefficient of housing-cost-bearing capacity $\{\beta\}$ and the coefficient of housing supply cost in each residential zone i $\{c_i\}$, as expressed by Eq.36. The negative sign at the right side of Eq.13 is put in order to follow the law of demand that the amount demanded of housing increases as its prices go down. And, it is assumed that the housing-cost-bearing capacity per capita $\{\Pi\}$ is the remainder between the per capita income $\{I\}$ and the per capita income without any housing expenditure $\{I_0\}$:

$$\Pi = I - I_0 \quad (14)$$

Integrating Eq.13, the per capita housing demand price $\{P\}$ can be expressed as follows:

$$P = -\beta'' \Pi \ln(x/{}_{\Pi}x) \quad (15)$$

Where, ${}_{\Pi}x$ is the amount demanded of housings per unit land area available for housing that a group of households, whose housing-cost-bearing capacity per capita are Π, asks for at the friction cost of the work trips $\{T\}$, when the per capita housing demand price is zero. The x is the amount demanded of housings when the per capita price is $\{P\}$, and the ${}_{\Pi}x$ is that when the per capita price is zero. Therefore,

$$x \leqq {}_{\Pi}x \quad (16)$$

Next, the relationship between the ${}_{\Pi}x$ and Π is sought for. Differentiating Eq.15 with respect to Π, when the amount demanded of housing $\{x\}$ is fixed, the marginal demand price to the housing-cost-bearing capacity $\{\Pi\}$ is obtained as follows:

$$\frac{\partial P}{\partial \Pi} = -\ \beta''\ \ln \frac{x}{{}_{\Pi}x} + \beta''\ \Pi\ \frac{1}{{}_{\Pi}x}\frac{\partial\,{}_{\Pi}x}{\partial \Pi} \tag{17}$$

It is supposed that this $\partial p/\partial\Pi$ rises with the capacity $\{\Pi\}$, when the amount supplyed of housing per unit land area available for housing at the friction cost of the work trips {T} is fixed. Hereupon, the $\partial p/\partial\Pi$ is assumed to rise in proportion to the capacity $\{\Pi\}$. Then, the $\partial p/\partial\Pi$ is expressed as follows:

$$\frac{\partial P}{\partial \Pi} = 2\ b\ (\ \beta''\ )^2\ \Pi \tag{18}$$

Where, b is a constant. And $2b(\beta'')^2$ is set up in due consideration of the following operations.

When Eq.17 corresponds with Eq.18, the following equation must be brought about(see the appendix 1):

$$\frac{1}{{}_{\Pi}x}\frac{\partial\,{}_{\Pi}x}{\partial \Pi} = b\ \ \beta'' \tag{19}$$

Solving this differential equation, we can obtain,

$${}_{\Pi}x = C_1\ \ \exp\ (\ b\ \beta''\ \Pi\ ) \tag{20}$$

Where, C_1 is a constant of integration. This C_1 can be determined by a boundary condition at the point that the friction cost of work trips {T} is zero. At the point, when the per capita housing price {P} is zero, the amount demanded of housings per unit land area available for housing is represented by ${}_0x$, and the housing-cost-bearing capacity per capita is represented by ${}_0\Pi$. Then, C_1 is

$$C_1 = \frac{{}_0x}{\exp\ (\ b\ \ \beta''\ \ {}_0\Pi\ )} \tag{21}$$

Substituting Eq.21 into Eq.20, we can obtain

$${}_{\Pi}x = {}_0x\ \exp\ \{\ -\ b\ \ \beta''\ \ ({}_0\Pi\ -\ \Pi\ )\ \} \tag{22}$$

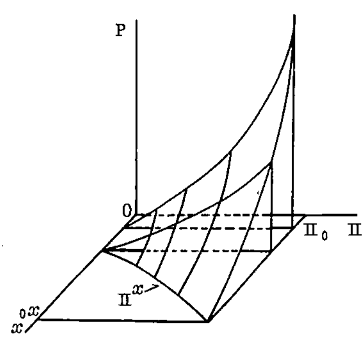
And, substituting Eq.22 into Eq.15, the per capita housing demand price {P} is obtained as follows:

$$P = b\ (\ \beta''\ \Pi\ )^2 -\ \beta''\ \Pi\ \ \{\ \ \ln\ (\frac{x}{{}_0x}) + b\ \beta''\,{}_0\Pi\ \} \tag{23}$$

Equation(23) represents the relationship between the per capita housing demand price {P} and the housing-cost-bearing capacity per capita $\{\Pi\}$. Fig.7 shows the conceptional picutre of the curved surface of housing demand which

Fig.7

The curved surface of housing demand (when the friction cost of work trips is zero).



is represented by Eq.23.

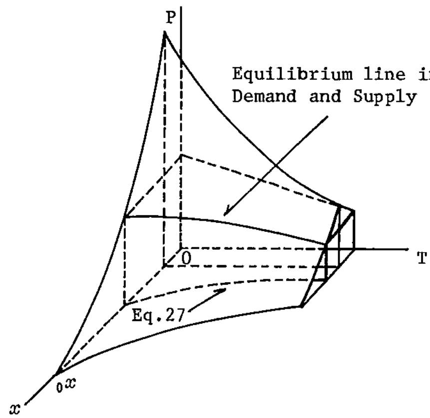
From equation(2),(14) and (23), the relationship between the per capita housing demand price {P} and the friction cost of the work trips {T} it can be shown that:

$$P = \frac{\beta''}{\gamma} \{ \ln (\frac{A}{a_0}) - \upsilon T - 1 - \gamma I_0 \}(-\ln \frac{x}{_0x} - \frac{b\,\beta''\,\upsilon}{\gamma} T) \quad (24)$$

Fig.8 shows the conceptional picture of the relationships among the factors: the per capita housing demand price {P} , the amount demanded of housings per unit land area available for housing {x} and the friction costs of the work trips {T}. And, substituting Eq.6 into Eq.24, the relationship between the per capita housing demand price {P} and the travel time to work {t} can be obtained.

Fig.8

The effects of the friction costs of work trips {T} on the housing demand price {P} and the amount demanded of housings {x}.



4.3 SUPPLY FUNCTION FOR HOUSING

Generally, a rich man tends not only to increase the amount which he is willing to buy of anything, but to choose a good of high quality. Therefore, even if the amount which he gets is fixed, he may pay much money for high quality. This tendency may be also found in house getting behaviour. The housing supply cost {C}, on the basis of this recognition, is assumed to be in proportion to the housing-cost-bearing capacity per capita {Π} and set up as follows:

$$C = c\,A\,\Pi \quad (25)$$

Where, c is an index that reflects the residential environment and the geographical features at each residential zone. This c is called the housing-cost-bearing capacity coefficient in this paper. And A stands for the average per capita floor space.

5. BASIC EQUATION OF WORK TRIP DISTRIBUTION

When every employed person can be assumed to make up a household and have a residence, it is reasonable that the distribution of residence densities of employed persons who go to a particular work zone is equal to the distribution of generative densities of work trips to the particular work zone. In this chapter, this generative density's distribution with respect to the friction cost of the work trips is introduced on the premise that the housing suppliers always try to maximize their profits.

A profit ordinarily is defined as the remainder between the price of a good and its cost. Applying this definition to housing transactions, the total profit per unit land area available for housing which the housing suppliers in the residential zone i may get from the employed persons who go from zone i to the particular work zone j, $\{R_{ij}\}$ can be represented as follows:

$$R_{ij} = (P_{ij} - C_i) N x_{ij} \quad (26)$$

Where, P_{ij} is the per capita housing demand price at the residential zone i which is offered by the employed persons who are employed at the work zone j.

C_i is the per capita housing supply cost at the residential zone i.

N is the average number of household members.

x_{ij} is the generative density of work trips at the residential zone i to the work zone j.

Substituting Eq.24 and Eq.25 into Eq.26, and differentiating Eq.26 with respect to x_{ij} , the generative density of work trips $\{x_{ij}\}$ which brings the maximum profit to the housing suppliers at the residential zone i is obtained as follows:

$$x_{ij} = {}_0x \exp(-1) \exp\left(-\frac{c_i \bar{A}}{\beta''}\right) \exp\left(-\frac{b\,\beta''\,\upsilon}{\gamma} T_{ij}\right) \quad (27)$$

Where, c_i is the index that refects the residential environments and the geo graphical features at the zone i.

$\bar{A}$ is the average per capita floor space.

This Eq.27 is the basic equation that represents the distribution pattern of work trips to a particular work zone j. Then, the distribution pattern takes the form of a negative exponential distribution. The indexes and the coefficients which are contained in Eq.27 can be estimated from the results of a pertinent survey to them.

Next, the relationship between the generative density of work trips $\{x_{ij}\}$ and the travel time to work $\{t_{ij}\}$ is introduced. Substituting Eq.6 and Eq.12 into Eq.27, this relationship is obtained as follows:

$$x_{ij} = {}_0x \exp(-1)\exp\left(-\frac{c_i\bar{A}}{\beta''}\right)\exp\left(-\frac{b\beta''\upsilon}{\gamma}T_0'\right)\exp\left(-\frac{b\beta''\eta}{\gamma}t_{ij}\right) \quad (28)$$

If now the following terms are employed:

$$K_j = {}_0x \exp(-1) \quad (29)$$

$$\lambda_i = \exp\left(-\frac{c_i\bar{A}}{\beta''}\right) \quad (30)$$

$$\tau_0 = \exp\left(-\frac{b\,\beta''\,\upsilon}{\gamma} T_0'\right) \quad (31)$$

$$\xi = \frac{b\,\beta''\,\eta}{\gamma} \quad (32)$$

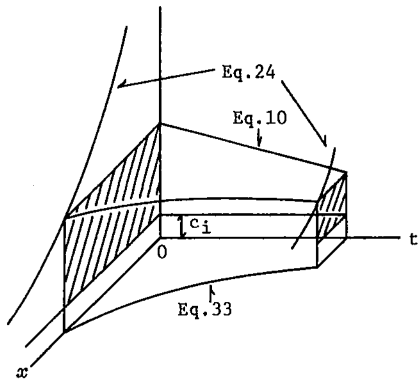
Then, we finally have:

$$x_{ij} = K_j \lambda_i \tau_0 \exp(-\xi t_{ij}) \quad (33)$$

This equation illustrates the phenomenon which the generative densities of work trips to a particular work zone $\{x_{ij}\}$ decrease, along a negative exponential function, as the travel time to work $\{t_{ij}\}$ becomes longer. Fig.9 shows the process of the above theoretical developement.

Fig.9

An illustration of the relationships among three factors: the per capita housing demand price $\{p_{ij}\}$, the generative density of work trips to a particular work zone $\{x_{ij}\}$ and the travel time to work $\{t_{ij}\}$. (The areas of hatching stand for the profit $\{R_{ij}\}$).



As mentioned in the chapter 2, the generative densities of work trips to a partiuclar work zone are not on a line, but dispersed as shown in each figure of Fig.2,.....,Fig.5. It is supposed that these dispersions may be caused not only by the differences of the λ_i and the τ_0 in Eq.33, but by the measurement errors of the travel time to work, too. These points will be considered in chapter 7.

Multiplying the land area available for housing at the zone $\{_R S_i\}$ by the generative density of work trips $\{x_{ij}\}$, the volume of work trips going from the residential zone i to the work zone j $\{X_{ij}\}$ is obtained as follows:

$$X_{ij} = K_j \lambda_i \ {}_R S_i \ \tau_0 \exp (- \xi \ t_{ij}) \tag{34}$$

6. EQUILIBRIUM HOUSING PRICE

In this chapter the equilibrium housing price per capita is examined. Substituting Eq.27 into Eq.24, the equilibrium housing price per capita $\overline{P}_{ij}$ is obtained as follows:

$$\begin{aligned} \overline{P}_{ij} &= (\beta'' + c_i \bar{A}) \left[\frac{1}{\gamma} \{ \ln(\frac{\bar{A}}{A_0}) - \upsilon T_{ij} - 1 \} - I_0 \right] \\ &= (\beta'' + c_i \bar{A}) \ \Pi \end{aligned} \tag{35}$$

This equation shows that $\overline{P}_{ij}$ is equal to the product of ($\beta'' + c_i \bar{A}$) and the housing-cost-bearing capacity per capita $\{\Pi\}$, and decreases as the friction cost of work trips $\{T_{ij}\}$ becomes larger.

The actual per capita expenditure on housing is equal to the per capita housing demand price expressed by Eq.4. Therefore, the per capita housing demand price must consist with the equilibrium housing price per capita expressed by Eq.35. Then, the following related equation must be formed between the coefficient $\{\beta\}$ and the index $\{\beta''\}$:

$$\beta'' = \beta - c_i \bar{A} \tag{36}$$

This equation explains that the index $\{\beta''\}$ is determined by the coefficient $\{\beta\}$ and the coefficient $\{Ci\}$.

The coefficient $\{\beta\}$ is supposed to be a ratio of the per capita expenditure on housing $\{P\}$ to the per capita capacity $\{\Pi\}$. And this ratio may be determined by following the law of equal marginal utilities per yen(money term). If now this $\{\beta\}$ can be regarded as a stable coefficient, the index $\{\beta''\}$ is considered to be an index which depends on the coefficients $\{c_i\}$.

7. CONSIDERATIONS

7.1 ON THE INDEXES AND THE COEFFICIENTS

The index K_j in Eq.29 stands for the generative density of work trips at the work zone j itself where the friction costs of work trips can be assumed to be nearly zero, when $\lambda_i = 1$ or $c_i=0$, and $\tau_0 = 1$ or $T_0'=0$. This K_j is an index which depends on the number of employed persons at the work zone j and the accessibility to the work zone j.

The index λ_i in Eq.30 depends on the housing-cost-bearing capacity coefficient $\{c_i\}$ at the residential zone i. The generative density $\{x_{ij}\}$ of work trips from the residential zone i to the work zone j decreases as this coefficient $\{c_i\}$ becomes larger. It is supposed that the better the land conditionis, the smaller the coefficient $\{c_i\}$ becomes.

The index τ_0 in Eq.31 expresses the total average inconvenience in the access and the egress of the work trips from zone i to zone j. This index, other things being equal, decreases as the total average friction cost $\{T_0\}$ of the access and the egress increases. Therefore, even if the travel time to work $\{t_{ij}\}$ is fixed, the generative density $\{x_{ij}\}$ of work trips from the residential zone i to the work zone j decreases as the $\{T_0\}$ increases.

The index ξ in Eq.32 represents the gradient of the generative densities distribution of work trips to a particular work zone. It is considered that the gradient varies with transportation means. In Fig.2, ... ,Fig.5, the gradients of the distributions have been nearly the same. It is considered that these phenomena have generated under the condition that most of the employed persons in the area have gone to work by trains.

7.2 THE ESTIMATION METHODS OF THE INDEXES AND THE COEFFICIENTS

It is to be desired that the measurements of the indexes and the coefficients involved in a theory are quite within the bounds of possibility. In this section, this possibility is explained.

It is possible that all of the indexes and the coefficients in this study are measured by the particular surveys to them. For example, β and I_0 can be found by a survey of the relationship between the per capita income and the per capita expenditure on housing. Other indexes and coefficients similarly can be obtained. However, K_j and τ_0 are the indexes which are determined by the number of employed persons at the work zone j, the accessibility to the zone j or the transportation conditions between the residential zone i and the work zone j, etc. as expalined in the proceeding section. Table-1 indicates an example of these indexes and coefficients. They are what have been obtained by a survey on residential location choices carried out in the TOKYO Metropolitan Area in 1970.

8. CONCLUSION

The purposes of this study have been to analyse empirically the distribution of work trips from possible residential zones to a particular work zone and introduce theoretically the distribution pattern found there. These pur-

Table-1 An example of the indexes and coefficients.

Index or coefficient	Equation to use	Numerical Values or remarks
β	Eq.4	0.1875
I_0	Eq.4	5,000(yen per capita)
ν	Eq.5	This index is dependent on the per capita income I.
A_0	Eq.8	2,200(square meters per capita)
γ	Eq.8	0.390×10^{-4} (capita per yen)
η	Eq.9 or Eq.10	0.858×10^{-2} (per minute)
a	Eq.12	This index is dependent on the per capita income I.
c_i	Eq.25	This c_i of each zone is different. But the average value is about 0.500×10^{-2} (per square meters)
$\bar{A}$		11 (square meters per capita) (Rented dwellings-owned privately)
ξ	Eq.33	0.06907752 (per minute) (In the case of railway)
β''	Eq.36	0.132
b	Eq.32	0.185 (capita per yen)
λ_i	Eq.30	The average value is about 0.6592
K_j	Eq.29	The index of each work zone is different
τ_0	Eq.31	The index of each work zone is different

poses have been achieved. Namely, it has been found that the generative densities of work trips to a particular work zone decrease, along a negative exponential function, as the travel time to work becomes larger. And this distribution pattern has been theoretically introduced on the premise that all the households have lived in rented dwellings(owned privately), every household has only one worker, and all the housing suppliers always have tried to maximize their profits.

The equation of the work trip distribution introduced in this study is represented by Eq.34. This equation is quite different from the gravity type models which have the number of trips generated and the number of trips attracted as a given condition. In the model in this study, how many persons live at each residential zone is not an important factor. This model has been introduced from the angle: how many employed persons at a particular work zone want to get their dwellings in any given residential zone ? Namely, in this study, the population at each residential zone is considered as not a cause, but an effect. Then, the factors which have some potent effect on the volume of the work trips from a residential zone i to a work zone j have been considered to be the number of employed persons at the work zone j, the friction cost $\{T_{ij}\}$ of the work trip between zone i and zone j, the sum $\{T_0\}$ of friction costs of the access and the egress, the housing-cost-bearing capacity coefficient $\{c_i\}$ and the land area available for housing $\{_R S_i\}$ at zone i.

The results of this study will be of use for the estimations of some phenomena related to the work trips. These phenomena are population distribution, trip distribution, residential land values, etc.

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APPENDIX 1.

From Eq.15, we obtain:

$$\frac{P}{\Pi} = - \beta'' \ln \frac{x}{\Pi^x} \qquad (i)$$

The remainder between Eq.17 and Eq.i is:

$$\frac{\partial P}{\partial \Pi} - \frac{P}{\Pi} = \beta'' \; \Pi \; \frac{1}{\Pi^x} \frac{\partial \Pi^x}{\partial \Pi} \qquad (ii)$$

On the other hand, integrating Eq.18 with respect to Π , we obtain:

$$P = b \; (\; \beta'' \;)^2 \; \Pi^2 \qquad (iii)$$

Where, the boundary condition is:

$$P = 0, \text{ when } \Pi = 0$$

From Eq.18 and Eq. iii we have:

$$\frac{\partial P}{\partial \Pi} - \frac{P}{\Pi} = b \; (\; \beta'' \;)^2 \; \Pi \qquad (iv)$$

So that, from Eq.ii and Eq.iv, we finally obtain:

$$\frac{1}{\Pi^x} \frac{\partial \Pi^x}{\partial \Pi} = b \; \beta'' \qquad (v)$$