A COMBINED RESIDENTIAL-LOCATION AND TRANSPORTATION NETWORK EQUILIBRIUM MODEL

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1. INTRODUCTION

This paper presents a formulation and a solution methodology of a model which combines residential-location into the transportation network equilibrium model. The model presented here will enable us to predict the impacts of transportation decisions such as investments for construction of new transportation facilities,changes in transit fares,etc. on residential location,and also the impacts of changes in residential location or employment location accompanied with the supply of housing,new location of firms on the use and performance of the transportation system.

It has been pointed out for a long time that one of the main defects of the traditional transportation model is in the mutual-independency among the models used at each stage of prediction of travel demand;trip generation,trip distribution,modal split and trip assignment. In order to overcome this theoretical weakness that the conventional transportation model has,combined transportation models based on the user equilibrium concept have been developed so far(1,2,3). Specifically, the unified transportation equilibrium model proposed by Safwat and Magnanti (3) involves the simultaneous prediction of trip generation,trip distribution,modal split,and trip assignment on large-scale networks. Although it is formulated as an equivalent convex optimization program, the model achieves a practical compromise between behavioral and computational aspects of modeling the problem and is easier and faster in computation than models formulated by the variational inequality approach. Likewise, Miyagi and Katoh (4) and Miyagi (5) have shown that using the conjugate theory a unified transportation model,consistent with the random utility theory, can be derived as a dual problem for a maximization problem of the total expected maximum utility. However, while the combined model proposed by Miyagi is constructed on the implicit assumption that trip assignment is performed by a logit-type network loading, modal split in Safwat and Magnanti's approach is formulated as a Wardrop user equilibrium model of path choice,not as a logit-type modal choice which is commonly assumed in practical studiés. Perhaps the most important problem neglected in these two approaches is that these models do not treat the economic framework explicitly,which is inevitable in formulating a combined land use and transportation model.

A few attempts have been demonstrated to combine both the activity allocation model and the transportation network equilibrium model into one general model in order to produce conditions of equilibrium in activity and travel distributions. This class of models is called the combined model approach following Berechman and Gordon (6). Examples of the combined model approach are provided by Boyce (7) and Boyce and Southworth(8),where Lowry model is used as activity allocation model and it is shown that Wilson's four types of interaction models (9) are expressed by equivalent mathematical optimization programs. Los (10) uses the Herbert-Stevens(11) model as its activity component and extends it so as to involve the simultaneous prediction of modal split and flow in networks. Problems arising in these approaches are that they ignore the existence of service trips or the interactions between service trip and traffic congestion. As the results, trip generation is independently determined from the congestion level of networks. Traffic congestion would affect the production of service trips more seriously than the work trip production. Recently, Prastacos (12) shows that the probabilistic choice framework for the Lowry model proposed by Coelho and Williams (13) and Wilson et al. (14) can be derived within the mathematical optimization framework (the similar result is also shown in Miyagi et al.(15)),in which he also assumes that interzonal travel costs are exogenously provided and that public modes are used only for work trip purpose.

Other comprehensive land-use/transportation interaction models are reported by the International Study Group on Land-Use/Transportation Interaction (ISGRUTI for short)(16). Among them,only DORTMUND makes trip generation,destination and mode choice and car ownership all dependent on travel costs,and therefore sensitive to congestion (16),however, the equilibration as the whole systems is not achieved based on the user equilibrium concept.

With these in mind, this paper aims at providing a combined model for predicting residential location and travel demand within a single mathematical programming framework ,in which the interactions among locational choice,travel demand including work and service trips,and performance of transportation systems are explicitly dealt with. This will be done by synthesizing a modified version of Safwat and Magnanti's model(3) with the conventional spatial interaction model proposed by Wilson et al (14). This synthesis brings about the new problem that a combined modal split and trip assignment can not be constructed as a mathematical optimization program because trip distributions generated by the synthesized model are no longer fixed variables but fluctuate linked together with spatial interactions of activities. In order to resolve this problem, coupling variables that link trip distribution to modal split are introduced.

This paper is organized as follows: In the second section, the relations between the economic base assumption and travel demand is described. Models presented there are not new ones and are those having been proposed elsewhere so far,however,they provide the equilibrium conditions of the models which will be presented in the third section. In the third section, the problem is formulated and the uniqueness of solution is proved. Although the model is described in terms of mathematical optimization programs, it includes coupling variables, and thus does not provide a separable optimization program. To resolve this difficulty, surrogate variables are introduced to uncouple the system, and a revised form of the program is presented. A solution methodology is also presented for prediction in the fourth section, where it is shown that by introducing the surrogate variables the total Lagrangian problem can be decomposed into two subprograms; one is provided as the spatial interaction submodel and the other as the combined modal split and assignment submodel, and that by interchanging information between master program and subprogram in terms of the coordination of the system, the whole program is easily computed.

2. ECONOMIC FRAMEWORK AND TRANSPORTATION MODELS:EQUILIBRIUM CONDITIONS

In this section, the relationships between economic base assumption and transportation models are described. Transportation models presented here constitute the equilibrium conditions of the combined models developed in the next section. All the models presented here are those that have been proposed so far.

2.1 Economic Framework and Its Relations to Trip Generation and Distribution

The heart of Lowry's activity allocation model is two spatial interaction models which generate patterns of population and service employment,consistent with the framework of economic base theory,in which distribution of basic employment is exogenously provided.

Wilson et al (14) have shown that the economic base assumption embeded in Lowry model can be condensed into the following closed set of simple equations (subscripts denoting work and service put on travel time are dropped here in relation to models developed later):

$$x^*_{ij} = \frac{E_j A^w_i \exp(-\beta^w \bar{u}_{ij})}{\Sigma_i A^w_i \exp(-\beta^w \bar{u}_{ij})} \qquad (1a) \qquad P_i = \kappa \, \Sigma_j \, x^*_{ij} \qquad (1b)$$

$$y^*_{ij} = \frac{\sigma P_i A^s_j \exp(-\beta^s \bar{u}_{ij})}{\Sigma_j A^s_j \exp(-\beta^s \bar{u}_{ij})} \qquad (2a) \qquad E^s_j = \Sigma_i \, y^*_{ij} \qquad (2b)$$

$$E_j = E^b_j + E^s_j \qquad (3)$$

where the variables and parameters are defined as follows:

x^*_{ij} : number of workers in i who are employed in zone j
y^*_{ij} : number of service job in zone j created by service demand in zone i
A^w_i : attractiveness-measure of zone i for the residential location
A^s_j : attractiveness-measure of zone j for the location of service activities
E^b_j, E^s_j, E_j : basic,non-basic,and total employment in zone j
κ : an inverse of activity rate in zone i
σ : service activity rate
$\bar{u}_{ij}$: travel time from zone i to j
β^w, β^s : parameters of the spatial interaction submodels.

The residence/workplace submodel and the residence/service submodel represented by Eqs.(1) and (2),respectively, are such that those satisfy the conditions

$$\Sigma_i x^*_{ij} = E_j \qquad (4a) \qquad\qquad \Sigma_j y^*_{ij} = \sigma P_i \qquad (4b)$$

The employment variables x^*_{ij} and y^*_{ij} are transformed into trip variables x_{ij} and y_{ij} by intervening the parameters η and ρ which denote the number of work and service trips generated by unit work and service employment, respectively. i.e.

$$x_{ij} = \eta \ x^*_{ij} \qquad (5a) \qquad\qquad y_{ij} = \rho \ y^*_{ij} \qquad (5b)$$

Wilson et al (14) have shown that (x_{ij}),(y_{ij}) satisfying the economic base assumptions are condensed into interlocked doubly constrained gravity models:

$$x_{ij} = a^w_i b^w_j P_i E_j \ A^w_i \exp(-\beta^w \bar{u}_{ij}) \qquad (6)$$

$$y_{ij} = a^s_i b^s_j P_i E^s_j \ A^s_i \exp(-\beta^s \bar{u}_{ij}) \qquad (7)$$

where

$$a^w_i = \frac{\eta/\kappa}{\Sigma_i b^w_j E_j \exp(-\beta^w \bar{u}_{ij})} \qquad (8a) \qquad\qquad a^s_i = \frac{\rho\sigma}{\Sigma_i b^s_j E^s_j \exp(-\beta^s \bar{u}_{ij})} \qquad (8b)$$

$$b^w_j = \frac{\eta}{\Sigma_i a^w_i P_i A^w_i \exp(-\beta^w \bar{u}_{ij})} \qquad (8c) \qquad\qquad b^s_j = \frac{\rho}{\Sigma_i a^s_i P_i A^s_i \exp(-\beta^s \bar{u}_{ij})} \qquad (8d)$$

and that those may be underpinned by probabilistic choice theory at the microlevel.

Population and employment variables (P_i),(E^s_j) and (E_j) are defined internally by using relations (1b),(2b),(3) and (4). Even if the planning control parameters are included and bind the allocation mechanism as constraints,for example, the projected population of certain zones are imposed as upper bound for the estimated population of these zones,by a trivial manipulation of balancing factors the doubly constrained models can be applicable. In the case when ,in certain zones, the control parameters do not bind variable concerned,these zones can be treated using single constrained model.

In this formulation,it is implicitly assumed that the choice of shopping location is dependent on the location of residence,but the choice of residence itself is independent of the resultant spatial distribution of retail services. Coelho and Williams (13) have shown that the residence/workplace submodel containing an additional component related to the comparative advantage of shopping from zone i is represented by the following relations :

$$x_{ij} = \frac{\eta E_j A^w{}_{i\cdot} \exp(-\beta^w u_{ij})}{\Sigma_i A^w{}_{i\cdot} \exp(-\beta^w \bar{u}_{ij})} \tag{9a}$$

where

$$A^w{}_{i\cdot} = A^w{}_i \exp\left(\frac{\beta^w}{\lambda}\bar{s}_i\right), \quad \lambda = \eta/\rho\kappa\sigma \tag{9b}$$

and

$$\bar{s}_i = \frac{1}{\beta^s} \ell n\, \Sigma_j A^s{}_j \exp(-\beta^s \bar{u}_{ij}) \tag{10}$$

$\bar{s}_i$ is the expected maximum utility associated with shopping from residence location i,and a measure of attractiveness of zone i for shopping(service) purposes.

The relation between the economic base and spatial trip distribution described by a set of equations mentioned above seems to be commonly used to integrate land-use model into transportation models,however,such approach does not permit the incorporation of trip generation model, sensitive to the changes in the service levels in transportation system , into a sequel transportation model. For our purpose to construct a unified transportation model responsive the changes of transportation service levels, the approach proposed by Sofwat and Magnanti (3),in which trip generation in zone i is assumed to be given by a function of s_i, may be effective. Especially, service trips would be characterized by the sensitiveness to the traffic congestion,while work trips tend to be inelastic to the service levels of transportation. Under this assumption, the model for service trip,Eq.(2a), is rewritten as follow:

$$y_{ij} = (\alpha s_i + SO_i) \frac{A^s{}_j \exp(-\beta^s \bar{u}_{ij})}{\Sigma_j A^s{}_j \exp(-\beta^s \bar{u}_{ij})} \tag{11}$$

where SO_i denotes the composite effects that the socio-economic variables,which are exogenous to the transportation system,have on service trip generation from zone i. α is a coefficient of s_i defined by

$$s_i = \max.\ [0, \sum_{j \in D_i} A^s{}_j \exp(-\beta^s \bar{u}_{ij})] \tag{12}$$

where Di is a set of destinations that are accessible from origin i.

A demand model for service trip which combines trip generation model into trip distribution model,Eq.(11),no longer requires the trip production unit σ. If we believe that the trip production unit is invariant over time, trip generation estimated by $Y_i = \alpha s_i + SO_i$ can be converted to the stock variable by using the trip production unit. Otherwise, it is no longer valid to assume that the number of trips attracted to a given destination from all origin zones can be equilibrated to non-basic employment of a given zone by intervening the trip production unit.

2.2 Modal Split and Trip Assignment

We consider the region where some of the network O-D pairs are connected by transit and both transit and automobile modes are competitive. The link travel time on the transit network is assumed constant,independent of the automobile links. The transit level of service between origin i and destination j is represented by a single link connecting each O-D pair served by transit. While both automobile and transit flows are expressed in terms of persons per unit of time,in automobile network a vehicle occupancy factor must be used to convert person flow to vehicular flow over this network. For simplicity,it is assumed here that the automobile occupancy factor is 1. Thus, the combined modal split/assignment model considered here is the same as that by Florian (17).

The appropriate flow of transit and automobile travelers for each O-D pair is assumed to be estimated by the well-known logit formula:

$$q_{ij} = \frac{\bar{q}_{ij}}{1+\exp[\theta(u_{ij}-u'_{ij})]} \tag{13a}$$

and

$$q'_{ij} = \bar{q}_{ij} - q_{ij} \tag{13b}$$

where

$\bar{q}_{ij}$:total trips for O-D pair i-j
q_{ij} :auto trips for O-D pair i-j
q'_{ij}:transit trips for O-D pair i-j
u_{ij} :the minimum travel time by auto between O-D pair i-j
u'_{ij}:the minimum travel time by transit between O-D pair i-j
θ : a positive parameter estimated from data.

The sum of trips estimated by trip purposes must equal to the total trip $\bar{q}_{ij}$. Thus we have

$$\bar{q}_{ij} = x_{ij} + y_{ij} \tag{14}$$

Providing that the user equilibrium conditions hold over the automobile network,at equilibrium the following relations hold.

$$\begin{aligned} (c_{kij} - u_{ij})h_{kij} &= 0 \\ c_{kij} &\leq u_{ij} \\ \Sigma_k h_{kij} &= q_{ij} \\ c_{kij} &= \Sigma_a \ \delta_{akij} \ t_a(f_a) \end{aligned} \tag{15}$$

where

h_{kij},c_{kij}:the flow and travel time,respectively,on automobile route k connecting O-D pair i-j
δ_{akij}:it takes 1 if automobile link a is on the kth route connecting O-D pair i-j,and 0 otherwise

$t_a(\cdot)$:link performance function
f_a:link flow defined by $f_a = \Sigma_k \Sigma_{ij} \delta_{akij} h_{kij}$.

3. COMBINED RESIDENTIAL-LOCATION AND TRANSPORTATION MODELS

We shall show in this section that the previously stated travel demand models ,modal split function and traffic equilibrium conditions can be derived from synthesized equivalent optimization programs.

3.1 Mathematical Programming Framework

The following notation closely parallels Safwat and Magnanti's notation. Let $u^i = (u_{ij} : j \varepsilon Di)$ be the vector of travel time from origin i to destinations Di. Similarly,let $Y^i(u^i) = (y_{ij}(u^i) : j \varepsilon Di)$ denote the vectors of trips distributed from origin i as a function of travel time. Then according to Theorem proposed by Safwat and Magnanti (3), the demand models for service Eq.(11), has the inverses of the following form:

$$u_{ij}(Y^i) = \frac{1}{\beta^s} [\ell n\ A^s{}_j - \ell n\ y_{ij} + \ell n\ \Sigma_j\ y_{ij} - \frac{1}{\alpha} (\Sigma_j\ y_{ij} - \alpha SO_i)] \qquad (16)$$

of which Jacobian matrix is symmetric and negative definite. Consequently, if the inverse demand functions are provided by Eq.(16), the line integral formulation of Beckmann's model (18) can be used to create the objective function of the combined model we aim at developing. However,demand models for work trip considered here do not have the inverse functions.

In sequent sub-sections, we shall show two different optimization programs which correspond to cases of (1) population and total employment are exogenously provided, (2) population is internally defined. The first model is developed with the intention to synthesize the unified network equilibrium model with activity variables. Following the discussion made by Wilson et al (14),however,note that the same model is applicable to the case when population is endogenously determined.

(1) A model with population and total employment being exogenously given

Consider the following optimization problem [P].

$$\min.\ Z(s,x,y,q,q',h) = \Phi_1(s) + \Phi_2(x,y,q,q') + \Phi_3(h) \qquad (17a)$$

subject to

$$\begin{aligned} &\Sigma_j\ x_{ij} = \lambda_1\ P_i \quad , \quad \Sigma_i\ x_{ij} = \lambda_2\ E_j \\ &\Sigma_j\ y_{ij} = \alpha s_i + SO_i \quad , \quad x_{ij} + y_{ij} = q_{ij} + q'_{ij} \qquad (17b) \\ &q_{ij} = \Sigma_k\ h_{kij} \\ &s,x,y,q,q',h \geq 0 \end{aligned}$$

$$\Phi_1(s)= \frac{1}{\beta^s} \Sigma_i [\alpha s_i{}^2/2+\alpha s_i-(\alpha s_i+SO_i)\ \ \ell n(\alpha s_i+SO_i)] \tag{18a}$$

$$\Phi_2(x,y,q,q')= \frac{1}{\beta^w} \Sigma_i\Sigma_j\ x_{ij}(\ell n \frac{x_{ij}}{A^w{}_i} -1)+ \frac{1}{\beta^s} \Sigma_i\Sigma_j\ y_{ij}(\ell n \frac{y_{ij}}{A^s{}_j} -1)$$

$$+ \frac{1}{\theta} \Sigma\Sigma q_{ij}(\ell n \frac{q_{ij}}{x_{ij}+y_{ij}} - 1) + \frac{1}{\theta} \Sigma\Sigma\ q'_{ij}(\ell n \frac{q'_{ij}}{x_{ij}+y_{ij}} - 1)$$

$$+ \frac{1}{\theta} \Sigma\Sigma\ (xij+yij) + \Sigma\Sigma\ q'_{ij}u'_{ij} \tag{18b}$$

$$\Phi_3(h)= \Sigma_a \int_0^{f_a(h)} t_a(f)df \tag{18c}$$

and

$\lambda_1=\eta/\kappa,\ \ \lambda_2=\eta.$

Equation (18a) can be derived by performing the details of the line integral of Eq.(16),but constant terms associated with SO_i are omitted. In this optimization program,decision variables $\{q_{ij}\}$ and $\{q'_{ij}\}$ are coupled with other decision variable $\{x_{ij}+y_{ij}\}$ through the third and fourth term in Φ_2. Without the coupling terms,the problem would be separable. In order to uncouple the system,a surrogate variable $\{\bar{q}_{ij}\}$ and the corresponding constraints will be introduced. The resultant problem is expressed as follows:

[P1]

$$\min.\ Z(s,x,y,\bar{q},q,q',h)= \Phi_1(s)+ \Phi_2(x,y)+\Phi_3(\bar{q},q,q')+ \Phi_4(h) \tag{19a}$$

subject to

$$\begin{aligned} &\Sigma_j\ x_{ij}= \lambda_1\ P_i \quad (\nu_i), \qquad \Sigma_i\ x_{ij} = \lambda_2\ E_j \quad (\omega_j) \\ &\Sigma_j\ y_{ij}= \alpha s_i+SO_i \quad (\gamma_i), \qquad x_{ij}+y_{ij}= \bar{q}_{ij} \quad (\bar{u}_{ij}) \\ &\bar{q}_{ij}=q_{ij}+q'_{ij} \quad (\mu_{ij}), \qquad q_{ij}=\Sigma_k\ h_{kij} \quad (u_{ij}) \\ &s,x,y,\bar{q},q,q',h \geq 0 \end{aligned} \tag{20}$$

where $\{\bar{q}_{ij}\}$ is a surrogate variable for $(x_{ij}+y_{ij})$ which is introduced in order for the system to be separable,thus by itself plays no role for determining interzonal trip distribution. Greek letters in parentheses attached to each constraint denote the Lagrangian multipliers associated with each constraint. Here the third term of the objective function of [P] is renumbered as the fourth. The second sets of terms included in the objective function of [P] are devided into two separate terms and redefined as follows:

$$\Phi_2(x,y)= \frac{1}{\beta^w} \Sigma_i\Sigma_j\ \ x_{ij}(\ell n \frac{x_{ij}}{A^w{}_i} -1)+ \frac{1}{\beta^s} \Sigma_i\Sigma_j\ \ y_{ij}(\ell n \frac{y_{ij}}{A^s{}_j} -1) \tag{21a}$$

$$\Phi_3(\bar{q},q,q')= -\frac{1}{\theta}\Sigma\Sigma\,\bar{q}_{ij}(\ln\bar{q}_{ij}-1)+\frac{1}{\theta}\Sigma\Sigma\,q_{ij}(\ln q_{ij}-1)$$

$$+\frac{1}{\theta}\Sigma\Sigma\,q'_{ij}(\ln q'_{ij}-1)+\Sigma\Sigma\,q'_{ij}u'_{ij} \tag{21b}$$

For finite β^w, β^s and θ, $\{x_{ij}\}$, $\{y_{ij}\}$ and $\{q_{ij}\}$ are always positive. Thus, the equality sign will prevail in Kuhn-Tucker conditions for [P1]. Hence,

$$s_i-\ln(\alpha s_i+SO_i)+\beta^s\gamma_i=0 \tag{22a}$$

$$\frac{1}{\beta^w}\ln\frac{x_{ij}}{A^w_i}+\bar{u}_{ij}-\nu_i-\omega_j=0 \tag{22b}$$

$$\frac{1}{\beta^s}\ln\frac{y_{ij}}{A^s_j}+\bar{u}_{ij}-\gamma_i=0 \tag{22c}$$

$$\frac{1}{\theta}\ln q_{ij}-\mu_{ij}+u_{ij}=0 \tag{22d}$$

$$\frac{1}{\theta}\ln q'_{ij}+u'_{ij}-\mu_{ij}=0 \tag{22e}$$

$$-\frac{1}{\theta}\ln\bar{q}_{ij}+\mu_{ij}-\bar{u}_{ij}=0 \tag{22f}$$

$\bar{u}_{ij}$ is obtained by eliminating μ_{ij} from Eqs.(22d) to (22f) and is given by

$$\bar{u}_{ij}=-\frac{1}{\theta}\ln[\exp(-\theta u_{ij})+\exp(-\theta u'_{ij})] \tag{23}$$

Taking into account of this, and from Eq.(22b) together with the first and second constraints in (20), the following doubly constrained model is derived for work trip.

$$x_{ij}=a^w_i b^w_j P_i E_j A^w_i \exp(-\beta^w\bar{u}_{ij}) \tag{24a}$$

where

$$a^w_i=\frac{\lambda_1}{\Sigma_i b^w_j E_j\exp(-\beta^w\bar{u}_{ij})} \tag{24b}$$

$$b^w_j=\frac{\lambda_2}{\Sigma_i a^w_i P_i A^w_i\exp(-\beta^w\bar{u}_{ij})} \tag{24c}$$

As to service trip it can be easily shown that the same equation as Eq.(11) is obtained and that from Eqs. (22a) and (22c) together with the third constraint in Eq. (20),Eq.(12) is also derived. Likewise, by examining the Kuhn-Tucker conditions, the solution of [P1] can be readily shown to include Wardrop equilibrium conditions,a set of Eqs.(15).

(2) A model with population being internally defined

We shall show here that if population is dealt with as a variable in [P1], that is, internally defined, then the similar program as [P1] generates the another demand model for work trip defined by Eqs.(9) and (10). For simplicity, we shall assume, without a loss of generality, that the socio-economic variable is composed of solely population variable. Let us define somewhat different objective function from that of [P1], in which only Φ_1 is changed and expressed as follows:

$$\Phi'_1(s,P) = \frac{1}{\beta^s} \Sigma_i [\alpha s_i{}^2/2 + \alpha s_i + \alpha_1 P_i - (\alpha s_i + \alpha_1 P_i)\ \ell n(\alpha s_i + \alpha_1 P_i)] \tag{25}$$

It should be noted that Φ'_1 is not the one derived from the line integral of the inverse demand function for service trip because the higher order term associated with P_i is omitted in the above set of terms instead of P_i being assumed to be variable here. First-order partial derivatives provide

$$-\ \ell n\ (\alpha s_i + \alpha_1 P_i) + \frac{\lambda_1 \beta^s}{\alpha_1} \nu_i + \beta^s \gamma_i = 0 \tag{26}$$

Considering s_i is given by Eq.(22a), and substituting ν_i, which is obtained from the equation above, into Eq.(22b), the demand model for work trip can be readily shown to be

$$x_{ij} = \frac{\eta E_j A^w{}_{i\cdot} \cdot \exp(-\beta^w u_{ij})}{\Sigma_i A^w{}_{i\cdot} \cdot \exp(-\beta^w u_{ij})} \tag{27a}$$

where

$$A^w{}_{i\cdot} = A^w{}_i \ \exp(\frac{\alpha_1 \beta^w}{\lambda_1} \bar{s}_i) = A^w{}_i \exp(\frac{\alpha_1}{\lambda_1} s_i) \tag{27b}$$

Comparing (27b) with (9b), we can see that the only difference between these relations lies in coefficients imposed on $\bar{s}_i$. A parameter α_1, being estimated from data, may correspond to $\sigma\rho$ because $\sigma\rho$ represents the number of service trips per unit population and a parameter λ_1 is given by η/κ as previously defined. Thus, it can be said that these equations are completely equivalent in its implication.

3.2 Uniqueness

To prove that the equivalent mathematical program [P1] has a unique solution in the feasible region, it is sufficient to show that the objective function is strictly convex in the feasible region. This is done by proving that the Hessian of the objective function is positive definite. Since the objective function is separable with respect to decision variable included in each term of the objective function, the structure of the Hessian is " block diagonal," with each block given by the m x m matrix, in which m denotes the dimension of each variable and varies according to the number of each variable. The structure of the Hessian, H, is expressed as follows:

$$
H=\begin{bmatrix} H_s & & & & & & \\ & H_x & & & & & 0 \\ & & H_y & & & & \\ & & & H_{\bar{q}} & & & \\ & & & & H_q & & \\ 0 & & & & & H_{q'} & \\ & & & & & & H_f \end{bmatrix}
$$

where each Hessian has the structure that all the off-diagonal elements are zero and all the diagonal elements are given by the second derivatives. Therefore,expressing each Hessian by its representative element,we have

$$
\begin{aligned}
&H_s = \left(\frac{\alpha}{\beta^s} \left[\frac{Y_i - \alpha}{Y_i} \right] \right), \quad H_x = \left(\frac{1}{\beta^w x_{ij}} \right), \quad H_y = \left(\frac{1}{\beta^s y_{ij}} \right) \\
&H_{\bar{q}} = \left(-\frac{1}{\theta \bar{q}_{ij}} \right), \quad H_q = \left(\frac{1}{\theta q_{ij}} \right), \quad H_{q'} = \left(\frac{1}{\theta q'_{ij}} \right), \quad H_f = \left(\frac{dt_a(f_a)}{df_a} \right)
\end{aligned} \tag{28}
$$

Accordingly,as Safwat and Magnanti have proved,$\Phi_1(s)$ is strictly convex over the feasible region if $\alpha < SOi$,which is usually satisfied. Also, taking the quadratic forms of H_x and H_y, we can easily show that H_x and H_y are positive definite over $x \geq 0$ and $y \geq 0$. Thus $\Phi_3(x,y)$ is convex. Furthermore, if $t_a()$ is assumed to be strictly increasing,its integral is a strict convex,and thus $\Phi_4(h)$ is strictly convex with respect to link flow. Finally, taking the quadratic forms of H_q, $H_{\bar{q}}$ and $H_{q'}$, taking into account of the relations which hold among these variables,we have

$$
[\bar{q},q,q'] \begin{bmatrix} H_{\bar{q}} & & \\ & H_q & \\ & & H_{q'} \end{bmatrix} [\bar{q},q,q']^t = 0 \tag{29}
$$

From these observation,it is assured that the objective function is strictly convex. In addition, since all the constraints of the problem are linear,the feasible region is convex. Thus, the equilibrium problem defined by the mathematical program [P1] has a unique solution.

4. INTERACTIVE BALANCING METHOD FOR PREDICTING EQUILIBRIUM STATE

We shall propose two computation methods for predicting equilibrium flow described by the mathematical optimization program [P1]. The proposed method is also applicable to the second program,a model with population being internally defined, with a slight modification,thus omitted here for that program.

By examining the structure of the program we can see that Lagrangian for the original program can be decomposed into two sub-systems;the first system consisting of trip generation,distribution variables forms the usual spatial interaction problem and the second one is given as a combined modal split/assignment problem. The surrogate variable $(\bar{q}_{ij})$ and its dual variable $(\bar{u}_{ij})$ play the important role coordinating the overlapping sub-systems. The decomposition forms hierarchical multilevel structures. At the first-level the spatial interaction submodel and combined modal split/assignment submodel are independently solved under the condition that $\{\bar{q}_{ij}\}$ or $\{\bar{u}_{ij}\}$ is given. At the second-level the output generated from each subsystem are coordinated via $\{\bar{q}_{ij}\}$ or $\{\bar{u}_{ij}\}$. Thus the approach presented here may be called interactive balancing method and yields two distinct methods depending on the choice of either coordinator, the state variable $\{\bar{q}_{ij}\}$ or the dual variable $\{\bar{u}_{ij}\}$. The basic idea of this approach comes from the feasible and nonfeasible decomposition of unseparable nonlinear programming(20),however,a way of transferring variables between master and subsystems is somewhat different from those decomposition methods.

4.1 Interactive Balancing Method with State-Variable-Coordinator

The Lagrangian for the problem [P1],L, can be decomposed into the following two sub-Lagrangians,L1 and L2:

$$L(s,x,y,\bar{q},q,q',h,\boldsymbol{\nu},\boldsymbol{\omega},\boldsymbol{\gamma},\boldsymbol{\mu},\bar{u},u)= L_1(s,x,y,\boldsymbol{\nu},\boldsymbol{\omega},\boldsymbol{\gamma},\bar{u};\bar{q})+L_2(q,q',h,\boldsymbol{\mu},u;\bar{q}) \qquad (30a)$$

where

$$L_1 = \Phi_1(s)+\Phi_2(x,y) + \Sigma\nu_i(\lambda_1 P_i-\Sigma x_{ij})+\Sigma\omega_j(\lambda_2 E_j-\Sigma x_{ij})+\Sigma\gamma_i(Y_i-\Sigma y_{ij})+\Sigma\bar{u}_{ij}(x_{ij}+y_{ij}-\bar{q}_{ij}) \qquad (30b)$$

$$L_2 = \Phi_3(q,q';\bar{q})+\Phi_4(h)+\Sigma\mu_{ij}(\bar{q}_{ij}-q_{ij}-q'_{ij})+\Sigma u_{ij}(q_{ij}-\Sigma h_{kij}) \qquad (30c)$$

Note that the surrogate variable,$\{\bar{q}_{ij}\}$,as arguments in the sub-Lagrangians and Φ_3, is preceded by (;) to indicate that it should be viewed as a known parameter,at the first-level optimization. Necessary conditions for stationarity for L_1 and L_2 are provided by a set of Eqs.(22a) to (22c) and a set of Eqs.(22e),(22f) and (15),respectively. The total Lagrangian system is optimized at the second level. At the second level,the only unknown variable is $\{\bar{q}_{ij}\}$ and is determined by Eq.(22f). Since the Lagrangian multiplier included in Eq.(22f),$\{\mu_{ij}\}$,is given by Eq.(22d),(22e),we have the relation Eq.(23). Thus,after obtaining solution for subsystem 2, from which $\{u_{ij}\}$ and $\{u'_{ij}\}$ are obtained under the condition that total interzonal trips $\{\bar{q}_{ij}\}$ are given, using Eq.(23) trip generations and trip distributions by each purpose,being solutions of subsystem 1,are easily calculated. In this approach,there is no guarantee that the feasibility condition in Eq.(20), $\bar{q}_{ij}=x_{ij}+y_{ij}$, is satisfied. Accordingly, iterative calculation should be required to meet the condition.

The algorithm can be summarized as follows:

Step 0: Set counter n:=1.$\{\bar{q}_{ij}^n\}$ is assumed.

Step 1: Solve the subsystem L_2. This yields $\{q_{ij}^n\}$,$\{q'_{ij}{}^n\}$ and $\{u_{ij}^n\}$. $\bar{u}_{ij}^n$ is estimated using Eq.(23),and is used as input in estimating of $\{s_i^n\}$,$\{x_{ij}^n\}$ and $\{y_{ij}^n\}$,which are solutions of the subsystem L_1. The previous solutions are updated by the following formula:

$$\bar{q}_{ij}^{n+1} = (\sum_{k=1}^{n} \bar{q}_{ij} + (x_{ij}^n+y_{ij}^n))/(n+1).$$

Step 2: the algorithm can terminate if, for example, for a positive small constant

$$\Sigma_{ij} \frac{\bar{q}_{ij}^{n+1} - \bar{q}_{ij}^{n}}{\bar{q}_{ij}^{n}} \leq \Delta.$$

Otherwise, set n:=n+1 and go to step 1.

The solution in subsystem L_2 in step 1 can be found using algorithm such as the convex combination method and the double stage method which have been developed so far (20).

4.2 Interactive Balancing Method with Dual-Variable-Coordinator

In this approach, $\bar{u}_{ij}$ is assumed to be a known parameter. The total Lagrangian is decomposed as follows:

$$L(s,x,y,\bar{q},q,q',h,\nu,\omega,\gamma,\mu,\bar{u},u) = L_1(s,x,y,\nu,\omega,\gamma;\bar{u}) + L_2(\bar{q},q,q',h,\mu,u;\bar{u}) \quad (31a)$$

where

$$L_1 = \Phi_1(s)+\Phi_2(x,y) + \Sigma\nu_i(\lambda_1 P_i - \Sigma x_{ij}) + \Sigma\omega_j(\lambda_2 E_j - \Sigma x_{ij}) + \Sigma\gamma_i(Y_i - \Sigma y_{ij}) + \Sigma\bar{u}_{ij}(x_{ij}+y_{ij}) \quad (31b)$$

$$L_2 = \Phi_3(\bar{q},q,q')+\Phi_4(h)+\Sigma\mu_{ij}(\bar{q}_{ij}-q_{ij}-q'_{ij})+\Sigma u_{ij}(q_{ij}-\Sigma h_{kij})+\Sigma\bar{u}_{ij}(-\bar{q}_{ij}) \quad (31c)$$

Since the average interzonal travel time is given, estimation of trip generation and trip distribution by each purpose is an easy task. However, Solving the second subsystem is not straightforward since this problem yields the modal split/distribution/assignment equilibrium problem with asymmetric cost function ,which can not be formulated as an equivalent mathematical program (20). There may exist two approaches to resolve this difficulty; One is to use the diagonalization algorithm to solve the subsystem 2; Alternative is to use information from the total Lagrangian problem. In other words, it follows from the feasibility condition that $\bar{q}_{ij}=x_{ij}+y_{ij}$, in which x_{ij} and y_{ij} are obtained as output of the first Lagrangian problem at the first level. Therefore, using this information the total O-D trip, $\bar{q}_{ij}$, in the second subsystem may be possible to be treated as constant. By this way the second subsystem becomes the combined modal split/assignment model with fixed O-D trips. $\bar{u}_{ij}$ is obtained as output from the second subsystem and may differ from the predeterminate value. Thus the iterative scheme should be adopted so as to meet the equality condition between the updated and predeterminate values of $\bar{u}_{ij}$.

5. CONCLUDING REMARKS

This paper proposed models that synthesize Lowry type of land-use model with the unified network equilibrium model as proposed by Safwat and Magnanti (3). The model was formulated as equivalent mathematical programs and it was shown that its equilibrium conditions provide spatial interaction submodel, a logit type of modal split function and Wardrop user equilibrium conditions over the automobile networks and that the model has a unique solution. Furthermore, two computation methods were proposed based on the observation that the total Lagrangian problem corresponding to the proposed mathematical program can be decomposed into sub-Lagrangians ,in which the state variable(the total O-D trip) and the dual variable(the interzonal travel time) are used to coordinate the solution of master program and the solutions of two subprograms.

In this paper Lowry model was used as the activity component, however,Herbert-Stevens model (11) is also possible to be used. In addition,as was demonstrated by Boyce and Southworth (8), it is possible to disaggregate the model by several person-types who are locationally unconstrained workers,workers with fixed residences seeking jobs so on. It should be noted that land price can be incorporated into the residential-location model as one of the attractiveness elements of the sites.

Since the model was developed with main focus on the travel demand modeling consistent with Lowry's economic framework,it does not include other sectors' behavior such as land-owners and firms except residential-location choice behavior of individuals or households. In order to make the model more realistic, it may be necessary that the dynamic nature of the urban growth and the demand-supply equilibrium of land based on the land-rent theory should be taken into account in modeling the problem.

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